Sawteeth in a Burning Plasma

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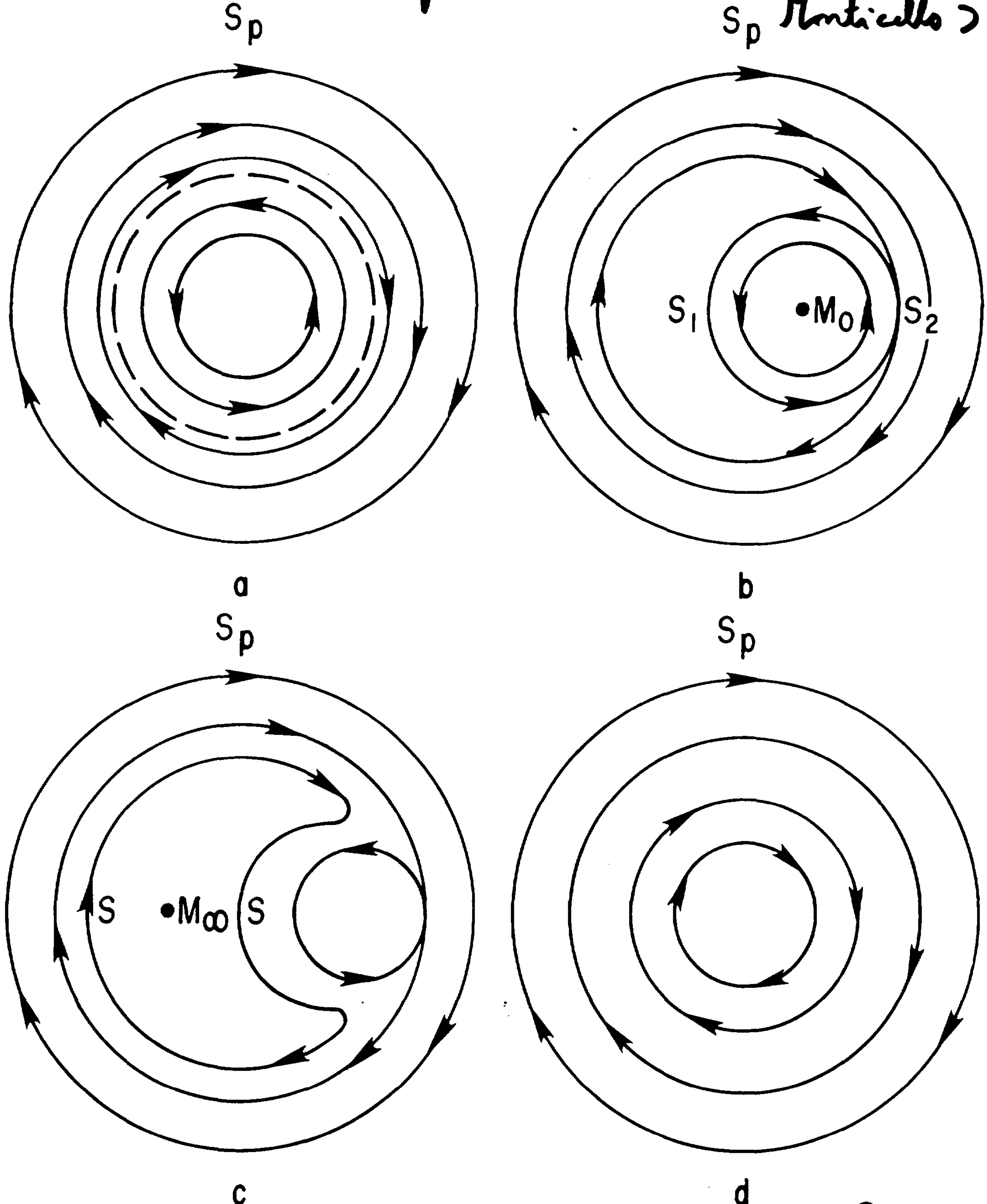
Quasi-ideal model of reconnection (Kadomtsev, 197[illegible] Monticello)



Contours of auxiliary field ($B_\theta - \frac{r B_z}{R q_s}$)

Crash problem : Kadomtsev's solution (1975)

- Nonlinear evolution of the $m=1$ resistive internal kink
- Crash timescale (Sweet-Parker analysis)

