MHD Stability of Axisymmetric Plasmas In Closed Line Magnetic Fields

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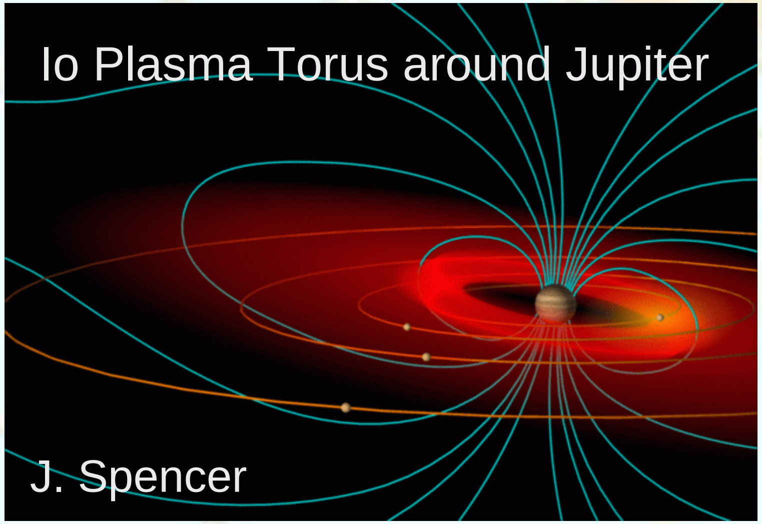
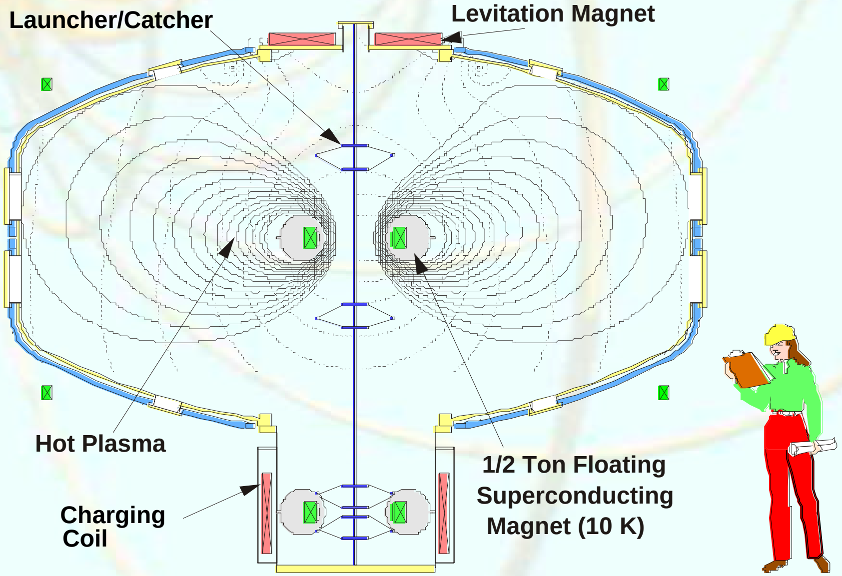
Motivation

