The physics issues that determine inertial confinement fusion target gain and driver requirements: A tutorial*

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This paper presents a simplified, tutorial approach to determining the gains of inertial confinement fusion (ICF) targets, via a basic, zero-dimensional (''0-D''), energy ''bookkeeping'' of input (parametrized by ICF drivers' coupling efficiencies to the target, and subsequent hydrodynamic efficiencies of implosion) versus output (thermonuclear burn efficiency and target fuel mass). Physics issues/constraints such as hydrodynamic instabilities, symmetry and implosion velocity requirements will be discussed for both the direct drive (driver impinging directly on the target) and indirect drive (x-ray implosion within a driver heated hohlraum) approaches to ICF. Supplementing the 0-D model with simple models for hohlraum wall energy loss (to predict coupling efficiencies) and a simple one-dimensional (1-D) model of the implosion as a spherical rocket (to predict hydrodynamic implosion efficiencies) allows gains to be predicted that compare well with the results of complex two-dimensional (2-D) radiation hydrodynamic simulations. © *1999 American Institute of Physics.* [S1070-664X(99)92205-X]

I. INTRODUCTION

The field of inertial confinement fusion (ICF) research involves the study of high energy density plasmas.[1,2] A major goal and application of ICF has always been the demonstration of and, ultimately, construction and operation of scientifically and economically feasible fusion energy power plants. The remote driver may be a laser or particle beam accelerator. The driver energy is focused onto a target, which implodes and releases fusion energy. The gain of that target must be sufficiently high, so that enough energy is produced, such that after it is converted to electricity, there is enough of it that can be recycled back into running the driver. Moreover, the unrecycled portion of energy produced must be sufficient to sell to the customer at competitive prices. Costs of constructing and operating the driver, fusion target chamber and target factory must, of course, all be taken into account.

In this paper we present a simplified tutorial approach to explaining what the gain and driver requirements are for ICF, and in particular focus on deriving what gains are feasible from ICF targets and how they scale with driver energy. The role of various physics constraints on gain is also presented. Thus, this paper is a simplified condensation of a complex field of study, whose state of progress has recently been summarized in great detail by Lindl.[3,4] This is done for both the direct drive (driver impinging directly on the target) and indirect drive (x-ray implosion within a driver heated hohlraum) approaches to ICF.

In Sec. II we discuss in the most general of terms the requirements for driver efficiency and target gain. In Sec. III we review the basic principles of ICF—confinement times, burn fractions, and the need for target compression/implosion. In Sec. IV we derive a simplified model of gain via a zero-dimensional (''0-D''), energy ''bookkeeping'' of input (parametrized by ICF drivers' coupling efficiencies to the target, and subsequent hydrodynamic efficiencies of implosion) versus output (thermonuclear burn efficiency and target fuel mass). We apply that model to various scale drivers and approaches to ICF. In Sec. V we explain some of the gain scaling with driver size. In Sec. VI we explore the issues of the efficiency of the coupling of the driver energy to the target. In Sec. VII we explain the sources of the differences between the hydrodynamic implosion efficiency of direct versus indirect drive. In Sec. VIII we explore the role of physics constraints such as hydrodynamic instability on target gain. In Sec. IX we summarize this tutorial review.

II. DRIVER EFFICIENCY AND TARGET GAIN REQUIREMENTS

Imagine a certain amount of electrical power $P_{\rm in}$ providing the input power to run a driver (e.g., a laser or heavy ion beam accelerator). Suppose that driver converts that input power to driver output that impinges on the target, with an efficiency η_D. Now let the target respond to that drive by delivering high gain, G. Thus the output fusion power from the target is $G\eta_D P_{\rm in}$. Let us denote by η_{Th} (''thermal to electric'') the efficiency by which the fusion target chamber and subsequent turbines, etc., convert that target fusion output to electricity. Thus the power plant produces a $P_{\rm out}$ that is available to go out to the grid. However, we must recycle back a fraction f of that power to run the driver. Thus we have $P_{\rm in}=fP_{\rm out}=f\eta_{\rm Th}G\eta_D P_{\rm in}$. For consistency then we must have $f\eta_{\rm Th}G\eta_D=1$. Since we want f to be sufficiently small as to not impact the cost of electricity [namely $(1-f)P_{\rm out}$ is available to the customers], we take as a requirement that $f<1/4$. In addition, a typical value for $\eta_{\rm Th}$ is about 0.4. This leads to the requirement, then, that $\eta_D G>10$.

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This criterion is clearly a rough figure of merit and nothing more. For example, inclusion of an efficiency factor of 1.1 for neutron multiplication in the tritium breeding chamber wall blanket, and assuming a somewhat higher η_{Th} of 0.45, leads to $\eta_D G > 8$.

It might be instructive to take a specific example. Suppose 0.3 GW is the input to a 10% efficient driver. Then 0.03 GW impinges on a target. Suppose that is in the form of a 6 MJ pulse of energy five times a second. Let's suppose the target gain is 100 (thus fulfilling the $\eta_D G > 10$ requirement). Then five times a second, 600 MJ of energy is produced by each of the five targets illuminated during that time frame. That can be five targets dropped into the fusion target chamber once every 200 msec, or perhaps five separate chambers, each with a target dropped into it once per second. In either scenario, 3 GW of fusion power is produced by these targets. With an η_{Th} of 0.43, 1.3 GW of electrical power is now available to the grid. We take 0.3 of that 1.3 GW and send it back to provide input power to the driver, as per the beginning of our example, leaving 1.0 GW available for this power plant to send out on the grid to its customers.

Of course the economic considerations that will determine if the cost of electricity so produced can compete in the marketplace, such as issues of the costs of constructing and operating the driver, fusion target chamber and target factory, must all be taken into account. The additional societal advantages of these fusion power plants, in the realm of global environmental impact and safety, will also play a role in determining the future success of these endeavors.

III. ICF BASICS

A. Inertial confinement time

Before proceeding to gain calculations we need to derive some quantities that are basic to any ICF approach. The first is confinement time. Inertial confinement is as minimal a confinement as there is. It simply reflects the fact that an assembled fusing fuel has inertial mass, which takes some finite time to disassemble when driven to do so by its own high pressure. In general, inertial mass, m, is the coefficient of resistance of a body to motion (acceleration, a) when that body is subject to a force, F, which we know as the equation $F = ma$. We also know that that body will move a distance $d = (1/2)$ a t^2 in a time t. Solving for t, $t = (2d/a)^{1/2} = (2dm/F)^{1/2}$, and the greater the inertial mass, the longer the time to move a distance d. To find a disassembly time for an assembled, compressed sphere of fusing fuel, of radius R, density ρ, and temperature T, let's take d to be of order R. The mass m scales as ρR^3, and the force F is basically the pressure times the area which will scale as ρT times R^2. Thus the confinement time (''disassembly time'') t will scale as $(R\rho R^3/\rho T R^2)^{1/2} = R/T^{1/2}$ or the radius over a sound speed, wherein the radius R represents to a large degree the ''inertial'' mass and source of confinement.