$$\tau_K = (\tau_A \tau_R)^{1/2} \equiv \tau_{SP}$$

- For tokamaks in the 1970's

$$\tau_A \sim 10^{-7}\,s \quad , \quad \tau_R \sim 10^{-1}\,s$$

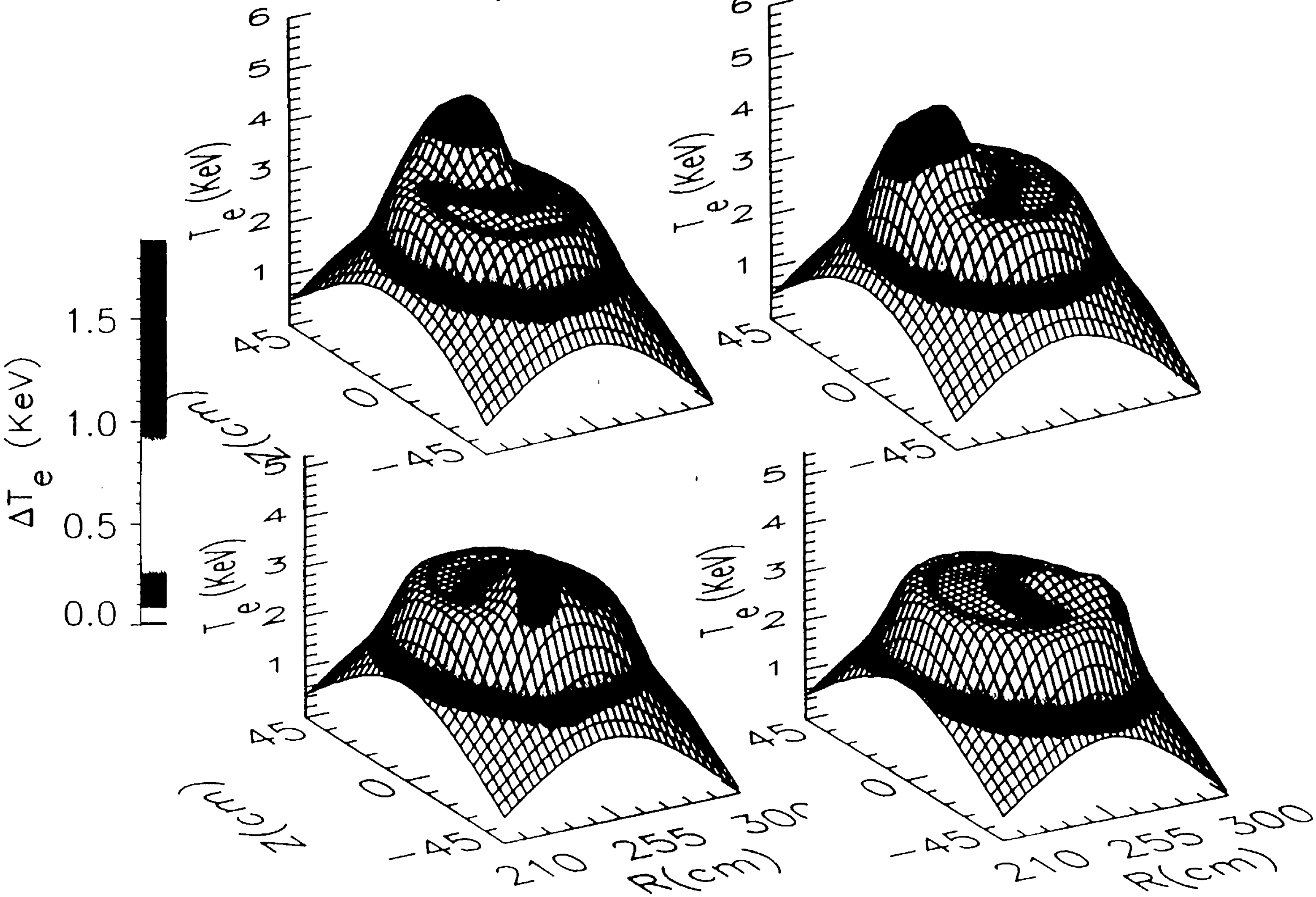
$$\Rightarrow \quad \tau_K \sim 100\,\mu s \, , \quad \checkmark \text{ with experiments}$$

For JET and TFTR

$$\tau_K \sim 2 - 10 \quad ms$$

whereas observations yield crash times $\sim 20 - 100\ \mu s$

Yamada and the TFTR Group (1994)



Intervals of ~ 120 μs

Wesson et al. (1991)
Evolution of plasma displacement of
point of maximum X-ray emission in JET

