SUSTAINMENT OF PLASMA ROTATION BY ICRF

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OUTLINE

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- **Representative Experimental Results**
- **Model Overview**
- **Angular Momentum Diffusion Equation**
- **ICRF Model and Torque Density**
- **ORBIT Code and Collision Upgrade**
- **Non-Dimensional Rotation Velocity**
- **Diamagnetic Scaling of Rotation Velocity**
- **Parameter Studies**
- **Resonant Surface Scan for Alcator C-Mod**
- **Conclusions**

BACKGROUND AND MOTIVATION

- **Alcator C-Mod and JET observe development of co-current plasma rotation in ICRF-heated discharges.**

- **ICRF heating introduces zero (or negligible) angular momentum to the plasma.**

  - **Experiments have a symmetric $k_{||}$ - spectrum and contribute no net angular momentum.**

  - **Even if the $k_{||}$ spectrum lauched is one sided, the angular momentum input is small** ($k_{||}$ = n/R ; n≈12 for C-Mod)

$$(\Delta T)_{RF} = \text{RF Torque} = M\,\Delta v_{||}\,R = n\;\Delta E/\omega$$

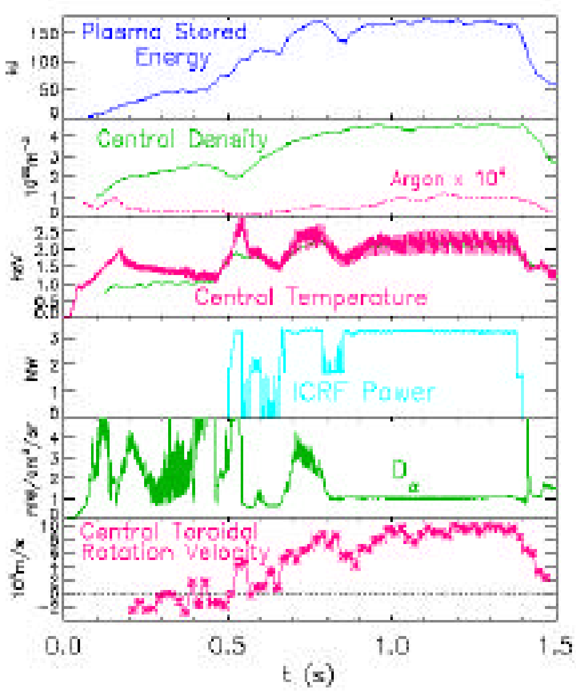
$$(\Delta T)_{NBI} = \text{typical NBI torque} = \Delta E\,R\,v_{beam}^{-1}$$

$$\frac{\Delta T_{RF}}{\Delta T_{NBI}} = \frac{n\,v_{beam}}{\omega\,R} \approx \frac{1}{15} << 1$$

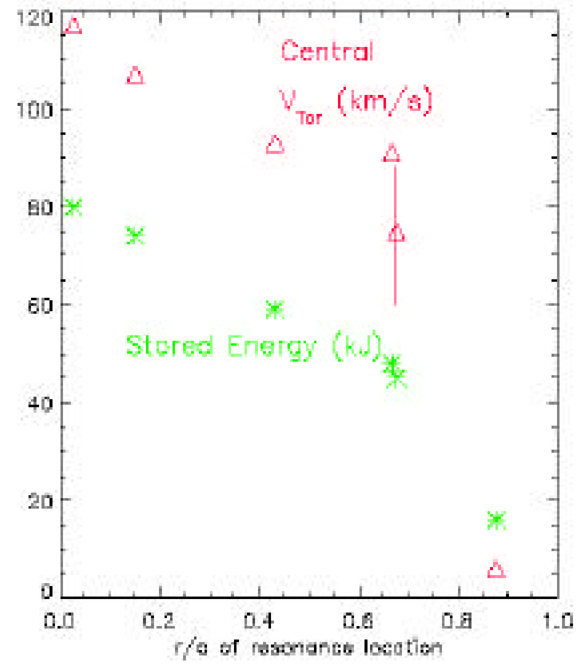
- **What is the mechanism for developing toroidal rotation and how does it scale?**

REPRESENTATIVE EXPERIMENTS

1. **Paper by J. E. Rice, et al. [*Nuclear Fusion* 39 (1999) 1175] reports rotation observations and scaling.**



5.7 T, 1.0 MA D(H) Discharge



Resonance location scan with B varying and q_{95} = 4.7

MODEL OVERVIEW - 1

1. Even though ICRF heating introduces no net torque, there remains the possibility of creating positive and negative torque density regions.