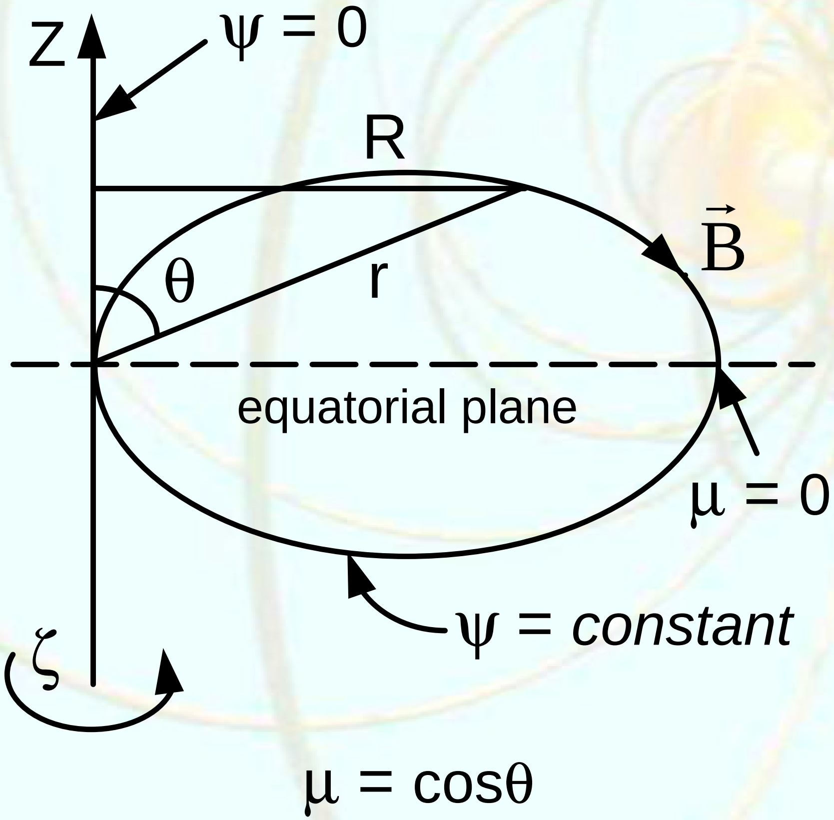
- **Astrophysical Plasmas:**
  dipolar fields of stars and planets can confine high-β plasma (β ~ 2 for the plasma in the Io plasma torus)



- **Levitated Dipole Experiment**
  - first plasma this year
  - high density regime: $N \sim 10^{13}\text{cm}^{-3}$, $T \sim 100$ eV, $B \sim 1$ kG, $R \sim 2.5$ m



