Who will save the tokamak – Harry Potter, Arnold Schwarzenegger, Shaquille O'Neal or Donald Trump?

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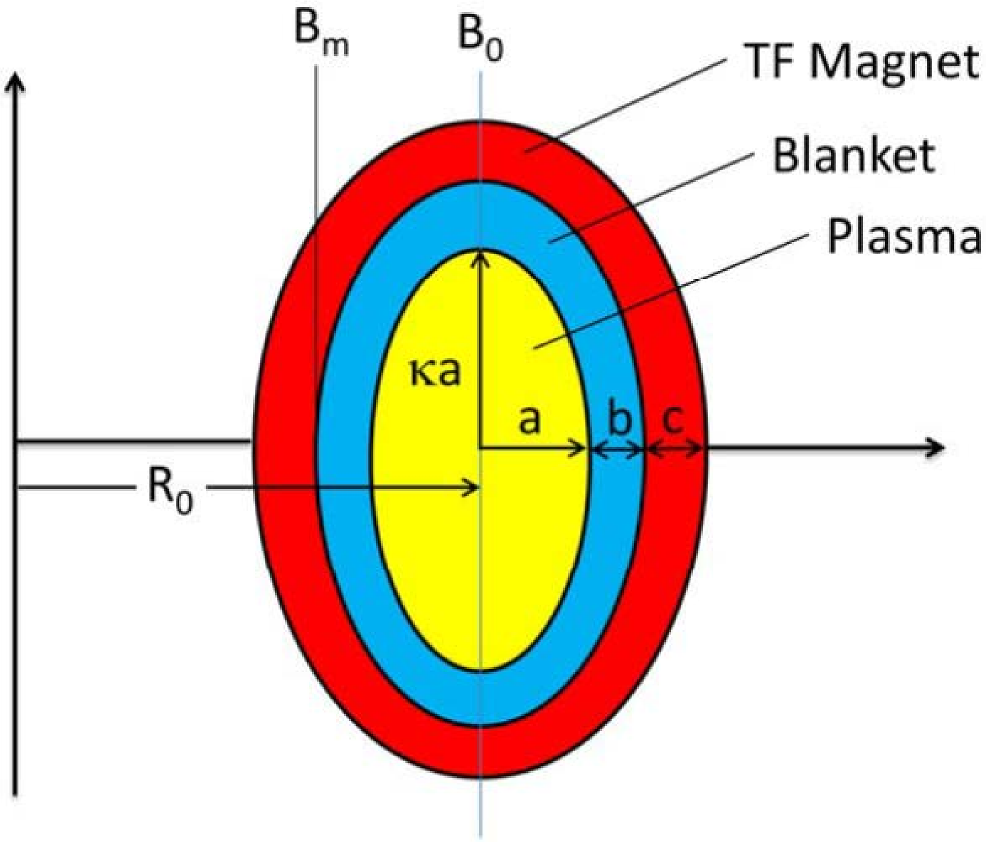
Why does the tokamak need saving?

- Standard tokamak does not scale to a reactor
- Design determined by nuclear physics and engineering constraints
- Minimal plasma physics enters the design
- But, the plasma physics is incompatible with the engineering design

Design a reactor and prove this!



Goal – Minimize the Cost

- Total cost = Capital + Operating + Fuel ≈ Capital
- Capital = Fixed + Nuclear island
- Fixed = $K_F P_E$
- Nuclear Island = $K_I V_I$
- Cost/Watt = $K_F + K_I \dfrac{V_I}{P_E}$
- Minimize

$$\frac{V_I}{P_E} = \frac{2\pi^2 R_0}{P_E}\left[\varepsilon_B (a+b)(\kappa a+b) - \kappa a^2 + (1-\varepsilon_B)(a+b+c)(\kappa a+b+c)\right]$$

$$\varepsilon_B = (a+b)/R_0$$

Design Goals

Quantity	Symbol
Minor radius of the plasma	a
Major radius of the plasma	R_0
Thickness of the blanket/shield and first wall	b
Thickness of the magnets	c
Plasma temperature	T
Plasma density	n
Plasma pressure	p
Energy confinement time	τ_E
Magnetic field at $R = R_0$	B_0
Normalized plasma pressure	β