2. Describe plasma response to torque density by an angular momentum diffusion equation.

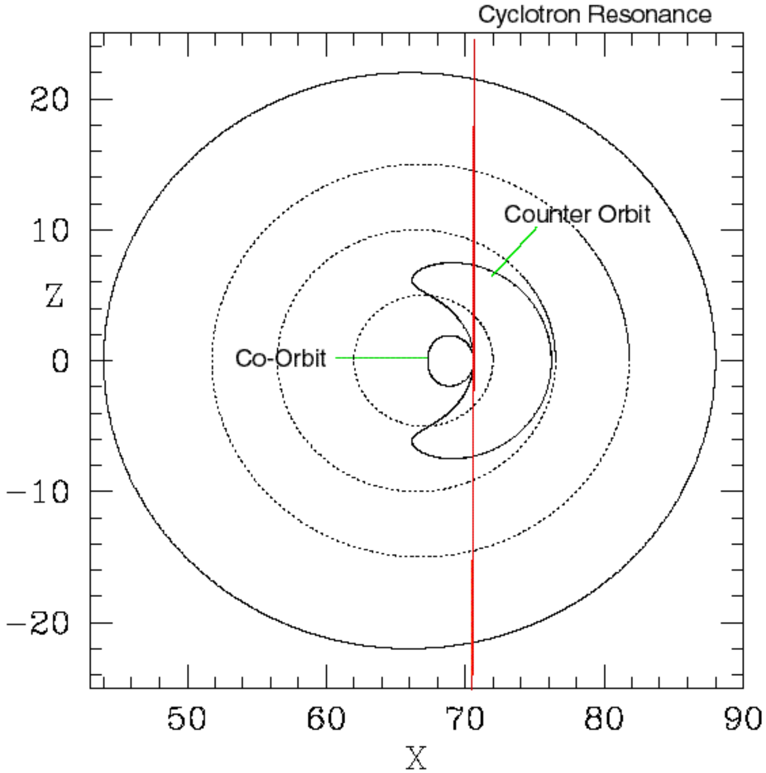
- **Separated torque density regions lead to finite central rotation**

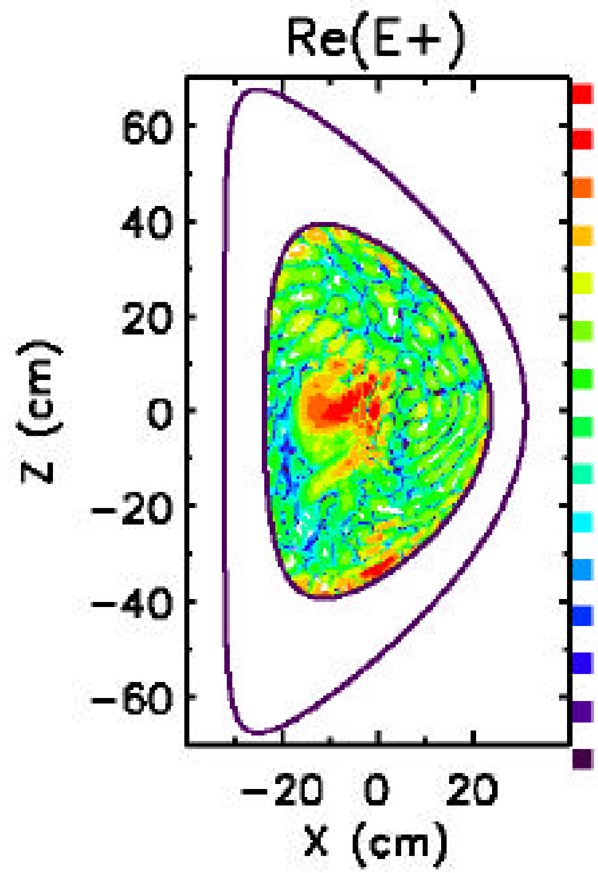
3. Model ICRF heating by the introduction of energetic particles and the removal of an equal number of cold particles.

- **Particles are introduced at a particular flux surface — the resonance location— with equal numbers of co- and counter- velocities so there is no angular momentum input.**

- **Two ICRF models detailed below.**

REPRESENTATIVE INITIAL ORBITS





(Plot by P. Bonoli)

• **Fast-wave refraction leads to midplane heating.**

MODEL OVERVIEW - 2

4. Follow particles by ORBIT with ion-ion pitch angle and drag collisions

- **Record particle's flux-surface when energy $\rightarrow$ 0.**

- **Particle's displacement from originating flux-surface drives a radial neutralizing current and a $j_r B_\theta$R torque density in the background plasma**

  - **Continuous creation of energetic particles drives steady j_r current**

- **ORBIT also computes the torque density imparted to the background by energetic-ion collisions**