A more sophisticated calculation of confinement time involves the hydrodynamic concept that in an assembled high pressure fuel, surrounded by a vacuum, a rarefaction wave will propagate at the speed of sound, C_S, into the fuel from the vacuum boundary, communicating to the interior the fact that there is a vacuum out there to which it is free to expand. Thus at every point r' within the sphere, we can define a local confinement time $\tau_C(r') = (R - r')/C_S$, at which time outward motion will begin. We can now find a global confinement time, τ_C, by mass averaging $\tau_C(r')$ over the entire sphere, which for simplicity we take as having uniform density ρ. Thus,

$$\begin{aligned}\tau_C &= \int_0^R \rho[(R - r')/C_S] 4\pi r'^2 dr' / [(4/3)\pi\rho R^3] \\ &= (3/C_S R^3)[(Rr'^3/3) - (r'^4/4)]_0^R = R/4C_S. \qquad (1)\end{aligned}$$

This result seems reasonable because in a sphere of uniform density, half the mass is in the outer 20% of the radius, thus the mass averaged confinement time τ_C is substantially less than simply R/C_S.

B. Burn fraction

We now ask the question: how much of the fuel is burned before it disassembles? Let's assume we have a 50–50 mixture of deuterium and tritium, and denote their number densities by n_{D} and n_{T}, respectively. Then if we denote the deuterium–tritium (DT) reactivity (reactions per cm^3 per sec averaged over a Maxwellian distribution), as $\langle\sigma v\rangle_{\mathrm{DT}}$, then the rate at which the tritium is burned up is given by

$$dn_{\mathrm{T}}/dt = -n_{\mathrm{T}} n_{\mathrm{D}} \langle\sigma v\rangle_{\mathrm{DT}}.$$

Since the total fuel number density, at any time t, is $n = 2n_{\mathrm{T}} = 2n_{\mathrm{D}}$, we can rewrite this as

$$dn/dt = -(n^2/2)\langle\sigma v\rangle_{\mathrm{DT}}.$$

This can easily be integrated from time zero to τ_C and we obtain

$$(1/n) - (1/n_0) = (\tau_C/2)\langle\sigma v\rangle_{\mathrm{DT}},$$

where n_0 is the initial number density of the assembled fuel.

If we define the burn fraction f_b as

$$f_b = 1 - (n/n_0)$$

and use Eq. (1) for τ_C, and relate initial number density n_0 to initial mass density ρ, by $n_0 = \rho/m_{\mathrm{DT}}$, where m_{DT} is the mass of a ''DT'' nucleus (2.5 AMU), then after straightforward algebraic manipulation we obtain

$$f_b = \rho R/(\rho R + \beta(T)), \qquad (2)$$

where $\beta(T) = 8 m_{\mathrm{DT}} C_S/\langle\sigma v\rangle_{\mathrm{DT}}$ and takes on a minimum value of about 6.0 g/cm^2 for optimal burn conditions of about 30 keV ion temperatures. Thus from Eq. (2) we can derive that to get a reasonable burn up fraction of the fuel of about 1/3, we need to achieve an assembled fuel that has a ρR product of 3 g/cm^2. Without achieving such a burn fraction, sufficiently high gains will be very difficult to be achieved, since there are other inefficiencies (to be discussed at length in this paper) that must be overcome. This $\rho R = 3$ g/cm^2 criterion can be rewritten in terms of number density and confinement time, $n\tau_C$, as $n\tau_C = 2\times 10^{15}$. Thus the ICF ''Lawson criteria'' is a factor of 20 greater than that of magnetic fusion energy (MFE) due to the inefficiencies in assembling the fuel that ICF must overcome.

C. The need for compression/implosion

The need for ICF targets to achieve high compression now follows immediately. If we require (ultimately for achieving reasonable efficiency to meet the high gain of order 100 requirements) an f_b of 1/3, namely a ρR product of 3 g/cm^2, then, assuming for simplicity an assembly of uniform density ρ, we can recast the mass of the assembly in the form $M=(4\pi/3)\ \rho R^3=(4\pi/3)(\rho R)^3/\rho^2$ and fix ρR at 3 g/cm^2. Then, if we use uncompressed DT fuel with a ρ of 0.21 g/cm^3, we obtain a mass of 2.6×10^3 g. The energy per gram produced by DT fusion, ϵ_{DT} is

$$\epsilon_{\mathrm{DT}}=17.6\ \mathrm{MeV}/5\ \mathrm{AMU}=3.4\times10^{11}\ \mathrm{J/g}. \tag{3}$$

Thus, this massive target will produce (with an f_b of 1/3) an output of 2.9×10^{14} J or the equivalent of 70 kilotons of TNT—a catastrophic amount of output!

If instead we compress the DT 1000-fold to density 190 g/cm^3, then the mass will be 5×10^{-3} g, and a yield of 5.5 $\times10^8$ J which is readily containable, and was in fact used in our example in Sec. II of a 600 MJ output from each ICF target (which are shot at five times a second). A spherical implosion geometry is the easiest way (in the sense of minimizing the convergence ratio defined as the ratio of initial to final radius) to achieve a compression of such a target to a 1000-fold initial density, since, for a fixed mass, the density will scale as R^3 as opposed, say, to cylindrical implosions with its R^2 scaling, or planar geometry with its R^1 scaling.

Since, as we shall see, much of the mass is ablated during the implosion, spherical convergence ratios of at least 20 are to be expected in order for the final assembled fuel mass to reach the requisite high density. Moreover, as we shall see, the final assembly will be a dense shell of fuel surrounding a hot, relatively lower density spherical bubble, whose radius is equal to about half the total radius of the assembly. Thus, in the example of the previous paragraph, to maintain the dense fuel ρR at 3 g/cm^2 requires doubling the density from 190 g/cm^3 to closer to 400 g/cm^3. The convergence ratio (''CR'') of over 20 implies the need for excellent symmetry (at the 1% to 2% time integrated drive uniformity level), because spherical compression magnifies (by a factor of CR) imperfections of drive from the original outer surface when the fuel assembles at a much smaller radius.

D. The means of compression/implosion

Target implosions can be viewed as rockets directed spherically inward. Driver energy couples to the outside of the target (with efficiency η_C) and heats a thin outer layer, which expands outward, much like the exhaust gasses of a rocket. In a rocket-like reaction, the remainder of the target implodes inward and is mostly now in the form of kinetic energy of implosion (with efficiency η_H). Upon convergence to the center, the kinetic energy is reconverted (with excellent efficiency, η_a, which we'll take, for now, to equal 1.0) to internal thermal energy of the high density assembled fuel that is ready to burn. Thus the process of implosion involves a total efficiency $\eta_T=\eta_C\eta_H$.