Geometry : General Remarks

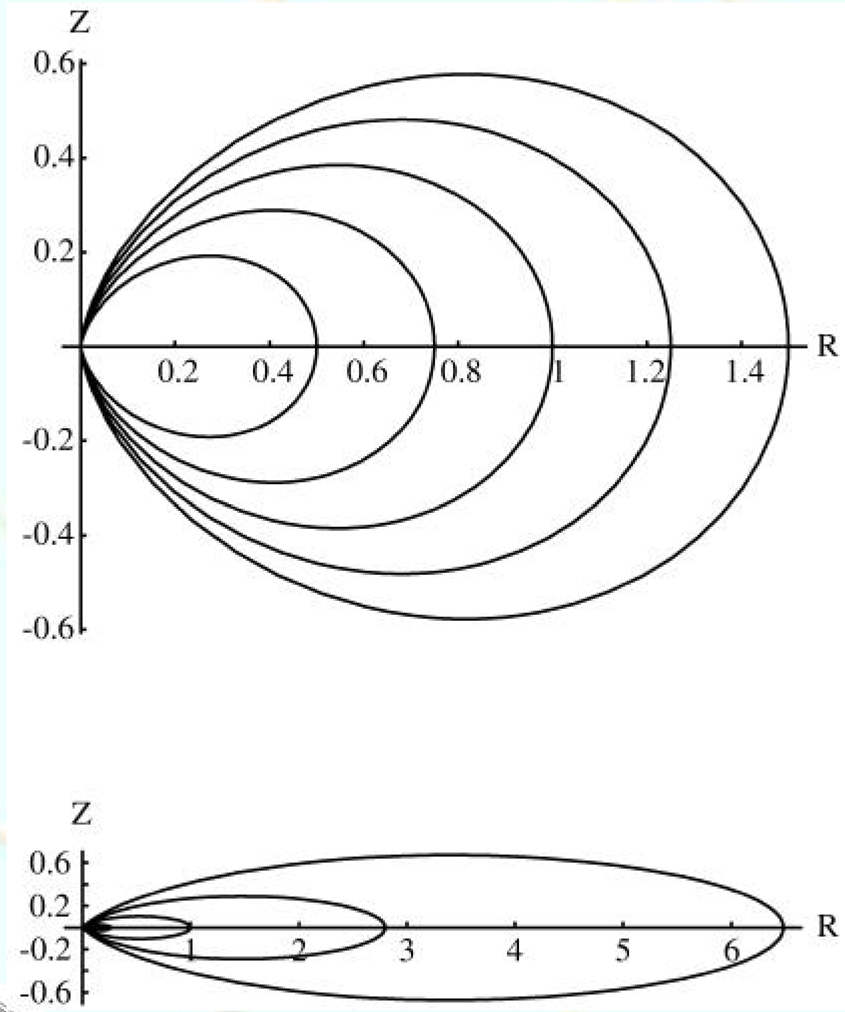


- Axial symmetry
- Closed field lines
- Flux surfaces, ψ = *constant*, are surfaces of rotation about the symmetry axis, *Z*
- Equatorial plane symmetry $\Rightarrow$ symmetric (even) and antisymmetric (odd) modes
- Toroidal equilibrium currents
- Poloidal equilibrium magnetic field $\Rightarrow J_{||} = 0 \Rightarrow$ no kinks
- Unfavorable curvature $\Rightarrow$ ballooning modes

Point Dipole Ideal MHD Equilibrium

- Model: point magnetic dipole
- Krasheninnikov *et al.* (1999) suggested looking for a separable solution to the Grad-Shafranov equation in a form $\boxed{\psi = \psi_0 h(\mu)(R/r)^{\alpha}}$ with $h(\mu)$ and $\alpha=\alpha(\beta)$ the eigenfunction and eigenvalue to be determined ($\alpha=1$ in a vacuum) by boundary conditions
- Plasma pressure is $p=p_0(\psi/\psi_0)^{2+4/\alpha}$
- 2nd order nonlinear differential equation for $h(\mu)$
- $\beta \ll 1$ limit: $1 - \alpha = (512/1001)\,\beta$
- $\beta \gg 1$ limit: $\alpha = 1 / \beta^{1/2}$

Flux Surfaces For Different b



$\beta = 0$

$\beta = 20$

- Flux surfaces for the point dipole equilibrium by Krasheninnikov *et al.*
- Flux surfaces become more and more elongated as β increases

Ideal MHD Ballooning Stability from Energy Principle

- Modes with toroidal mode number n>>1 are the most unstable
- Integro-differential ballooning equation for shear Alfvén modes:

$$\vec{B}\cdot\vec{\nabla}\left(\frac{\vec{B}\cdot\vec{\nabla}\xi_\psi}{R^2B^2}\right)+4\pi\left(2\frac{\vec{\kappa}\cdot\vec{\nabla}p}{R^2B^2}+\frac{\rho\,\omega^2}{R^2B^2}\right)\xi_\psi=16\pi\Gamma p\frac{\vec{\kappa}\cdot\vec{\nabla}\psi}{R^2B^2}\frac{\left\langle\frac{\vec{\kappa}\cdot\vec{\nabla}\psi}{R^2B^2}\xi_\psi\right\rangle}{1+4\pi\Gamma p\left\langle B^{-2}\right\rangle}$$

field line bending | pressure drive | inertia | plasma + magnetic compression

- Shear Alfvén modes are stabilized by
  - even: field line bending + compression
  - odd: field line bending only