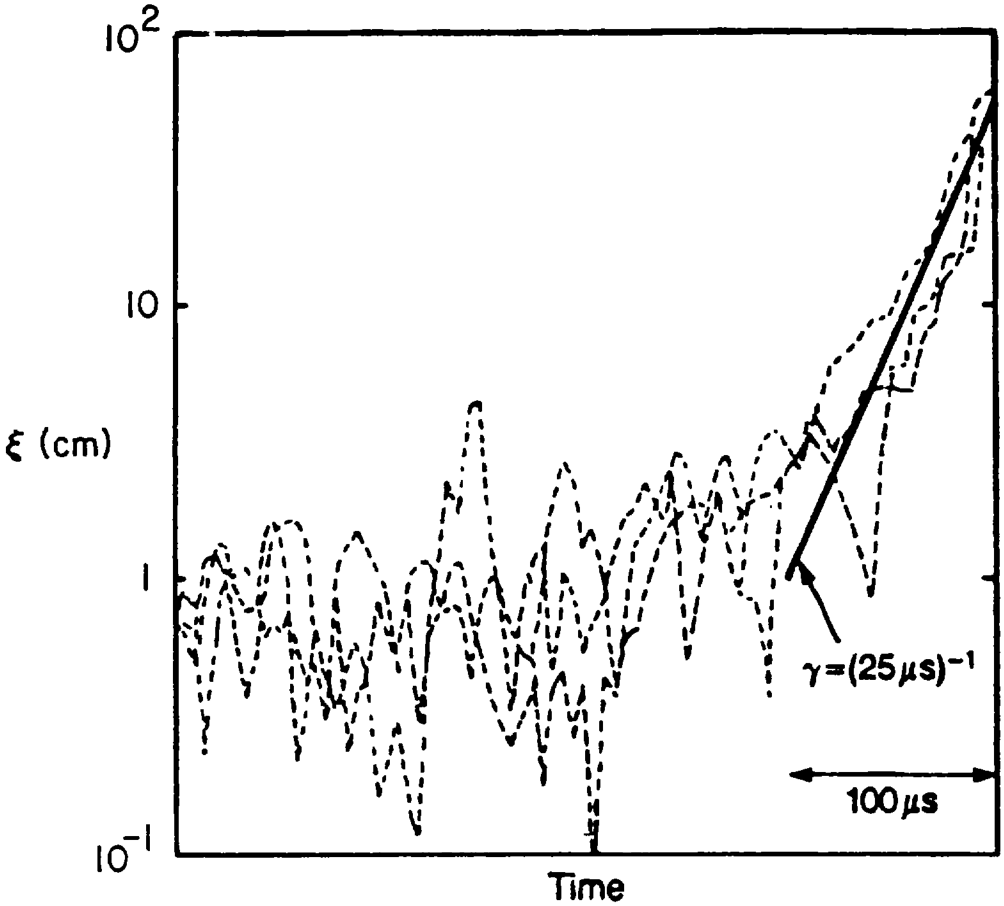
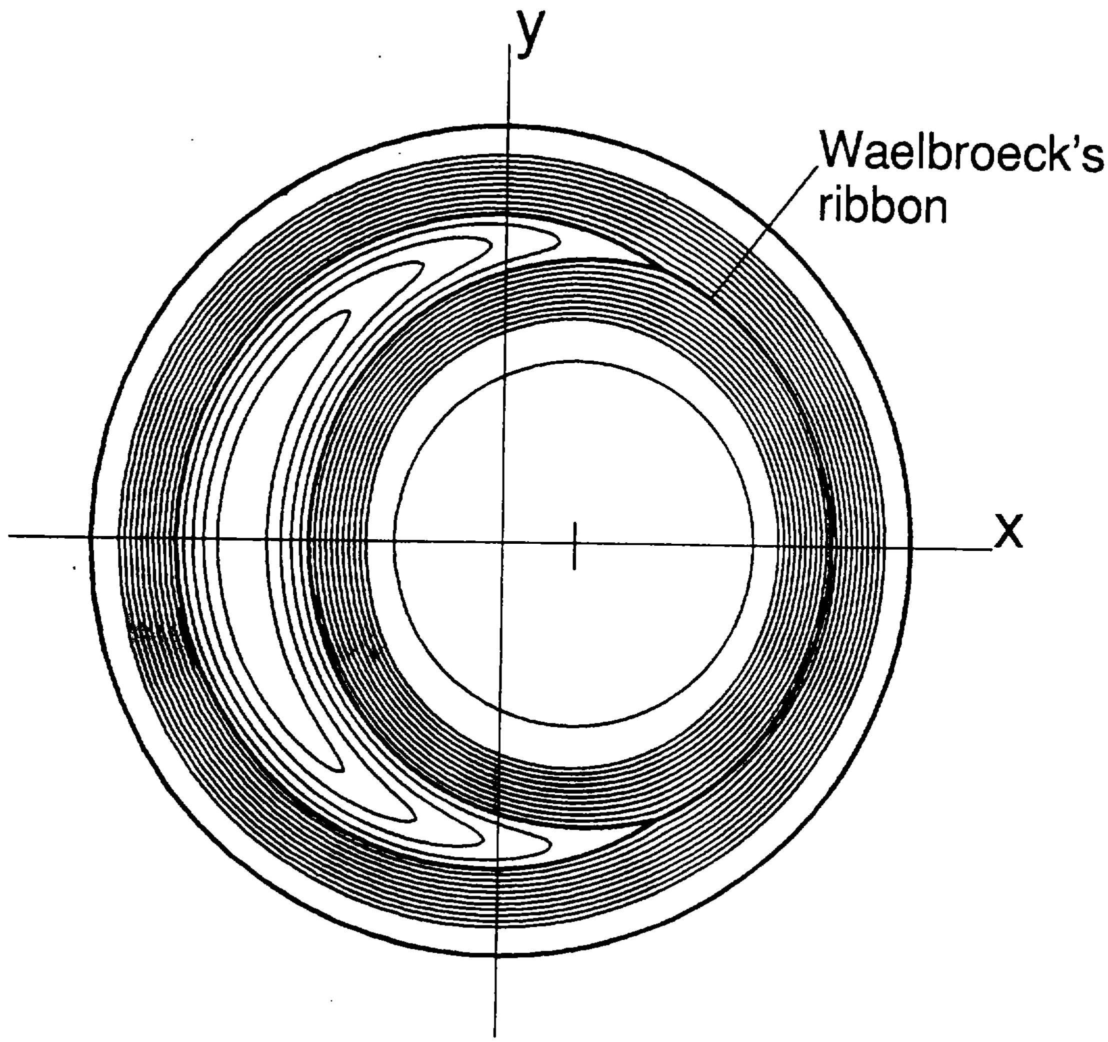