Engineering and Nuclear Physics Constraints

Quantity	Symbol	Limiting Value
Electric power output	P_E	1000 MW
Maximum neutron wall loading	P_W	4 MW/m^2
Maximum magnetic field at the coil	B_{max}	13 T
Maximum mechanical stress on the magnet: Total = 650 MPa, Tensile = 500 MPa	σ_{max}	500 MPa
Maximum superconducting coil overall current density	J_{max}	25 MA/m^2
Thermal conversion efficiency	η_T	0.4
Maximum RF recirculating power fraction	f_{RP}	0.1
Wall to absorbed RF power conversion efficiency	η_{RF}	0.4
Temperature at $[\sigma v / T^2]_{max}$	$\overline{T}$	14 keV
Fast neutron slowing down cross section in Li-7	σ_{sd}	2 barns
Slow neutron breeding cross section in Li-6	σ_{b0}	950 barns at 0.025 eV

The Design – No Plasma Physics Required

- Elongation: Good for cost, good for plasma physics
  Limited by engineering, limited by plasma physics

$$\kappa = 1.7$$

- Blanket/shield: limited by nuclear cross sections

$$b = 1.2\, m$$

- Wall loading: relation between R_0 and a : $P_W A = P_n$

$$R_0 = \left[\frac{1}{4\pi^2} \frac{E_n}{E_F} \frac{P_E}{\eta_T P_W} \left(\frac{2}{1+\kappa^2} \right)^{1/2} \right] \left(\frac{1}{a} \right) \sim \frac{1}{a}$$

- Coil thickness: relation between c and a

$$c = c_\sigma + c_J = R_0 \left\{ 2(1-\varepsilon_B) - \left[(1-\varepsilon_B)^2 - \alpha_M \right]^{1/2} - \left[(1-\varepsilon_B)^2 - \alpha_J \right]^{1/2} \right\}$$

$$\alpha_M = \left(\frac{B_0^2}{2\mu_0 \sigma_{\max}} \right) \ln \left(\frac{1+\varepsilon_B}{1-\varepsilon_B} \right) \qquad \alpha_J = \frac{2B_0}{\mu_0 R_0 J_{\max}}$$

Minimize V_I / P_E with respect to a

- Set $B_0 = (1 - \varepsilon_B) B_{\max}$

Quantity	**Symbol**	$B_{\max} = 13$
Cost function (m^3/MW)	V_I / P_E	0.79
Magnetic field at $R = R_0$ (T)	B_0	5.7
Elongation	κ	1.7
Blanket thickness (m)	b	1.2
Minor radius (m)	a	1.49
Total magnet thickness (m)	c	0.65
Major radius (m)	R_0	4.8
Aspect ratio	R_0 / a	3.2
Plasma volume (m^3)	V_P	357

Engineering DEMANDS on the Plasma Physics

- Profiles: $T = 2\overline{T}(1-\rho^2) \quad p = 2.5\overline{p}(1-\rho^2)^{3/2} \quad n = 1.5\overline{n}(1-\rho^2)^{1/2}$
- Temperature: Maximize $\sigma v / T^2$ $\quad \overline{T} = 14\ keV$
- Pressure: $P_E = \eta_T P_F = \frac{\eta_T E_F}{16}\int p^2 \frac{\langle \sigma v \rangle}{T^2} d\mathbf{r} \quad \rightarrow \quad \overline{p} = 7.3\ atm$
- Beta: $\beta = \frac{2\mu_0 \overline{p}}{B_0^2} = 5.6\%$
- Density: $p = 2nT \quad \rightarrow \quad n = 1.35 \times 10^{20}\ m^{-3}$
- Energy confinement time: $P_\alpha = \frac{3}{2\tau_E}\int p\, d\mathbf{r} = \frac{3}{2}\frac{V_P \overline{p}}{\tau_E} \quad \rightarrow \quad \tau_E = 1.0\ \sec$