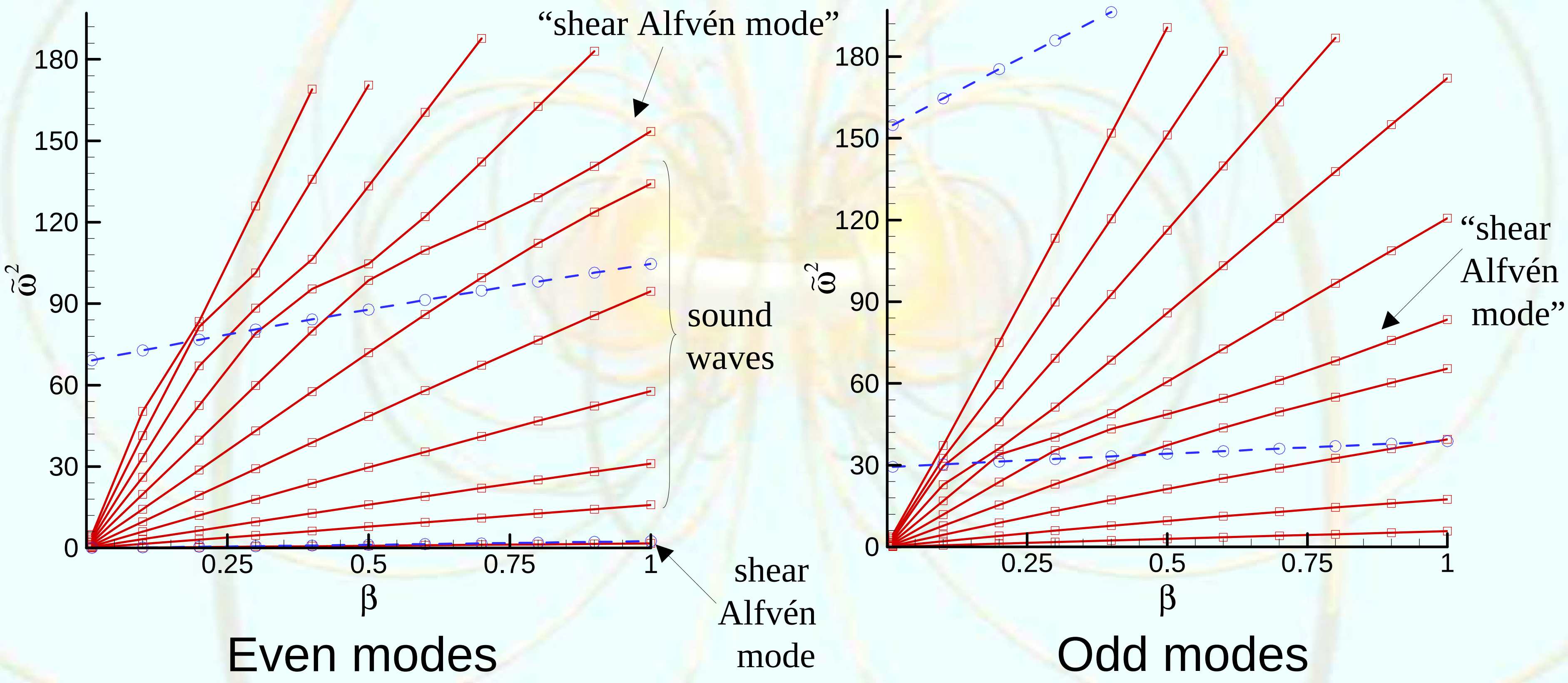
$\vec{\kappa}$ = curvature, Γ=5/3
ξ_ψ = radial displacement
$\langle\ldots\rangle$ = field line average

- For the point dipole equilibrium both even and odd shear Alfvén modes are stable for arbitrary β