The target coupling can occur by two principal methods. In direct drive, a laser or particle beam directly impinges onto the target. Excellent coupling can be achieved. For example, using 1/3 μm light, at relevant irradiances of 10^{15} W/cm^2, absorption in the neighborhood of 80% can be achieved. A challenge for direct drive is to achieve good symmetry via multiple smoothed beams.[5]

The indirect approach involves a target capsule at the center of an enclosure called a hohlraum. In ICF hohlraums (on Nova,[6] mm scale gold cylinders), laser light enters the hohlraum interior through laser entrance holes located in either end cap of the cylinder. The light is absorbed at the cylinder walls, converting laser light into soft x-rays. The hohlraum is made of a high atomic number material such as gold, which maximizes the production of x-rays. These x-rays are rapidly absorbed and reemitted by the walls setting up a radiation driven thermal wave[7] diffusing into the walls. Most of the x-rays are ultimately lost into the walls, some escape out the laser entrance holes, and the rest are absorbed by the target capsule in the center of the hohlraum and drive its implosion. Typically this coupling to the capsule is a less than 1/2 of the total energy (about 0.2 for a power plant scale laser heated hohlraum), so coupling for indirect drive is relatively poor compared to direct drive. On the other hand, as we shall discuss later in this paper, x-ray drive, compared to direct drive, provides for more hydrodynamically efficient ''rocket'' implosions (20% vs. 10%), and for implosions that are more hydrodynamically stable.

IV. GAIN SYSTEMATICS

A. Volume ignition

We first consider the simple-minded approach to target gain. Namely, arrange to heat the entire fuel to about 10 keV, and allow it to fuse. What gain will ensue? From Eq. (3), the fusion output for DT is 3.4×10^{11} J/g. The energy per gram needed to heat DT to 10 keV is (3/2) (4) (10 keV)/5 AMU or 10^9 J/g. The factor of 4 comes from the presumption that we are heating four particles—the deuteron, the triton, and each of their electrons. Assuming an $f_b=1/3$, the gain we'd expect is then of order (1/3) (340) times the total coupling efficiency, $\eta_T=\eta_C\eta_H$. For direct drive that would be of order (0.8) (0.1) or 8%, and for indirect drive it would be of order (0.2)(0.2) or 4%. Thus gains of either 9 or 4.5 are to be expected in this approach. Since driver efficiencies are expected to be of order 25% for heavy ion beam accelerators and 10% for lasers, gains of order 40 to 100 are needed. Thus this approach produces gains that are insufficient by an order of magnitude.

B. Propagation from a hot spot

In this approach we only heat a small fraction of the mass to 10 keV. This usually occurs naturally in the center of the implosion as shocks converge at the center and reflect off the center to further heat the low density DT gas that is present there. Typically this hot spot has density of order 50 g/cm^3 and temperatures of order 10 keV, and takes up nearly half the radius of the assembled fuel. The hot spot is surrounded by high-density cold fuel, at a density of about 500 g/cm^3. Thus the fraction of mass heated to 10 keV is of order $\rho_{\rm HS} R_{\rm HS}{}^3 = (\rho_F/10)(R_F/2)^3$ or about 1% of the main fuel mass.

If the $\rho_{\rm HS} R_{\rm HS}$ of this hot spot is of order 0.3 g/cm^2 which, in a 5 to 10 keV plasma, is the range of the 3.6 MeV alpha particles produced by the DT fusion reaction, then the alpha particles can stop within the hot spot and help self-heat itself to thermally ''run away'' to the 30 to 40 keV range.

At that point, the $\rho_{\rm HS} R_{\rm HS}$ of 0.3 g/cm^2 provides a sufficient f_b to produce enough alphas to stop in the adjacent shell (''AS'') of high density cold matter, with a thickness of an alpha range of $\rho_{\rm AS} R_{\rm AS} = 0.3\,{\rm g/cm}^2$ as well, and heat it up to 10 keV. To see how the energetics work out, consider the fact that such an adjacent shell has three times the mass of the hot spot: $M_{\rm HS} = (1/3) 4\pi R_{\rm HS}^2 (\rho_{\rm HS} R_{\rm HS})$, whereas the adjacent high density, thin shell mass is $M_{\rm AS} = 4\pi R_{\rm HS}^2 (\rho_{\rm AS} R_{\rm AS})$ but both $\rho_{\rm HS} R_{\rm HS}$ and $\rho_{\rm AS} R_{\rm AS}$ are equal to 0.3 g/cm^2. Thus $M_{\rm AS} = 3 M_{\rm HS}$. The hot spot produces 3.6 MeV alphas per fused DT pair, and there are $(M_{\rm HS}/5\,{\rm AMU})$ such potential pairs. The number fused is f_b times that potential, and f_b is, by Eq. (2), $0.3/(6+0.3)$ or about 5%. Thus $(180\,{\rm keV}) \times (M_{\rm HS}/5\,{\rm AMU})$ of alpha energy in produced by the hot spot. Those alphas stop in the adjacent ($\rho_{\rm AS} R_{\rm AS} = 0.3\,{\rm g/cm}^2$) shell which has three times the mass, or $3(M_{\rm HS}/5\,{\rm AMU})$ DT pairs. To heat all of them up to 10 keV takes (3/2)(4)(10 keV) or 60 keV per DT pair, or $(60\,{\rm keV}) \times (M_{\rm AS}/5\,{\rm AMU}) = (180\,{\rm keV}) \times (M_{\rm HS}/5\,{\rm AMU})$ total, which matches the energy supplied to it by the hot spot, so a self-consistent picture emerges.

When this (''AS'') adjacent-to-the-hot-spot shell ''runs away'' it will produce more than sufficient alpha particle energy to supply the (''NS'') next shell (of thickness $\rho_{\rm NS} R_{\rm NS} = 0.3\,{\rm g/cm}^2$ equal to an alpha range) and heat all of it up to 10 keV and thus continue the propagating thermonuclear burn wave. In fact, the AS's energy production surplus is more than sufficient by about a factor of 3, because the mass of the (''NS'') next high density thin shell is about equal to this adjacent shell (''AS'') mass since both thin shells have the same ρR. (This is to be contrasted with the HS vs. AS comparison, where the HS produced only just enough energy to heat the AS, because AS had three times the mass of the HS.) Throughout this process the 14 MeV neutrons are streaming through the target to the chamber walls, as their range is of order 5 g/cm^2 which is quite a bit longer than the alpha range and even longer than the typical total target ρR of 3 g/cm^2. In summary, the fusion process itself, originating in a 1% of the mass hot spot, can initiate a process that burns the surrounding high density cold fuel.