Plasma Current and Bootstrap Fraction

- Requires engineering and plasma physics
- The current from empirical scaling ($H = 1$):

$$\tau_E = 0.145 H \frac{I^{0.93} R^{1.39} a^{0.58} \kappa^{0.78} \overline{n}^{0.41} B_0^{0.15} A^{0.19}}{P_\alpha^{0.69}} \quad \text{sec} \quad \rightarrow \quad I = 17.5\, MA$$

- Kink safety factor: $q_* = \frac{2\pi a^2 B_0}{\mu_0 R_0 I} \left(\frac{1+\kappa^2}{2} \right) = 1.47$

- LH current drive: $P_{CD} = \eta_{RF} P_{RF} = \eta_{RF} f_{RP} P_E = 40\, MW$

$$\eta_{CD} = \frac{R_0 \overline{n} I_{CD}}{P_{CD}} \approx \frac{1.2}{\mathrm{n}_{\|}^2} \qquad \mathrm{n}_{\|} \approx \frac{\omega_{pe}}{\Omega_e} + \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2} \right)^{1/2} \left(1 - \frac{\omega_{LH}^2}{\omega^2} \right)^{1/2} \quad \rightarrow \quad I_{CD} = 2.0\, MA$$

- Bootstrap fraction: $f_B = 1 - \frac{I_{CD}}{I} = 0.89$

How well does the plasma shape-up?

- Greenwald density limit:

$$\bar{n} < \bar{n}_G \equiv \frac{I}{\pi a^2} \quad \rightarrow \quad 1.35 < 2.51$$

✓

- Troyon beta limit:

$$\beta < \beta_T \equiv \beta_N \frac{I}{aB_0} \quad \beta_N = 2.8\% \quad \rightarrow \quad 5.6 < 5.8$$

✓

- Kink safety factor limit:

$$q_K \approx 2 < q_* = \frac{2\pi a^2 B_0}{\mu_0 R_0 I}\left(\frac{1+\kappa^2}{2}\right) \quad \rightarrow \quad 2 < 1.5$$

✗

The Maximum Bootstrap Fraction

- The maximum bootstrap fraction:

$$J_B(\rho) = -2.44\left(\frac{r}{R_0}\right)^{1/2}\left(\frac{p}{B_\theta}\right)\left(\frac{1}{n}\frac{\partial n}{\partial r} + 0.055\frac{1}{T}\frac{\partial T}{\partial r}\right) = \left[6.8\frac{\overline{p}}{\hat{a}^{1/2}R_0^{1/2}}\right]\frac{\rho^{3/2}(1-\rho^2)^{1/2}}{B_\theta}$$

$$b_\theta(\rho) = \frac{B_\theta(\rho)}{\mu_0 I / 2\pi\hat{a}} = \frac{1}{\rho}\left[\frac{(1+\alpha-\alpha x)e^{\alpha x} - 1 - \alpha}{e^{\alpha} - 1 - \alpha}\right]$$

$$f_{NC} = \frac{I_B}{I} = 268\left(\frac{a^{5/2}\kappa^{5/4}\overline{p}}{\mu_0 R_0^{1/2} I^2}\right)\int_0^1\left[\frac{\rho^{5/2}(1-\rho^2)^{1/2}}{b_\theta}\right]d\rho$$