Figure 2

Rosenbluth, Dagazian and Rutherford (1973)
— ideal internal kink

Waelbroeck (1989) — resistive kink



Early nonlinear stage of m=1 reconnection

- The linear $m=1$, $n=1$ instability slows nonlinearly to algebraic growth, $W \sim \eta t^2$

Fig.7

- Simulations: Park et al. (1984)

  Biskamp (1991)

Beyond the Kadomtsev model —

Collisionless Reconnection (Nonlinear)

Aydemir 1992

Wang and Bhattacharjee 1993, 1995

Kleva, Drake and Waelbroeck 1995

Rogers and Zakharov 1996

Grasso, Pegoraro, Porcelli and Califano 1999

Aydemir (1992) : recent numerical results

- Model : four-field MHD (Hazeltine et al. 1987)

- Four-field equations are a generalization of reduced MHD equations. They include FLR effects, as well as compressibility and electron adiabaticity

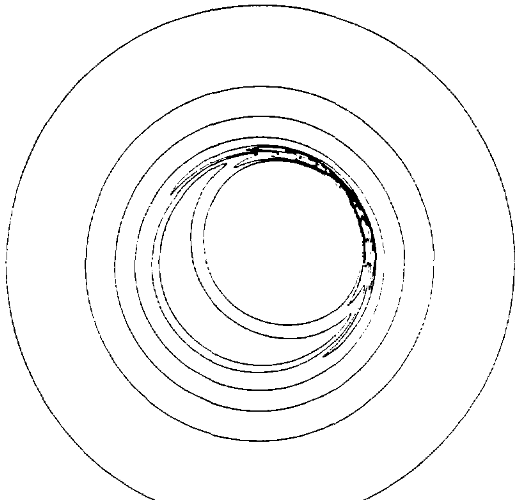
- Generalized Ohm's law

$$\vec{E} + \frac{\vec{v}_e \times \vec{B}}{c} = \frac{m_e}{ne^2}\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{J} - \frac{\nabla p}{nc}$$

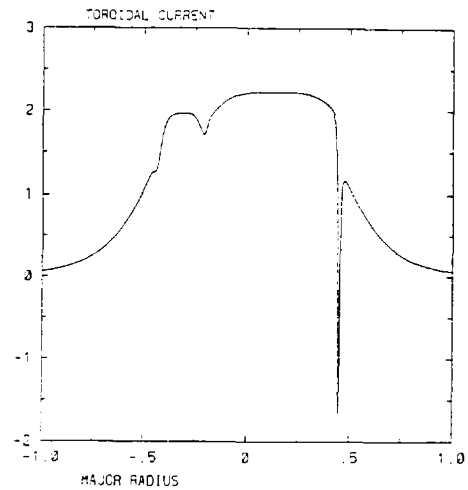
Electron inertia term: breaks field-lines

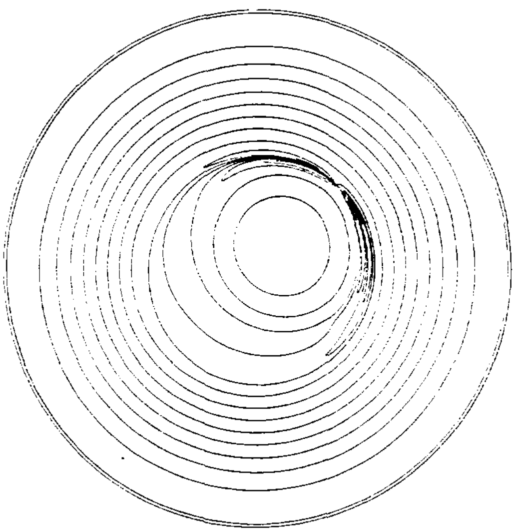
Electron pressure gradient: cannot break field-lines

Aydemir (1992)