Does this mean that gains are nearly infinite? No! It takes energy to compress the cold main fuel that surrounds the hot spot. The minimum amount of energy investment will occur if the fuel is kept on the lowest allowable isentrope known as the Fermi degenerate (''FD'') isentrope. It costs energy to compress this cold fuel because we are fighting ''quantum pressure,'' since as we compress we are squeezing the electrons into interparticle distances that are smaller than their De Broglie wavelengths. Hence by the uncertainty principle, each electron's momentum will increase as its wavelength is squeezed shorter and shorter, and this momentum increase acts like a pressure. The formula for the pressure P_F (in cgs units) for the cold main fuel is

$$P_F = \alpha_{\rm FD} P_{\rm FD} = \alpha_{\rm FD} 2 \times 10^{12} \rho^{5/3}, \tag{4}$$

and the formula for the specific energy costs of cold compression is

$$\epsilon_F = \alpha_{\rm FD} \epsilon_{\rm FD} = \alpha_{\rm FD} 3 \times 10^5 \rho^{2/3} \ \ ({\rm J/g}). \tag{5}$$

In both formulas, we denote by a multiplier, $\alpha_{\rm FD}$ (which is greater than or equal to 1.0), the measure by which we have successfully stayed on the minimum FD isentrope. Thus for densities of order 1000 g/cm^3, $\epsilon_{\rm FD} = 3\times 10^7$ J/g. This is to be compared with the fusion output per gram of 3×10^{11} J/g from Eq. (3). Thus with the hot spot ignition and subsequent propagating burn ''doing the work'' of heating the dense surrounding fuel, we can expect an inherent gain of $\epsilon_{\rm DT}/\epsilon_{\rm FD}$ of order 10^4, times f_b, times the coupling efficiencies $\eta_T = \eta_C \eta_H$. This value for gain is indeed sufficient to achieve the goal of gains of order 100.

C. Gain formula

Formally, then, based on the above discussion, by justifiably neglecting the hot spot mass and energy, we can derive the gain (G) by taking the yield (Y), which is f_b times $\epsilon_{\rm DT}$ times the fuel mass, M_F, and dividing it by the incident driver energy, E_D, a fraction η_T of which remains in internal energy of the assembled fuel and used, at a cost of $\epsilon_F M_F$, to compress that fuel to high density:

$$\begin{aligned} G &= Y/E_D = f_b \epsilon_{\rm DT} M_F / [(\epsilon_F M_F)/\eta_T] \\ &= \eta_T f_b \epsilon_{\rm DT}/\epsilon_F = \eta_C \eta_H f_b \epsilon_{\rm DT}/\epsilon_F, \end{aligned} \tag{6}$$

or, using Eqs. (3) and (5),

$$G = 10^4 [(\rho/1000)^{-2/3}/\alpha_{\rm FD}\ \ {\rm IgnMrgn}] \eta_C \eta_H f_b, \tag{7}$$

where we have added an extra gain degradation term, the ignition margin (''IgnMrgn''). We normally take it to be a factor of about 2, and it represents the strategy of actually providing about twice the necessary energy to the final fuel assembly (in the form of kinetic energy of the imploding rocket), in order to overcome nonidealities of the implosion due, for example, to the final assembly not being perfectly spherical as the result of asymmetries and hydrodynamic instability growth. This is equivalent to taking η_a (the efficiency of reconverting the kinetic energy of the imploding shell into thermal energy of the assembled fuel, as described in the first paragraph of Sec. III D) to be 0.5, not 1.0 as had been assumed previously.

Table I is the principal result of this paper. It uses Eq. (7) and applies it to targets designed to be driven by the National Ignition Facility[8] (NIF) and for future power plant scale targets, which we denote by ''HiY'' (high yield). These HiY targets are assumed to be driven by drivers with input energies of 5 to 10 MJ, about an order of magnitude higher than NIF input energies of 1 to 2 MJ. Table I considers both direct

TABLE I. Predicted gain [derived from Eq. (7)] versus driver scale/ICF approach.

Scale	(MJ)	Approach	$(10^4$	$\rho_{1000}{}^{-2/3}$	/	α_{FD}	IgnMrgn)	η_C	η_H	f_b	$=G$
NIF	1–2	Indirect	$(10^4$		/	1.5	2.0)	0.1	0.2	0.2	=13
NIF	1–2	Direct	$(10^4$		/	3.0	2.0)	0.8	0.1	0.2	=26
HiY	5–10	Indirect	$(10^4$	2.0	/	1.5	2.0)	0.2	0.2	0.3	=80
HiY	5–10	Direct	$(10^4$	2.0	/	2.0	2.0)	0.8	0.1	0.3	=120

drive (DD) and indirect drive (ID) approaches at both driver energy scales. The remainder of this paper is principally devoted to explaining the choices for all of the entries in Table I. However, before proceeding to those details let us summarize the results of Table I. NIF scale targets have yields of order 10, which agrees with detailed numerical simulations.[8] The HiY targets have an order of magnitude larger gains, or order 100, sufficient to drive power plants. Direct drive, by virtue of its higher η_c, produces somewhat larger gains, though we will discuss the issues that challenge the direct drive approach in subsequent sections as well.

V. GAIN SCALING WITH DRIVER SIZE

A. Fuel density decreases with driver size

In either DD or ID approach, we see from Table I, that, independent of the coupling efficiencies, the gain coefficient increases from NIF to HiY because the density decreases for the HiY scale. To explain this result, we must consider what the optimal fuel assembly arrangement is at any given driver scale. Namely, given a final fuel assembly of hot spot surrounded by cold dense fuel, which has a total internal energy $E_T = \eta_T E_D$, what is the optimal energy (and mass) allocation to the hot spot versus the main fuel region. To do so we make a key assumption that the hot spot and main fuel have come into pressure equilibrium with each other. Detailed simulations confirm that this isobaric assumption is generally valid. The optimum configuration can be found by means of a simple model.[9,10]

Let us take the hot spot radius, R_{HS}, as the quantity to be varied as we find the optimal configuration (which will be characterized by R^*_{HS}).

The hot spot must satisfy the ignition criteria discussed in Sec. IV B. Namely it **must** have a temperature T of order 10 keV, and **must** have a $\rho_{\text{HS}} R_{\text{HS}}$ product of 0.3 g/cm^2. Thus, as we vary R_{HS}, the quantity ρ_{HS} will vary as R_{HS}^{-1}. The mass of the hot spot, $M_{\text{HS}} = (1/3) 4\pi R_{\text{HS}}^2 (\rho_{\text{HS}} R_{\text{HS}})$, will vary as R_{HS}^2, as will E_{HS}, the energy in the hot spot, which is proportional to $M_{\text{HS}} T$. For convenience we denote $E_{\text{HS}} = c R_{HS}^2$. Finally, the pressure in the hot spot, P_{HS}, which is proportional to $\rho_{\text{HS}} T$, will vary as R_{HS}^{-1}.

Now we consider the main, cold, high density, near Fermi degenerate fuel region. The isobaric assumption tells us that the pressure there, P_F, equals P_{HS} so it too will vary as R_{HS}^{-1}. Then, by Eq. (4), this tells us that the density in the main fuel, ρ_F, will vary as $R_{\text{HS}}^{-3/5}$. Now the energy available to compress the main fuel is $E_F = E_T - E_{\text{HS}}$. We set that equal to $\epsilon_F M_F$, and use Eq. (5) for ϵ_F. Thus

$$M_F \propto (E_T - E_{\text{HS}}) \rho_F^{-2/3} \propto (\eta_T E_D - c R_{\text{HS}}^2) R_{\text{HS}}^{2/5}. \tag{8}$$

Since we get yield by burning fuel, we'd like to maximize M_F subject to the constraint that the total fuel assembly has a particular $\eta_T E_D$, given an initial driver energy scale E_D. [To actually optimize gain[9,10] we would maximize $G \propto f_b M_F / E_D$ (and E_D is fixed), but the simplified discussion presented here gets to the heart of the scaling without delving into more convoluted algebra.]