Effects of Resistivity on Plasma Stability

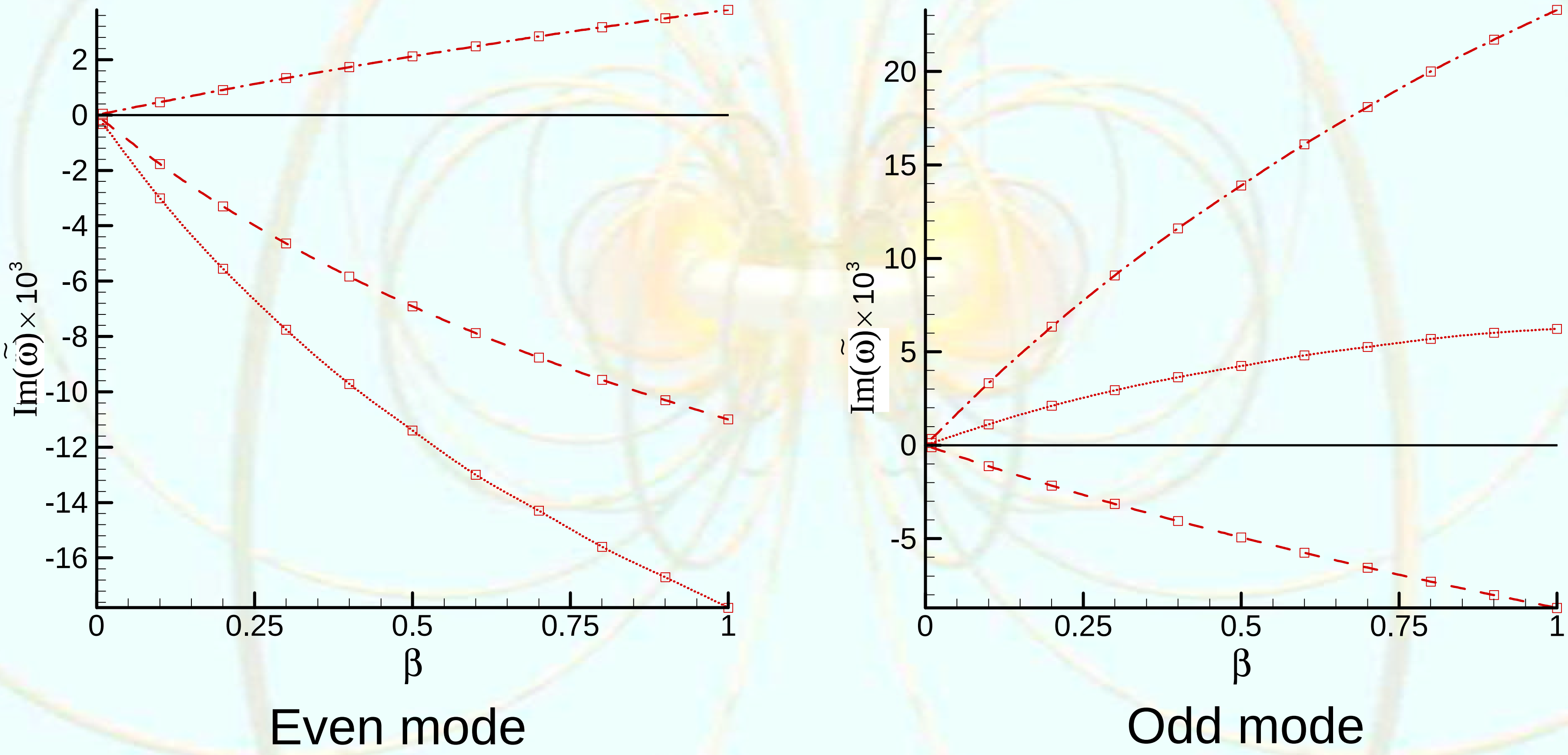
- Parallel resistivity reduces stabilizing field line bending energy by allowing plasma to slip through field lines
- This can result in two kinds of "resistive instabilities"
  - strong purely growing resistive modes at ideal odd mode stability boundaries with $\omega \sim i(\tau_A^{-2/3}\tau_R^{-1/3}) \propto i\eta_{||}^{1/3}$
  - slow growth/decay of ideally stable MHD modes away from ideal stability boundaries: $\mathrm{Im}(\omega) \sim \tau_R^{-1} \propto \eta_{||},\eta_{\perp}$
- Here, $\tau_A \equiv (4\pi\rho R^2/B^2)^{1/2}$ and $\tau_R \equiv (4\pi R^2/c^2 n^2 \eta_{||})$ are Alfvén and resistive times, respectively
- Use linear resistive MHD theory with anisotropic resistivity, $\eta_{||,\perp}=\eta_{||,\perp}(\psi)$, to study such instabilities for the point dipole equilibrium
- To do everything correctly must retain effects of sound waves