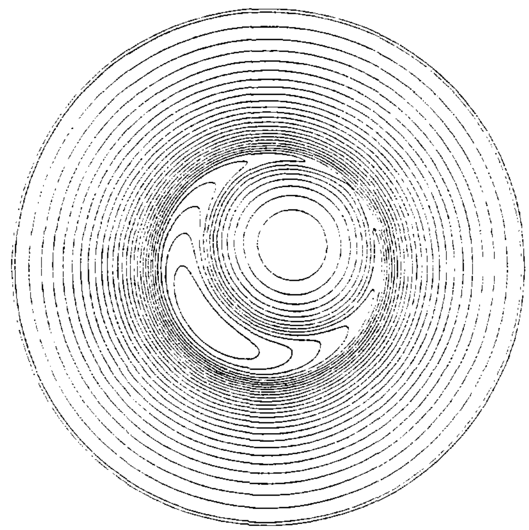


t= 2.32143E+02 TOROIDAL CURRENT

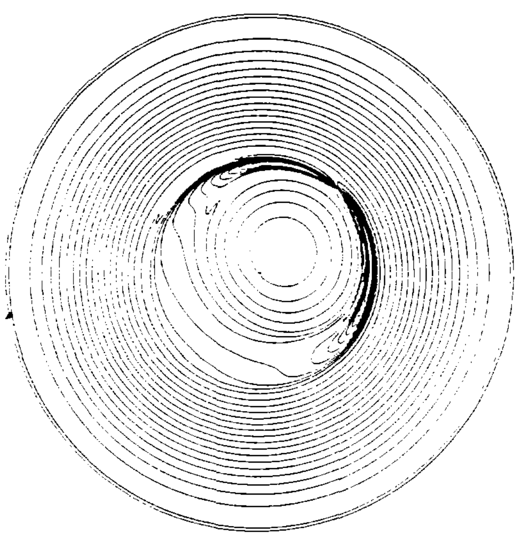




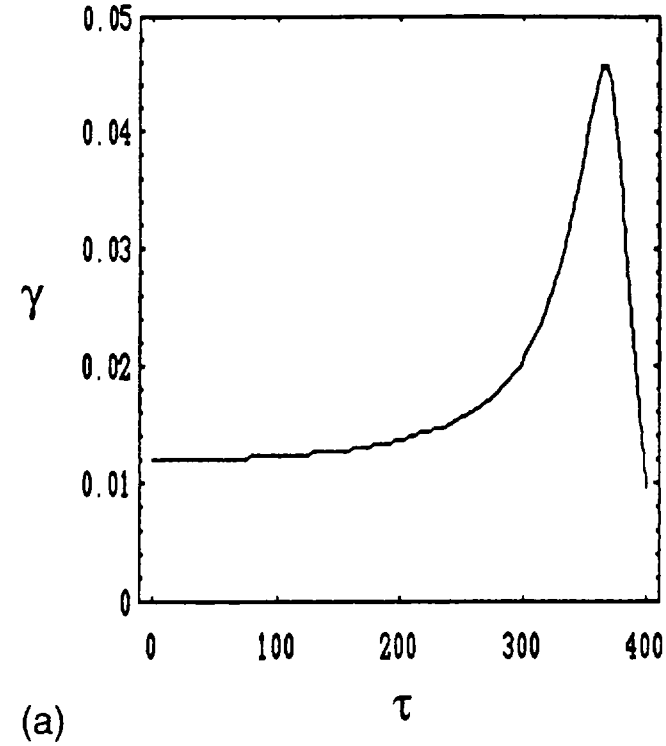
t= 2.32143E+02 PRESSURE



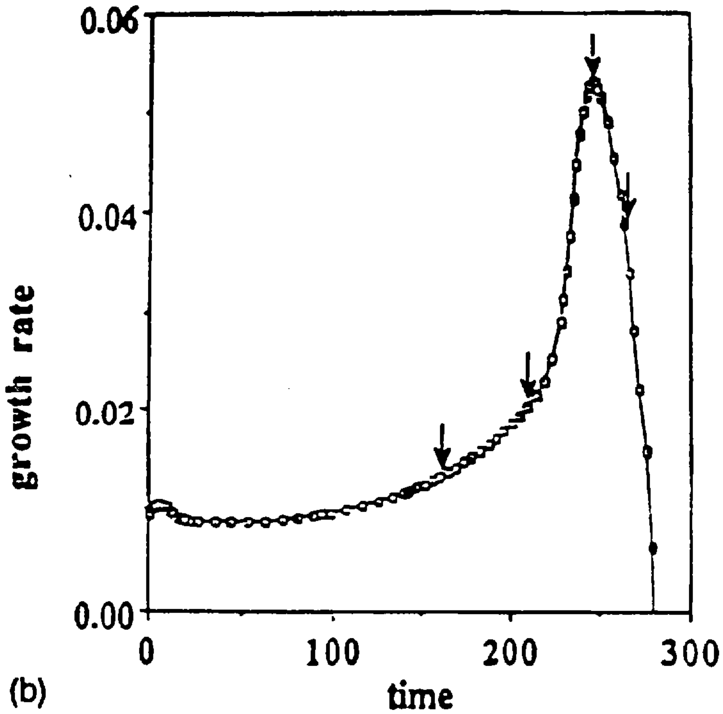
t= 2.32143E-02 HELICAL FLUX