$$f_B < f_{NC} \quad \rightarrow \quad 0.89 < 0.39$$

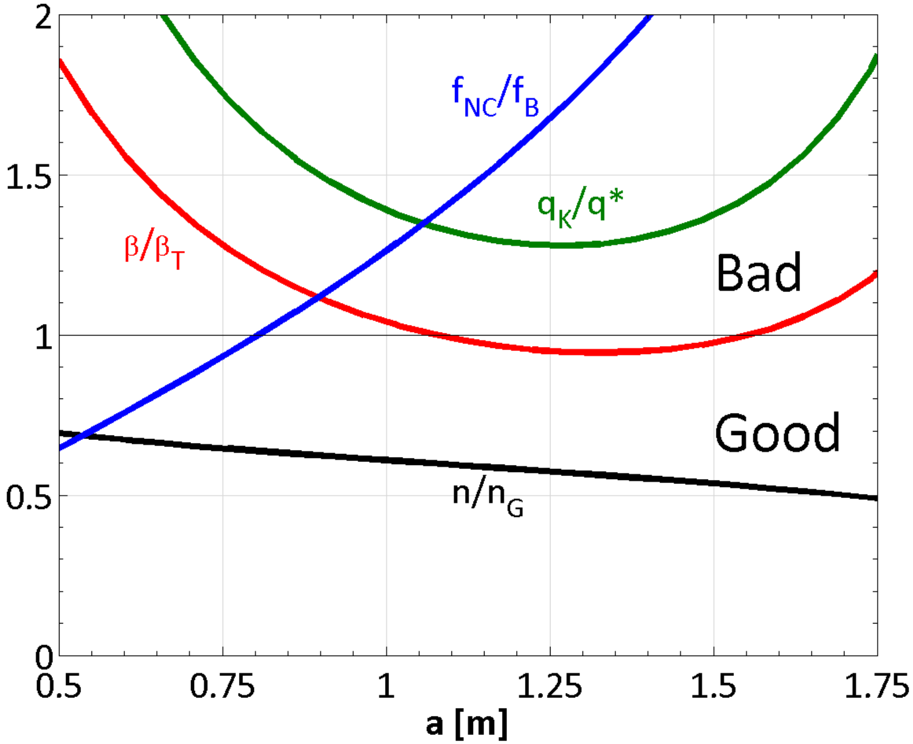
X

- The problem:

Plasma needs too much current for ignition

Is there a simple way out?

- Forget about minimizing V_I / P_E
- Is there any value of *a* that satisfies all constraints?