From Eq. (8) we see that there will be an optimal R^*_{HS}. If R_{HS} is too big, the first term in Eq. (8) makes M_F too small (because we've put too much energy into the hot spot). If R_{HS} is too small, then the second term in Eq. (8) makes M_F too small (because we've made the hot spot pressure too high, and therefore the cold fuel pressure is too high as well, leading to too high a cold fuel density, which means that, for a given $E_F = \epsilon_F M_F \propto \rho_F^{2/3} M_F$, the high ρ_F leads to a low M_F).

It is clear that, mathematically, optimizing Eq. (8) will lead to an R^*_{HS} that scales as $(\eta_T E_D)^{1/2}$. (Thus the optimal hot spot energy will be a fixed, small fraction of the total energy.) Thus the optimal hot spot density will scale as $(\eta_T E_D)^{-1/2}$. And most importantly the main fuel density ρ_F will scale as

$$\rho_F^* \propto R_{\text{HS}}^{*\,-3/5} \propto (\eta_T E_D)^{-3/10}. \tag{9}$$

Thus as we increase the driver energy scale, the optimal hot spot and main fuel densities in the assembled fuel decrease. Indeed, detailed simulations [see Fig. (35) of Ref. 3)] show that peak densities at the NIF scale are of order 1000 g/cm^3, whereas they are about 1/2 that for the power plant driver scale. Since $G \propto \rho_F^{-2/3}$, the gain coefficient, independent of coupling efficiencies, increases with driver scale.

In preparation for Sec. V C below, we note that the optimal fuel mass, M_F^*, will, by virtue of Eq. (8) and the $R^*_{\text{HS}} \propto (\eta_T E_D)^{1/2}$ result, scale as

$$M_F^* \propto (\eta_T E_D) R_{\text{HS}}^{*\,2/5} \propto (\eta_T E_D)^{1.2}. \tag{10}$$

B. The fast ignitor utilizes burn at low fuel density

At this point we take a slight detour from our analysis of Table I, and consider the ''fast ignitor'' approach to ICF.[11] In that approach, once a fuel assembly is achieved, a high power, short pulse laser or particle beam impinges on the outside of the assembled dense fuel, heats a small spot on the outside to 10 KeV, and it acts as an ignition hot spot and starts a propagating burn into the main fuel. The advantage of this approach over the conventional approach described above is that now the hot spot is **not** in pressure equilibrium with the main fuel. Both the conventional and fast ignitor hot spots have similar pressures, but the fast ignitor assembled main cold dense fuel can be at much lower pressure than the conventional approach. Namely, it can be at lower density [recall Eq. (4)]. Thus for a given $E_F = \epsilon_F M_F \propto \rho_F^{2/3} M_F$, the low ρ_F leads to a high M_F, Namely, there is more fuel mass available to burn, and thus higher gain. This is represented, of course, by the $G \propto \rho_F^{-2/3}$ scaling of Eq. (7).

So, by virtue of using an auxiliary fast ignition driver to break the isobaric constraint, fast ignitor targets can have substantially higher gains than the conventional central hot spot ignition via imploding shocks approach to ICF. (Of course there are many challenging issues of properly coupling the ultra high power auxiliary source into the target and creating that external hot spot.) In summary, fast ignitor targets act like ''little big men'' in that at small driver scale they have the higher gain of large driver targets, because they burn lower density fuel just as the high yield, large scale driver targets do.

C. Burn fraction increases with scale size

Returning to Table I we also see that f_b increases from the NIF driver scale targets to the HiY scale. Having done the hard work in Sec. V A above, we can easily derive this result. For a low $\rho_F R_F$ (compared to 6 g/cm^2) to begin with (as is the case for the NIF scale), f_b will be roughly linearly proportional to $\rho_F R_F$. Since $M_F \propto \rho_F R_F^3$, then $R_F \propto (M_F/\rho_F)^{1/3}$, and therefore,

$$f_b^* \propto \rho_F^* R_F^* \propto \rho_F^{*2/3} M_F^{*1/3} \propto (\eta_T E_D)^{-2/10} (\eta_T E_D)^{(1.2/3)} \propto (\eta_T E_D)^{2/10}, \tag{11}$$

where we have used Eqs. (9) and (10). Thus we see from Eq. (11) that the burn fraction increases with driver scale.

Intuitively we would expect that larger targets driven by larger scale drivers will have larger $\rho_F R_F$, and therefore burn more efficiently, namely, have a higher f_b. In other words, we'd naively expect f_b to scale as $R_F \propto M_F^{1/3}$ which would scale as $E_D^{1/3}$. The above detailed derivation and discussion simply tempers that intuition with the fact that the densities at the larger scale are somewhat lower, leading to Eq. (11)'s result of $f_b \propto E_D^{2/10}$.

Summarizing the gain scaling with driver scale, from Eq. (7), G scales as $\rho_F^{-2/3} f_b$, which, by Eqs. (9) and (11) leads to

$$G \propto (\eta_T E_D)^{2/10} (\eta_T E_D)^{2/10} \propto (\eta_T E_D)^{4/10}. \tag{12}$$

Thus an order of magnitude in driver scale naturally leads to about a factor of 3 in increase in gain. If the coupling, $\eta_T = \eta_C \eta_H$, happens to increase with scale as well (an example of which we shall see in Sec. VI), we get a double advantage. From Eq. (7) we see an explicit dependence on η_T, namely an obvious increase of gain with improved coupling, and from the scaling of optimal targets with driver scale, where it is the coupled energy into the final fuel assembly, $\eta_T E_D$, that is the relevant quantity, we get the further increase in gain with η_T, as per Eq. (12). Alternatively, we can view the double advantage in another way. If η_T improves with scale, and it is $\eta_T E_D$ that is the relevant quantity, then we can have these higher gains with a smaller driver than had η_T not improved with scale. Smaller size drivers reduce initial capital costs and contribute to lower cost of electricity.

VI. COUPLING EFFICIENCY INCREASES WITH DRIVER SIZE

With regard to coupling efficiency, η_C, we see two important features from Table I. First that direct drive can have excellent coupling, of order 80%. This occurs with the use of 1/3 μm light, whose high frequency absorbs at a high critical density where classical, collisional inverse bremsstrahlung absorbs the laser light efficiently, even at the irradiances of order 10^{15} W/cm^2 which are required to produce the requisite implosion velocity (to be discussed below). This excellent coupling can be achieved at either driver scale.

The second feature of note is the relatively poor coupling efficiency for indirect drive at NIF scale (most of the energy soaks into the hohlraum walls, not into the capsule) and the increase of η_C to about 20% at the HiY driver scale. How does this improvement with scale size come about?