t= 2.32143E-02 Utn^2/2 + del*P



Wang and B., 1993, 1995



Aydemir, 1992

Figure 3

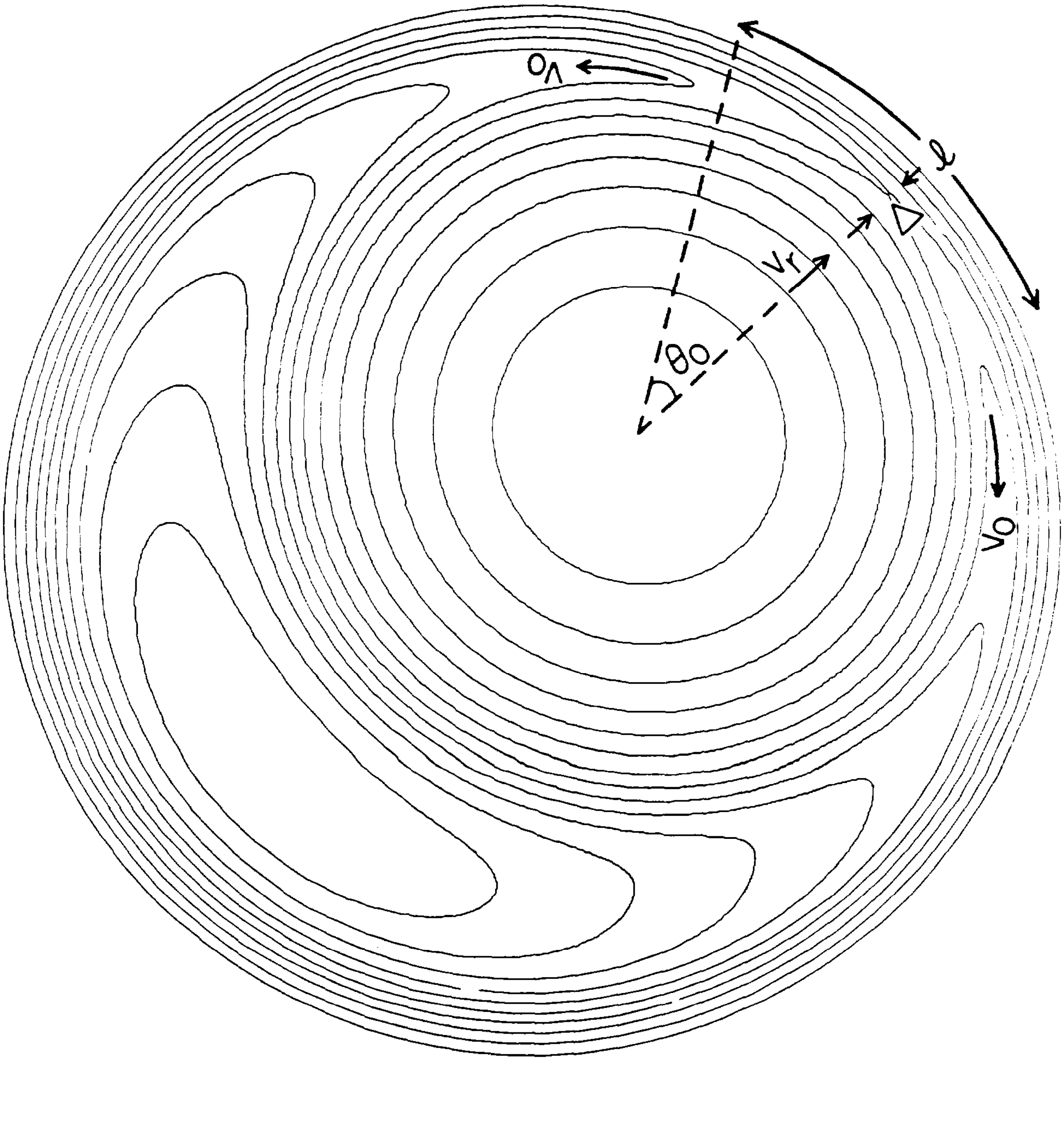


Figure 4

Physics of Near-Explosive Growth of Magnetic Islands

Conservation of mass (in an incompressible plasma)