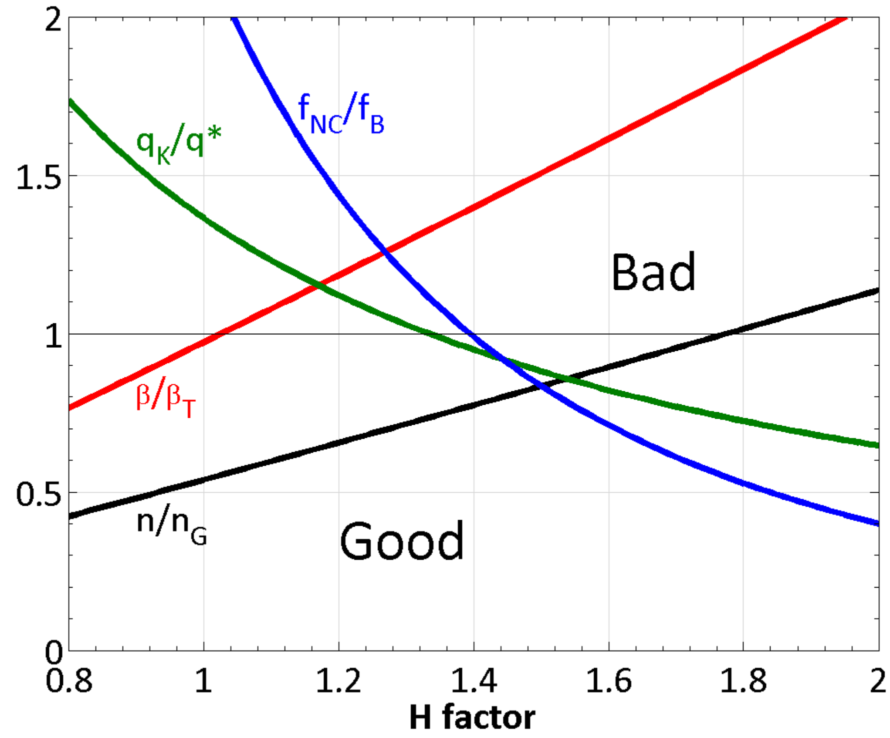


No!

- **The standard tokamak does not scale to a reactor!**

The Harry Potter Solution

- Keeps V_I / P_E fixed
- Raise H
- Lowers the required I
- Lowers the achievable n
- Lowers the achievable β
- Success requires



$$H = 1 \quad \rightarrow \quad H = 1.4$$

$$\beta_N = 2.8 \quad \rightarrow \quad \beta_N = 3.4$$

Robust, disruption free operation

YES?

Plasma Physics Strategy - Magic

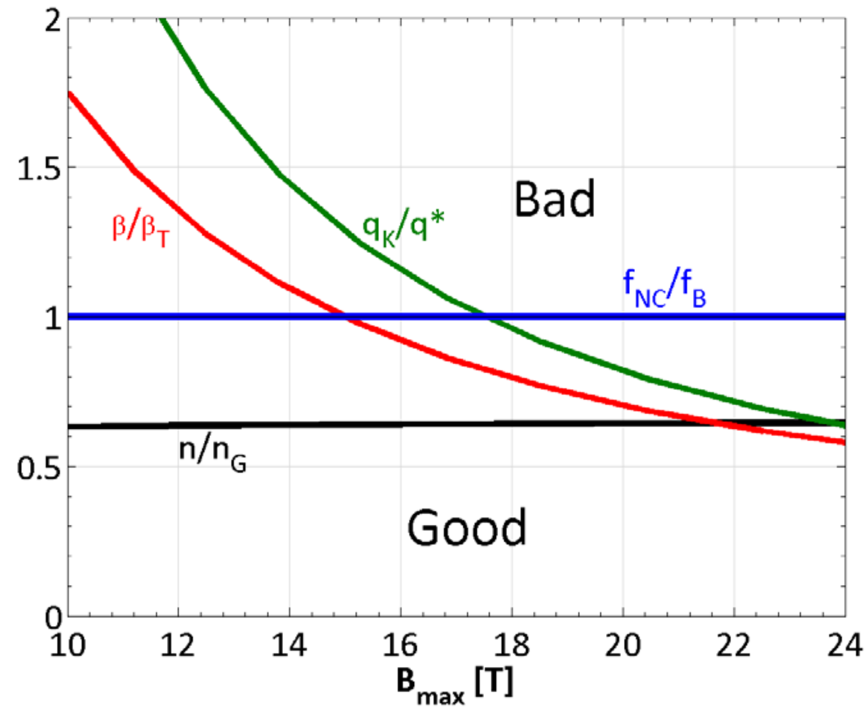
The Arnold Schwarzenegger Solution

- Raise B_{max}
- Improves plasma physics
- Raises V_I / P_E
- Forget optimization
- Set $f_B = f_{NC}$ as a constraint
- Success requires

$B_{max} = 13\,T \quad \rightarrow \quad B_{max} = 17.5\,T$

HTS already exist (YBCO)