Point Dipole Equilibrium: Ideal Modes



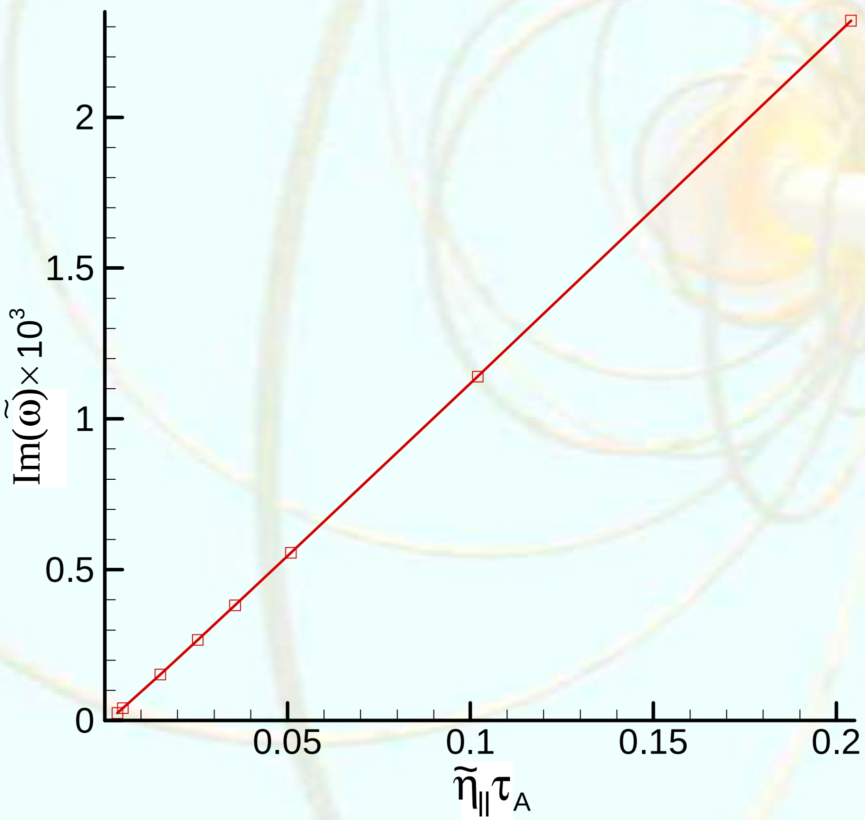
- Frequencies of shear Alfvén modes and sound waves obtained from linearized MHD equations are shown in red
- Frequencies of shear Alfvén modes obtained from the ballooning equation are shown in blue for comparison

Resistive Growth/Decay Rates for the Lowest Even and Odd Modes



- Resistivity results in slow growth or decay of ideally stable modes
- Resistive growth/decay rates due to $\eta_\perp$ only, $\eta_{||}$ only, and $\eta_\perp + \eta_{||}$, $\eta_\perp = 1.96\,\eta_{||}$, are shown in dashed, dashed-dotted and dotted lines, respectively

Scaling of Resistive Growth Rate for the Lowest Odd Mode with Resistivity



- For the lowest even and odd modes $\eta_{||}$ alone is destabilizing, $\eta_{\perp}$ is stabilizing, while the sum is destabilizing for the lowest odd mode and stabilizing for the lowest even mode
- These growth and decay rates scale linear with resistivity:

$$\mathrm{Im}(\tilde{\omega}) \propto n^2\eta_{||}, n^2\eta_{\perp}$$

Conclusions

- The stability of axially (and up-down) symmetric plasma in a closed poloidal magnetic field is investigated
- Ideal MHD energy principle is used first to derive a ballooning equation for (potentially the most unstable) shear Alfvén modes
- This equation is employed to show that the point dipole equilibrium by Krasheninnikov *et al.* is MHD stable for arbitrary β
- Next, the treatment is generalized to include effects of sound waves and plasma resistivity
- Ideally stable equilibria can be a subject to resistive instabilities with growth rates proportional to resistivity
- In particular, parallel resistivity results for the point dipole equilibrium by Krasheninnikov *et al.* in a slow resistive growth of the lowest (always ideally stable) odd mode: $\mathrm{Im}(\omega) \propto \eta$