ℓ : length
Δ : width
of layer

$$\ell V_r \sim \Delta V_\theta$$

Conservation of energy (that is, work done by the pressure gradient in the poloidal direction increases the poloidal flow energy)

$$V_\theta \sim \xi / \tau_A$$

Writing $V_r = d\xi/dt$ and $W = 2\xi$ obtain island equation

$$\tau_A \frac{dW}{dt} = \frac{\Delta}{\ell} W$$

$$\tau_A \frac{dW}{dt} \doteq \frac{\Delta}{\ell} W$$

- If $\Delta \downarrow$ with time, W can slow down with time ($W \sim \eta t^2$ with resistivity)
- If Δ is constant, W can grow exponentially
- In the presence of electron pressure, it can be shown (Wang and B. 1993, Rogers and Zakharov 1996)

$$\Delta \sim W$$

$$\Rightarrow \qquad W \sim (t_c - t)^{-1}$$

Island Equation (including electron inertia and pressure gradient)

$$X = \frac{W}{2 r_s} \ll 1$$

$$\frac{dX}{dt} \simeq \frac{2 d_e \omega_A}{r_s \sin\vartheta_0} X + \frac{\omega_A^2}{2\Omega_i \sin\vartheta_0} X^2$$

$$\sin\vartheta_0 \simeq \left(\frac{\omega_A}{\Omega_i}\right)^{1/2}, \quad \omega_A \equiv \frac{v_A}{r_s}$$

Solution

$$X(t) = \frac{X(0)\exp(2\gamma_c t)}{1 - \frac{\gamma_0}{2\gamma_c} X(0)\left[\exp(2\gamma_c t) - 1\right]}$$

Two-phases

(i) Initially exponential $X(t) \simeq X(0)\exp(2\gamma_c t)$

(Ottaviani and Porcelli, 1993)

(ii) Then near-explosive $\sim (t_c - t)^{-1}$

What arrests explosive growth?

- Island grows to large size and reduces $B_{*\theta}$.
- Thin-island approximation $W/2r_s \ll 1$, breaks down
- Reduction of $B_{*\theta}$ can be calculated by a simple correction term

$$\psi_* \simeq \frac{1}{2}\psi_0''\,(x + \xi(x,\theta))^2$$

Here $\xi(x,\theta) = \xi\cos\theta$ $\qquad$ $\xi = 0 \quad x > 0$; $\xi = \text{constant}, x <$

$$W = 2\xi$$

From $B_{*\theta} = \frac{\partial\psi_*}{\partial r} = \left.\frac{\partial\psi_0}{\partial r}\right|_{r-\xi}$

Expanding to second order

$$B_{*\theta} \simeq B_p \frac{W}{2r_s}\left(1 - \frac{W}{2r_s}\right)$$

Master Island Equation

Three phases : slower exponential phase,

nearly-explosive phase,

decay.

$$\frac{dX}{d\tau} = \frac{2 d_e}{r_s} X \left\{ \frac{\Omega_i}{\omega_A} (1-X) \right\}^{1/2}$$

$$+ \frac{1}{2}\left(\frac{\omega_A}{\Omega_i}\right)^{1/2} X^2 (1-X)^{3/2}$$

Here $X \equiv W/(2 r_s)$, $\tau \equiv \omega_A t$

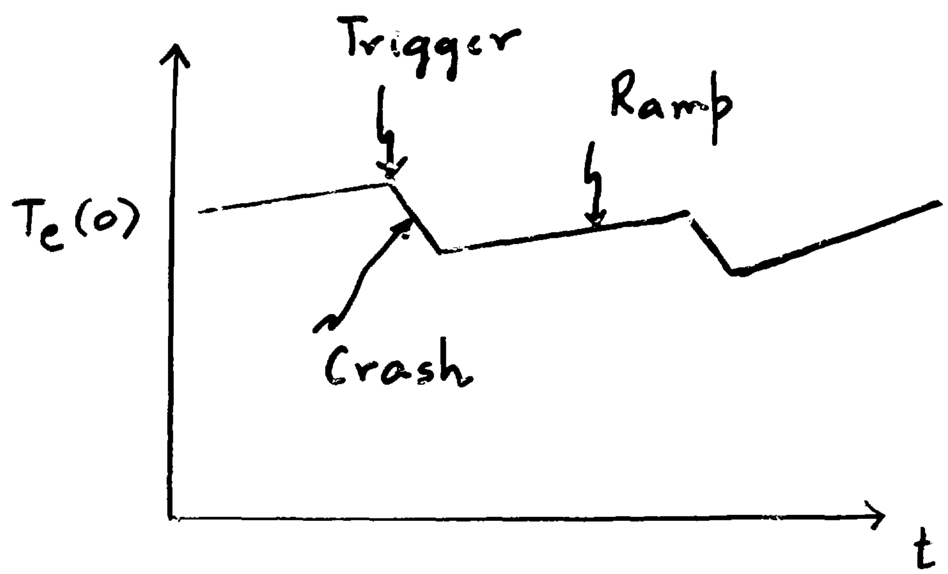
$$\omega_A \equiv \left(\frac{V_A}{R}\right) q' r_s$$

$$\gamma \equiv \frac{d}{d\tau} \ln W$$

$$W = 2\xi$$

What are sawtooth oscillations?

- Occur in the central plasma core of a tokamak



✓ Crash problem: How rapid is the crash?