$V_I / P_E = 0.79 \quad \rightarrow \quad V_I / P_E = 1.27$

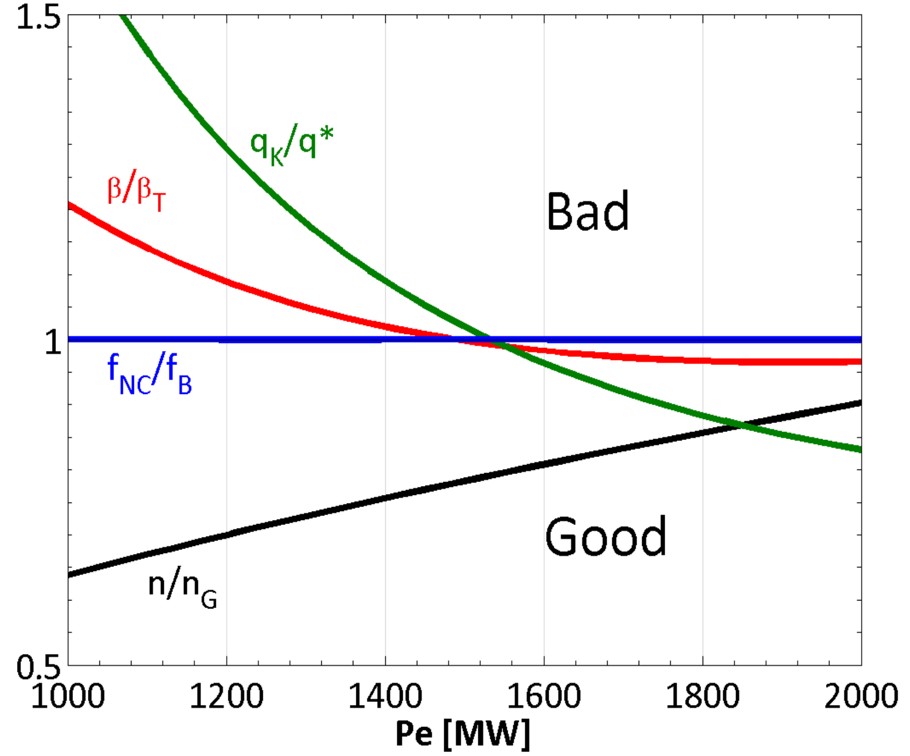


YES!

Engineering Strategy – Strong B

The Shaquille O'Neal Solution

- Raise P_E
- Keep standard $H, \beta_N, B_{\max}$
- Forget optimization
- Set $f_B = f_{NC}$ as a constraint
- A larger plant
- V_I / P_E about the same
- Success requires