  - **Total volume-integrated applied torque vanishes to $2 \cdot 10^{-4}$ accuracy.**
  - **Torque arsing from particle loss imparts counter-current rotation**

5. Compute non-vanishing central rotation from torque density

6. Investigate scaling of central rotation and sensivity to initial conditions

- **Particle energy and pitch, resonance location and q.**

ANGULAR MOMENTUM DIFFUSION EQUATION-1

1. General Form of angular rotation rate Ω response to torque density τ

$$\frac{\partial}{\partial t}\left(M\, n R^2 \Omega\right) = \nabla \cdot \left\{ n\, M\, R^2 \chi_M \;\nabla\, \Omega \right\} + \tau$$

2. Steady-state axisymmetric version

$$\frac{1}{V'} \frac{\partial}{\partial \psi} \left\{ V' \left\langle n\, M\, R^2 \chi_M \left(\quad \psi \right)^2 \right\rangle \frac{\partial \Omega}{\partial \psi} \right\} = -\left\langle \tau \right\rangle$$

$$V' = \oint \frac{d\ell\, 2\pi R}{\psi} = \frac{\partial V}{\partial \psi}$$ **and <> denotes magnetic surface average**

3. $\langle \tau \rangle$ is torque density on bulk plasma and has two sources:

- **$j_r B_\theta$ torque arising from radial curents which neutralize energetic particle displacements**

- **Collisional Angular momentum transfer from energetic particles.**

ANGULAR MOMENTUM DIFFUSION EQUATION-2

4. First integral of angular momentum equation

$$V' \left\langle n\, M\, R^2 \chi_M \left(\quad \psi \right)^2 \right\rangle \frac{\partial \Omega}{\partial \psi} = -\int_0^{\psi} \langle \tau \rangle V'\, d\psi = T(\psi)$$

- $T(\psi)$ **= torque exerted inside ψ-surface**
- **No net torque condition: $T(\psi_{max}) = 0$**

5. Apply no-slip boundary condition at surface

- **Field lines outside separatrix line-tied to vessel; toroidal rotation not permitted**

6. Torque proportional to rate of creation of energetic particles $\dot{N}$ and angular momentum transferred per particle.

$$\langle \tau \rangle \quad \dot{N}$$

ANGULAR MOMENTUM DIFFUSION EQUATION-3

7. Angular rotation rate (use toroidal flux Φ as independent variable)

$$\Omega(\Phi) = \int_{\Phi}^{\Phi_{max}} \frac{d\Phi}{q\,V'} \frac{T(\Phi)}{\left\langle n\,M\,R^2\,\chi_M\,\left(\quad\psi\right)^2\right\rangle}$$

8. Conclude:

For regions of separated postive and negative torque density,
T(Φ) is non-zero and toroidal rotation can develop, even though the total torque $T(\Phi_{max})=0$

ION-CYCLOTRON HEATING: MODEL 1

1. The ion-cyclotron heating process changes a particle's prependicular energy, while leaving $v_{||}$ and the canonical angular momentum unchanged.

- **No net angular momentum is introduced**

2. Our ICRF models replace cold particles by energetic particles constrained to have an equal number of co- and counter velocity particles.

- **ICRF Model 1 creates particles at interesection between midplane and cyclotron resonance surface. Pitch is low: $v_{||}/v = (0.25 - 0.40)$**

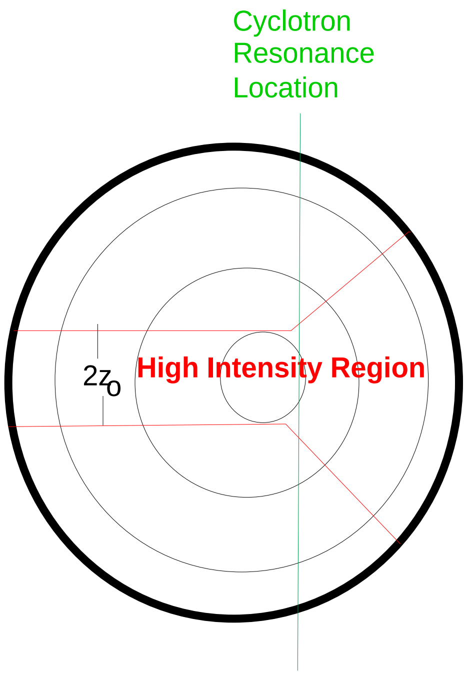
  Mimics ICRF heating for particles with orbits tangent to the resonant surface at the midplane where wave intensity is high.