Laser light, E_D, enters a hohlraum, via a laser entrance hole (''LEH''), is absorbed efficiently (nearly at 100%) in the high Z walls, and about 80% of that absorbed energy converted to x-rays, E_{RAD}. (Symbolically written as $E_{\mathrm{RAD}} = \eta_X E_D$ where η_X is about 0.8). The x-ray energy can flow into the capsule, E_{CAP}, out the laser entrance holes, E_{LEH}, and soak into the high Z walls of the hohlraum, E_W.

Before writing down formulas for these quantities let us introduce convenient ''radiation hohlraum units (r.h.u).'' In r.h.u. T is measured in hectovolts (hundreds of eV), area in mm^2, time in nsec, mass in grams and energy (a bit clumsily) in hectojoules. With these units, the Stefan Boltzmann constant σ(of ''flux$=\sigma T^4$'' fame)$=1$, normalized irradiance is 10^{13} W/cm^2 ($=$hJ/mm^2 nsec$=10^2$ J/10^{-2} cm^2 10^{-9} sec) and, similarly, normalized power is 10^{11} W($=$hJ/nsec $=10^2$ J/10^{-9} sec). In these units, we find, for a hohlraum at temperature T, the free streaming absorption into the capsule, and loss out the LEH, are given simply by

$$E_{\mathrm{CAP,LEH}} = 1.0 A_{\mathrm{CAP,LEH}} T^4 t \quad (\mathrm{hJ}), \tag{13}$$

where $A_{\mathrm{CAP,LEH}}$ are the areas of the capsule and LEH, respectively.

The formula for the wall loss is a bit more involved. As described in Sec. III C, the photons diffuse into the wall, their rapid absorptions by the high Z ions followed by remission in a random direction, effectively a random walk scattering process. To describe this process a simplified energy equation would read as follows:

$$\partial(\rho C_P T)/\partial t = \partial/\partial x[(c\lambda_R/3)\partial/\partial x(aT^4)]; \tag{14}$$

namely the change in material energy is due to a divergence of a diffusive flux. The diffusive flux is, as usual, 1/3 times a free streaming velocity, c (the speed of light), times a mean free path (the Rosseland radiation mean free path, λ_R) times the gradient of an energy density, where here that energy density, aT^4, is that of the radiation field. The constant ''a'' that appears there is related to σ by $\sigma = ca/4$. Typically the specific heat C_P scales as $T^{1/2}$ (because it scales as number of freed electrons, namely the ionization state Z, and Z^2 scales as T) and fits to detailed opacity calculations has λ_R

scaling as $T/\rho\kappa$ where κ is a multiplier on the opacity. Then by dimensional arguments, Eq. (14) leads to an expression for the depth of the radiation heat wave:

$$(\rho x)^2/t \propto T^{3.5}/\kappa \quad \text{or} \quad (\rho x) \propto T^{1.75} t^{1/2} \kappa^{-1/2}. \tag{15}$$

We then expect the wall loss, E_W, to scale as $A_W(\rho x) C_P T$, and when calculated explicitly we get a coefficient of about 1/2. Thus

$$E_W \approx 0.5 A_W T^{3.2} t^{1/2} \kappa^{-1/2} \quad \text{(hJ)}. \tag{16}$$

The coupling efficiency can now be found as

$$\eta_C = E_{\mathrm{CAP}}/E_D = E_{\mathrm{CAP}}/(E_{\mathrm{RAD}}/\eta_X)$$
$$= \eta_X E_{\mathrm{CAP}}/(E_{\mathrm{CAP}} + E_{\mathrm{LEH}} + E_W).$$

Dividing this expression through by E_{CAP}, and defining $a_{\mathrm{LEH}} \equiv A_{\mathrm{LEH}}/A_{\mathrm{CAP}}$ and $a_w \equiv A_W/A_{\mathrm{CAP}}$, and $N_W \equiv 2T^{0.8} t^{1/2} \kappa^{1/2}$, and using Eqs. (13) and (16), we obtain

$$\eta_C = \eta_X/(1 + a_{\mathrm{LEH}} + [a_w/N_W]). \tag{17}$$

Typically a_{LEH} is of order 2, and a_W is of order 32. Thus very low coupling efficiencies could be expected, if not for the favorable scaling of N_W. The large value of a_w is a reflection of the symmetry constraint. In order to achieve the required good symmetry in a hohlraum via its natural ''geometric smoothing'' we operate with a wall radius to initial capsule radius ratio of about 4. (See Fig. 61 of Reference 3.) The value of a_W is of order 32 and not 16 because the effective absorption radius for the imploding capsule is somewhat smaller than its initial value. (During the time of the main drive pulse, and, hence, the time of the shell's main acceleration, the average radius of the imploding capsule's dense shell is about 3/4 of its initial radius.) As we proceed from the Nova scale to the NIF to HiY driver scales, we have N_W scaling from about 3.5 ($T=2$ heV, $t=1$ nsec) to 8 ($T=3$ heV, $t=3$ nsec) to 14 (t scaling as $E_D^{1/3}$ and an assumed improvement in κ by 1.25) and subsequently η_C scaling from about 0.06 to 0.1 to nearly 0.2. Some of the expected improvements in N_W and η_C as we scale up in driver size are due not only to the characteristic irradiation times increasing with size ($t \propto R \propto M^{1/3} \propto E_D^{1/3}$), but also weak but favorable scaling of η_X with pulse length, and improvements in wall opacities by using mixtures of materials.[12] A more rigorous discussion of this scaling can be found in Ref. 1. Thus we have motivated the scaling of η_C that appears in Table I.

VII. HYDRODYNAMIC IMPLOSION EFFICIENCIES: DIRECT VERSUS INDIRECT DRIVE

A. Required implosion velocity

Returning to Table I we now note that the hydrodynamic efficiency, η_H, of turning coupled thermal energy into kinetic energy of an imploding dense shell, is about twice as large for indirect drive as it is for direct drive. Why is that?

First let us remember that the goal is to assemble the fuel into the hot spot surrounded by dense cold fuel discussed in Sec. IV B. That energy is delivered to the ''assembly region'' by the imploding dense shell moving with velocity v_{imp}. Neglecting mass and energy of the hot spot, as per the discussion in Sec. IV B, and setting the kinetic energy $(1/2) M_F v_{\mathrm{imp}}^2$ of the shell equal to the required assembled energy $E_F (= \epsilon_F M_F)$ times an ignition margin factor of 2 [as per the discussion following Eq. (7)] we get, using Eq. (5), $v_{\mathrm{imp}} \approx 3.5 \times 10^6 \rho_F^{1/3}$ cm/sec, so for a fuel density of about 1000, the required $v_{\mathrm{imp}} \approx 3.5 \times 10^7$ cm/sec.

B. Ablation pressure and velocity scaling: Direct versus indirect drive

The fundamental difference between the dynamics of implosions directly driven by lasers and those driven by x-rays is that lasers absorb at relatively low electron density, n, corresponding to the critical electron density for the wavelength of that laser, whereas x-rays are absorbed deeper into the target at solid material densities, which, when ionized by the x-ray flux, are at very high electron densities. Thus even if the laser is at 1/3 μm light, the typical x-ray absorption region has electron densities nearly 100 times larger.