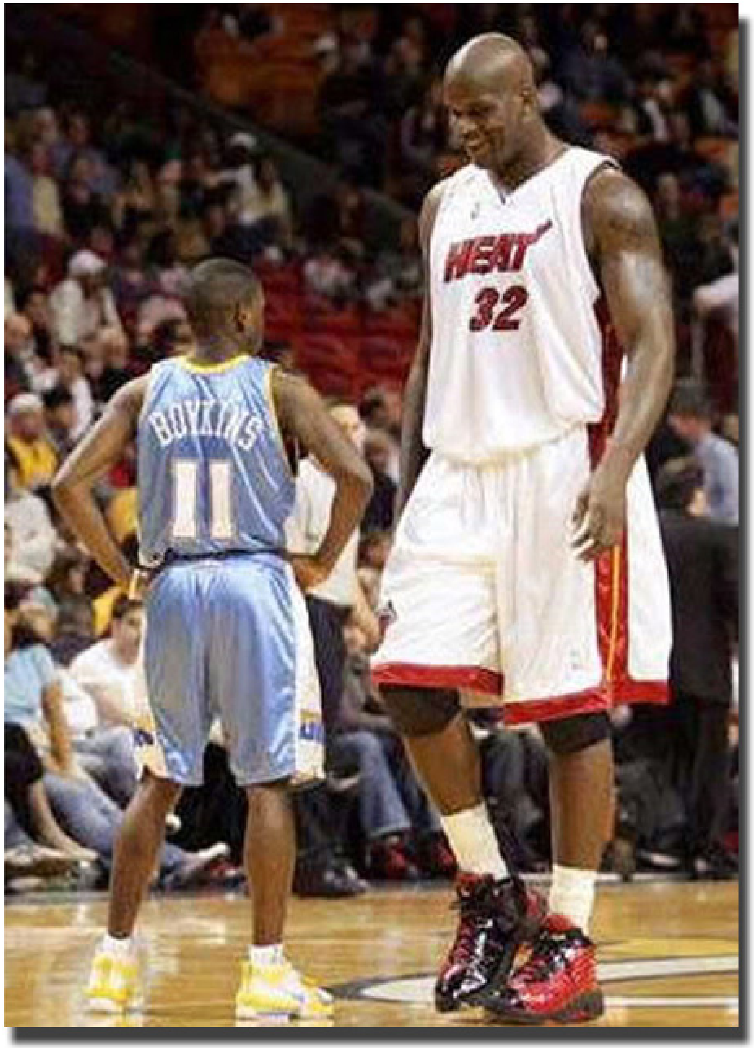


$$P_E = 1000\,MW \quad \rightarrow \quad P_E = 1530\,MW$$

$$V_I / P_E = 0.79 \quad \rightarrow \quad V_I / P_E = 0.90$$

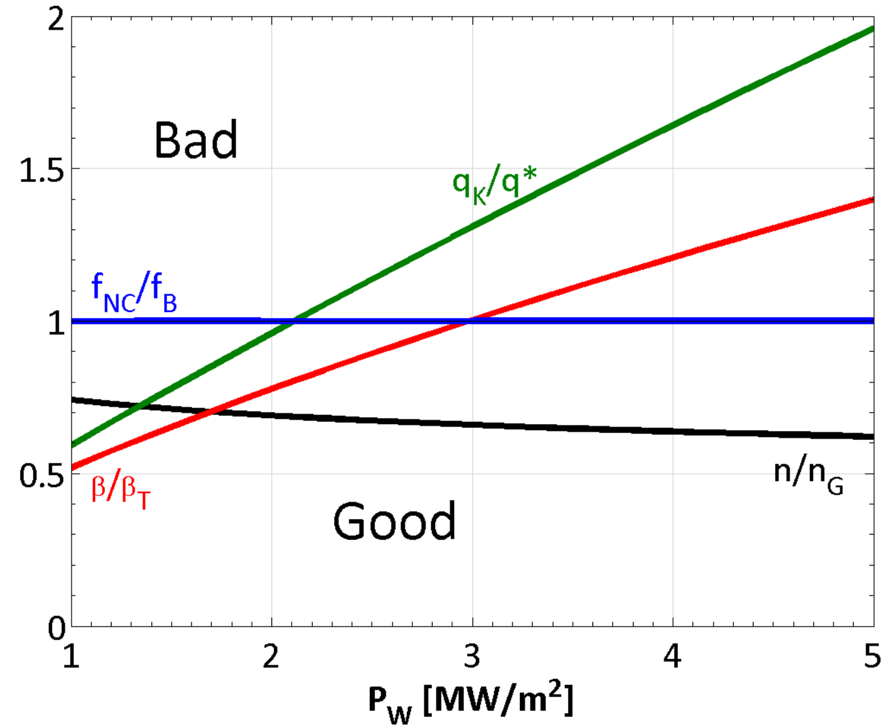
YES!

Utility Risk – Large Power Plant



The Donald Trump Solution

- Lower P_W
- Keep standard H, β_N, B_{max}
- Forget optimization
- Set $f_B = f_{NC}$ as a constraint
- A large, expensive plant
- Much larger V_I / P_E
- Success requires



$$P_W = 4\,MW/m^2 \;\rightarrow\; P_w = 2.1\,MW/m^2$$

$$V_I / P_E = 0.79 \;\rightarrow\; V_I / P_E = 1.83$$

YES!

Utility Risk – Large $/W

What if the tokamak doesn't work?

There is always the stellarator