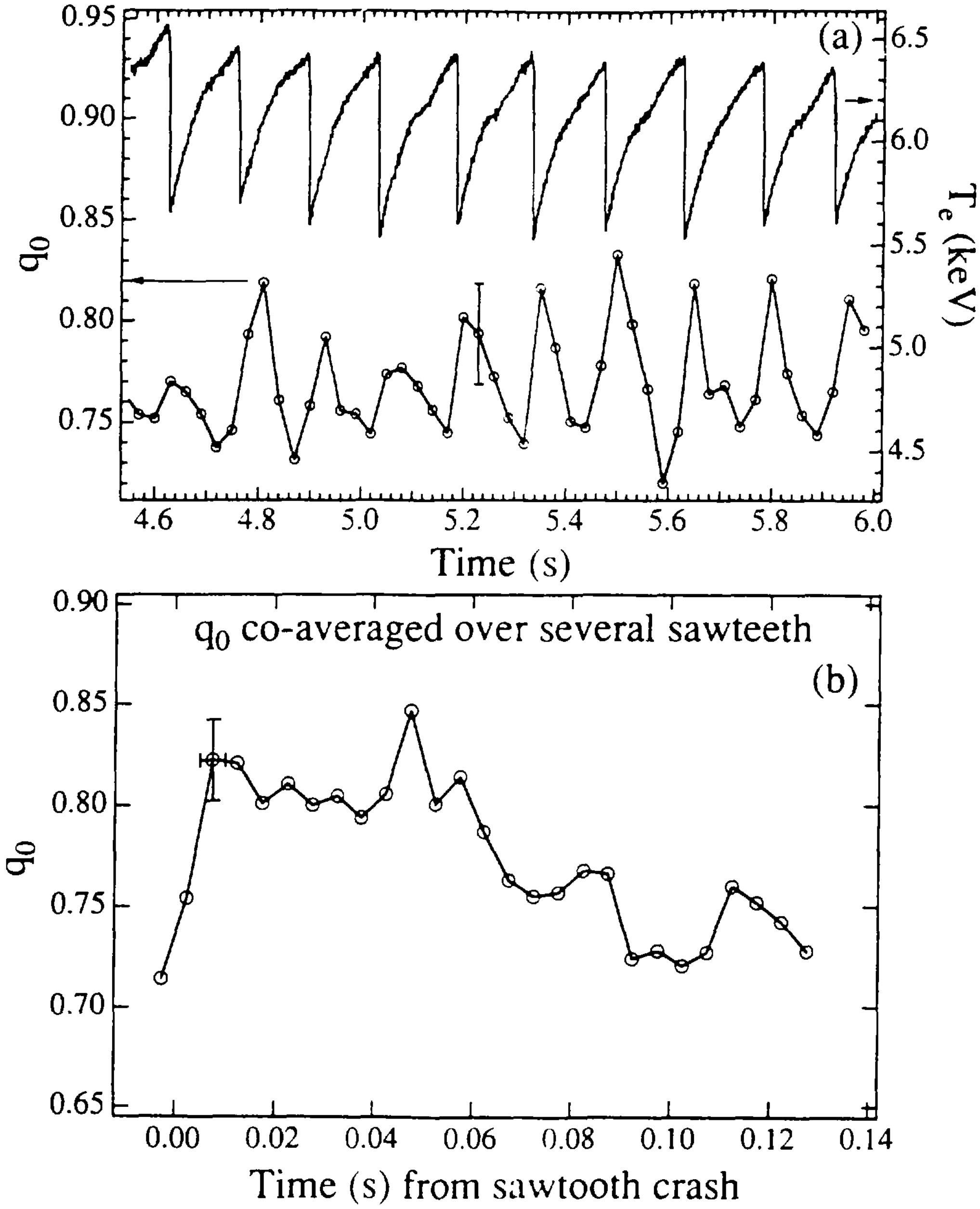
✓ Trigger problem: What causes the sudden onset from the ramp phase to the crash phase?

× "q(0)" problem: $q(0) < 1$ during crash.

Osborne et al. (1982)

Soltwisch (1988)

TFTR group (1994)



But, this is not universal:

Wroblewski et al. 1987

Weisen et al. 1989

Effect of sawteeth on a burning plasma (cf. ITER Physics Basis, Nucl. Fusion 1999)

- Redistribution of alpha particles within the plasma, although overall losses are small (Zweben et al. 1993)
- May produce seed islands triggering neoclassical tearing modes which, in turn, may affect plasma energy content (La Haye et al. 1998)/cause disruption
- Can purge accumulated core impurities and penetration of inwardly propagating impurities

In the absence of sawtoothing, density peaking and improved confinement (cf. Ödblom et al. 1996)

Effect of energetic particles on sawteeth (Porcelli et al. 1996)

- Sawteeth can be stabilized transiently by fusion alpha particles for periods that are long on the energy confinement time scale
- These periods are terminated by large crashes ("monster sawteeth") with a mixing radius larger than half of the minor radius. Can couple to neoclassical tearing, field errors, kinks, edge-localized modes
- Some current profile controlled by ECRH, ICRF appears necessary.