Using dimensional analysis, equating incoming, absorbed energy flux, I, to outward flowing ablated material energy, with velocity v (scaling as a sound speed, $T^{1/2}$), times its internal energy content nT, we get $I \propto nT^{3/2}$ or $T \propto (I/n)^{2/3}$. Then typical sound speeds in the ablation region will scale as $C_S \propto T^{1/2} \propto (I/n)^{1/3}$. By this scaling we'd expect about a factor of 5 difference in sound speeds between direct and indirect drive, and indeed at equal energy fluxes of 10^{15} W/cm^2, 1/3 μm laser light has a sound speed of about 10^8 cm/sec whereas x-rays produce an ablation region with a sound speed of about 2 10^7 cm/sec. The latter value corresponds to a temperature of about 300 eV, which not coincidentally is the T associated with a σT^4 flux of 10^{15} W/cm^2.

The pressures, P, will scale as $nT \propto n^{1/3} I^{2/3}$. Again, by this scaling we'd expect about a factor of 5 difference in pressures between direct and indirect drive, and indeed at equal energy fluxes of 10^{15} W/cm^2, 1/3 μm laser light has a pressure of about 90 MB, whereas x-rays produce an ablation region pressure of about 400 MB.

While C_S and P are all that we need to calculate η_H, for completeness, and for use later in this paper, let us calculate ablation velocity scaling. The ablation rate, dm/dt (where m is mass per unit area) can be found by noting that $P \propto C_S\, dm/dt$. Given the above results for C_S and P, then, $dm/dt \propto n^{2/3} I^{1/3}$. Since the dense shell should stay near the FD, isentrope, we demand that its density ρ scale as $(P/\alpha_{\mathrm{FD}})^{3/5}$, thus as $n^{1/5} I^{2/5} \alpha_{\mathrm{FD}}^{-3/5}$. Since we define the ablation velocity by $\rho\, V_{\mathrm{abl}} = dm/dt$, then by the above expressions for ρ and dm/dt, we obtain $V_{\mathrm{abl}} \propto n^{7/15} I^{-1/15} \alpha_{\mathrm{FD}}^{3/5}$. With this even stronger scaling with n, we expect a full order of magnitude difference in V_{abl}, between direct and indirect drive, and indeed at equal energy fluxes of 10^{15} W/cm^2, 1/3 μm laser light has a V_{abl} of about 10^5 cm/sec whereas x-rays produce a V_{abl} of about 10^6 cm/sec.

C. The rocket equation

Having derived expressions for C_S and P in the ablation region, we have what we need to calculate η_H. The force law $m\, dv/dt = -P$ can be reformulated (in terms of dm/dt) as $m\, dv/dm = -P/(dm/dt)$ which can be integrated to find $v = [P/(dm/dt)] \ln(m(t)/m_0)$. Here m_0 is the mass per unit area of the shell at time zero. This form is precisely the classical rocket equation. Defining $X \equiv m(t)/m_0$, and remembering that $P/(dm/dt) \approx C_S$ we rewrite this as

$$v = V_{\text{exh}} \ln X = C_S \ln X. \tag{18}$$

Already in this form, efficiency arguments can be made. We know from the classical rocket equation that the most efficient rockets have their exhaust velocity (as measured relative to the rocket) equal to the payload velocity (as measured in the lab frame). Simply put, if this condition is fulfilled, there is no wasted energy of kinetic motion, in the lab frame, of the useless exhaust velocity. Based on our discussions in Secs. VII A and B above, the required payload velocity is of order 3×10^7 cm/sec, and the exhaust velocity for direct drive is a mismatched 10^8 cm/sec whereas the exhaust velocity for indirect drive is a well matched 2×10^7 cm/sec. Thus we expect indirect drive to have a higher η_H.

We can be more quantitative. Rewriting time, t, in terms of mass, by using $m = m_0 - (dm/dt)t$ we obtain $t = [m_0/(dm/dt)]\,(1-X)$. Using that in

$$\begin{aligned}\eta_H \ \text{(ideal)} &= (1/2)mv^2/It \\ &= (1/2)m_0 X C_S^2 \ln^2 X / I[m_0/(dm/dt)](1-X) \\ &= (1/2)[(dm/dt)C_S^2/I][X \ln^2 X/(1-X)] \\ &\approx (1/2)[X \ln^2 X/(1-X)],\end{aligned} \tag{19}$$

where the bracketed quantity $[(dm/dt)C_S^2/l]$ is of order 1 by our simplified discussions in Sec. VII B above. In fact, the numerical simulations (that include more detailed physics, such as heat maintenance of the blowoff temperature, etc.) give something closer to

$$\eta_H = (1/3) \quad [X \ln^2 X/(1-X)]. \tag{20}$$

Solving Eq. (18) for X, given the required v_{imp}, from Sec. VII A and given the typical C_S for direct and indirect drive given in Sec. VII B, we obtain $X = 0.7$ and 0.17 for direct and indirect drive, respectively. Substituting those values of X into Eq. (20) leads to η_H of 0.1 and 0.2 for direct and indirect drive, respectively, precisely as reported in Table I. The value of $X = 0.17$ for indirect drive says that more than 4/5 of the shell is ablated before it gets to its terminal velocity of v_{imp}. Thus indirect drive shells are thicker than their direct drive counterparts, which has advantages for their surviving hydrodynamic instabilities which are discussed in the next section.

The question that could be asked is why have direct drive operate at the high I of about 10^{15} W/cm^2 that we have assumed. Clearly that has led to a high C_S (=a high V_{exh}) and therefore a low η_H. Why not operate at much lower irradiance? The answer lies in considerations of surviving hydrodynamic instabilities, which we now discuss.

VIII. HYDRODYNAMIC INSTABILITIES AS A CONSTRAINT ON GAIN

The Rayleigh Taylor (RT) instability is prevalent in ICF implosions. An inverted glass of water is in principal in equilibrium (the atmosphere's 14 lb/sq. in. can keep the water in the glass) but it is a RT unstable equilibrium. The dense water would ''prefer'' to lower the energy of the system by being lower in the gravitational potential than the lighter air it will soon replace (on its way to the soon-to-be-wet floor!). An ICF capsule is similar. The low density ablated material accelerates the dense shell. The shell feels a huge ''gravity'' much like the gee force an astronaut feels at launch time. Thus again we have dense matter in a ''gravity'' field wishing to exchange places with low density matter. The target crinkles on its way towards implosion. The instability is mitigated somewhat by the ablative acceleration process—the ablation tends to effectively burn-off or smooth the perturbations. Upon deceleration at the culmination of the implosion, the low density hot spot DT gas holds up the dense DT shell, again in an effective gravity. An unstable RT situation arises yet again, and the cold shell mixes into the hot fuel. Understanding these quantitatively is required to ascertain just how smooth an initial target must be, since initial small perturbations will grow due to the RT instability.

For an initial perturbation of wavelength λ, at the interface of a dense fluid of density ρ_1, on top of a less dense fluid of density ρ_2, in an effective gravity field g, the classical growth rate of the RT instability is given by $\gamma_{\text{CL}} = A_{\#}(2\pi g/\lambda)^{1/2}$, where the Atwood number, $A_{\#}$, is given by $(\rho_1-\rho_2)/(\rho_1+\rho_2)$. In ICF where the gravity is simply the reaction of the target due to the ablation driven acceleration, there is a stabilizing term due to the ablation: $\gamma = \gamma_{\text{CL}} - 2\pi\beta V_{\text{abl}}/\lambda$. (Here β is a factor between 1 and 3.) This ''ablation stabilization'' mitigating factor plays a very important role in having the target survive its implosion without completely breaking up.

Let us consider a thin dense shell of density ρ and thickness ΔR accelerating inward. Typical ICF targets reach their peak implosion velocity when they have moved inward to about $R/2$ (1/2 their initial radius) because most of the potential for compressional (''P dV'') work is used up by the time we've reached $R/2$, since 88% of the volume has been used by then. By the usual expression $v^2 = 2ad$, where a is the acceleration and d is the distance moved, setting $d = R/2$, and a to $F/m \approx (P\text{ Area}/\rho\Delta R\text{ Area}) = P/\rho\Delta R$, we obtain $P = \rho v^2/(R/\Delta R)$. We call the quantity $(R/\Delta R)$ the inflight aspect ratio (''IFAR'') of that shell. Insisting on that shell being nearly FD, using Eq. (4), we can eliminate ρ and obtain

$$(R/\Delta R) \propto v^2/P^{2/5}\alpha_{\text{FD}}^{3/5}. \tag{21}$$

With this result we are now in a position to answer the question posed at the end of Sec. VII C. To have direct drive's η_H match indirect drive, it would need to have a matching C_S, not one that is five times higher. Since (from Sec. VII B), $C_S \propto I^{1/3}$ that would require lowering I by 125. Since P scales as $I^{2/3}$ (also from Sec. VII B) that would lower P by 25. Then, since we must still achieve a given v_{imp}, Eq. (21) tells us that such a lowered P requires a IFAR larger by a factor of 3.6. So what's wrong with that?

Well, the classical RT instability will have an initial perturbation grow by a factor of $\exp(\gamma_{\text{CL}} t)$ after a time t. The perturbation wavelength of most concern is that of order the shell thickness, ΔR. Let's investigate that growth factor:

$$(\gamma_{\mathrm{CL}}t)^2=(2\pi/\lambda)at^2=(2\pi/\lambda)R=2\pi(R/\Delta R). \qquad (22)$$

Thus we see that large IFAR leads to large RT growth. Direct drive already has low P compared to indirect drive, so lowering I further will only exacerbate its natural high IFAR handicap. From Eq. (21) we see that a way to lower IFAR is to raise α_{FD}. Of course we know from Eq. (7) that raising α_{FD} will lower gain.

As mentioned above, the ablative stabilization term is crucial in lowering the RT growth. From Sec. VII B we saw that V_{abl}, is much smaller for direct drive than for indirect drive. We also saw that V_{abl} scales as $n^{7/15}I^{-1/15}\alpha_{\mathrm{FD}}^{3/5}$. Clearly with the weak scaling with I, the only strategy remaining for direct drive to increase its ablative stabilization is to raise α_{FD}. Again, from Eq. (7) that implies a penalty in gain. Present efforts[5] at direct drive target design are aimed at perhaps ''ruining the isentrope'' (high α_{FD}) only in the ablation region and not in the main fuel so as to avoid that gain penalty. In any event, these considerations explain why an α_{FD} greater than 1 appears in Table I, especially in connection with direct drive. (The α_{FD} factor of 1.5 for indirect drive reflects a conservative assumption that the implosion may not perfectly stay on the FD isentrope.)

Thus we have seen that direct drive, by virtue of its overall better coupling $[\eta_T$ of order $(0.8)(0.1)=8\%$ versus indirect drive $(0.2)(0.2)=4\%]$ has some advantages over indirect drive (both in terms of gain, and in terms of a smaller driver), but is challenged by the RT instability and the need to purposefully ''ruin the isentrope'' and raise α_{FD} at some cost in gain. Is there any approach that lets us have our cake and eat it, too?

Consider heavy ion fusion (HIF). Since the ion beams penetrate matter, they can drive hohlraums without LEHs. From Eq. (17), without that a_{LEH} loss term (of order 2) we can expect an η_C of order 0.3, a 50% improvement in the coupling coefficient for indirect drive. If we substitute 0.3 for 0.2 in the η_C column of Table I for the HiY scale, and consider that with this higher η_T, the driver scale can be reduced from 5 to 3.3 MJ, we'd expect a HIF design to produce a gain of about 120 with a 3.3 MJ driver. Indeed a HIF target gain of 130 at 3.3 MJ has been obtained recently in a detailed design.[13] It should also be noted that these high gains, coupled with the high expected driver efficiency for HIF of order 25%, lead to favorably large $\eta_D G$ values in excess of 30. It also appears[13] that adequate gains may be achieved at driver scales of under 2 MJ, thus going a long way towards reducing initial capital costs of a power plant.

IX. SUMMARY

We have reviewed the needs for ICF target gain and motivated the typical numbers expected for both the NIF driver scale and high yield target driver scale. The $\eta_D G > 10$ criteria, coupled with assumed driver efficiencies of 10%, lead to needed gains of order 100. The concept of hot spot ignition driving a propagating thermonuclear burn wave into cold dense near FD fuel assembly can lead to such high gains. The fast ignitor approach, by virtue of its breaking the constraint of an isobaric fuel assembly and thus operating at lower density, can, for a given energy scale, burn more fuel and thereby achieve higher gains.

We have shown that the gains of order 10 at the NIF driver scale scale up rather straightforwardly to the required gains of order 100 at a higher driver energy scale.

The gains achievable by indirect drive are constrained by their relatively poor coupling efficiencies. These are due to large hohlraum wall losses. Those, in turn, are due to the requirement of having the wall radius be about four times the capsule radius to ensure good geometric smoothing in order to provide the required drive symmetry for these high convergence targets. Thus symmetry constrains indirect drive gains, and research continues into creative ways to ''push the envelope'' and achieve good hohlraum symmetry through a variety of methods that would allow increased coupling efficiencies.

Direct drive, by virtue of its overall better coupling, might hold some advantage over indirect drive. However, we have seen that for direct drive, hydrodynamic instabilities are a constraint, leading to lower hydrodynamic efficiencies, and perhaps the need to pay a price in gain by purposefully ruining the isentrope. Ways to do so and not take a big penalty in gain is an area of active current research.

Heavy ion fusion has the target dynamics advantages of x-ray drive and, by virtue of it not having lossy laser entrance holes to contend with, has improved coupling, and therefore has shown some very promising results of high gain at small driver size.

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