3. Energetic particles then spatially diffuse via banana diffusion and collisionally transfer angular momemtum to the bulk plasma.

- **ORBIT code follows these processes via a Monte Carlo approach.**

ICRF MODEL 2

- **Creates particles at banana tips ($v_{\parallel} = 0$) distributed along cyclotron resonance surface in region of high wave intensity: $-z_{max} \le z \le z_{max}$**



- **Distribution with z weights midplane creation:**

$$\frac{dN}{dz} = \frac{\left(z_{max}^2 - z^2\right)}{z_{max}^2\left(2z_{max}^2 - z^2\right)^{1/2}}$$

- **Again, ORBIT code follows subsequent spatial diffusion and collisional angular momentum transfer.**

ORBIT CODE

1. ORBIT code has been developed to follow energetic particle orbits in toroidal confinement geometries of arbitrary cross section.

2. Hamiltonian formalism developed

- **Rigorous Hamiltonian form found:**
  - **R. B. White and M.S. Chance, Phys. Fluids 27, 2455 (1984)**
  - **R. B. White, Phys. Fluids B2, 845 (1990)**

$$dP_\zeta/dt = -\partial H/\partial\zeta \qquad d\zeta/dt = \partial H/\partial P_\zeta$$

$$dP_\theta/dt = -\partial H/\partial\theta \qquad d\theta/dt = \partial H/\partial P_\theta$$

3. Monte- Carlo collisions after A. Boozer et al. Phys. Fluids 24, 851 (1981)

4. Collision model: Energetic proton ion-ion collisions with cold deuterons.

$$\frac{d\langle\theta^2\rangle}{dt} = \nu_o\left(\frac{E_o}{E}\right)^{3/2} \qquad \frac{1}{E}\frac{dE}{dt} = -\nu_o\left(\frac{E_o}{E}\right)^{3/2}\frac{M_{proton}}{M_{deuteron}} \qquad \nu_o = \frac{2^{3/2}\pi n e^4 \ln\Lambda}{(M_p)^{1/2} E_o^{3/2}}$$

ORBIT CODE MODIFICATIONS

- Plasma divided into $5 \cdot 10^4$ bins in toroidal flux (magnetic surface label)

- For each time step, momentum transfer from particles to plasma through pitch angle scattering and drag recorded in each bin.

- Final particle momentum and density recorded in each bin

- Integrals over toroidal flux (bins) needed for angular rotation performed

- Angular momentum check accurate to 1 part in 5000.

NONDIMENSIONAL CENTRAL ROTATION RATE-1

1. Let Φ_o denote the toroidal flux value where energetic particles of energy E are introduced at a rate $\dot{N}$.

2. Let $v = \left(2E/M\right)^{1/2} \left(\Omega_a R_a\right)^{-1}$ denote a nondimensional particle speed.

3. ORBIT computes $F(\Phi)$ = fraction of particles ending up inside flux surface Φ and the integral T_1 of the $j_r \times B_\theta$ torque.

- **Φ_o is flux surface of creation for Model 1.**

$$T_1(\Phi) = \frac{1}{v} \int_0^{\Phi} \frac{d\Phi'}{q} \; G(\Phi') \qquad\qquad G(\Phi) = \begin{cases} F(\Phi) & \Phi \le \Phi_o \\ F(\Phi) - 1 & \Phi \ge \Phi_o \end{cases}$$

NONDIMENSIONAL CENTRAL ROTATION RATE-2

4. For Model 2, with a distribution of initial flux surfaces, the generalization is:

$$F = \frac{1}{N}\sum_{k=1}^{N} F_k \qquad\qquad G = \frac{1}{N}\sum_{k=1}^{N} G_k$$

where F_k, G_k **can be expressed in terms of the Heavyside function** Θ

$$F_k = \Theta\left(\Phi - \Phi_k\right) \qquad\qquad G_k = \Theta\left(\Phi - \Phi_k\right)\Theta\left(\Phi_{k_o} - \Phi\right) - \Theta\left(\Phi_k - \Phi\right)\Theta\left(\Phi - \Phi_{k_o}\right)$$

- Φ_{ko} **denotes the creation flux surface for the** k^{th} **Monte Carlo particle**

4. ORBIT also calculates v $T_2(\Phi)$ **= mechanical angular momentum deposited inside** Φ.

NONDIMENSIONAL CENTRAL ROTATION RATE-3

5. Standard circular tokamak formulas, an assumed constant momentum diffusivity $\chi_M = a^2/6\tau_M$, and $\dot{N} E\tau_E = P\tau_E = W$ are employed to calculate the central rotation frequency

6. On-axis rotation rate is expressed in terms of the nondimensional rotation rate I*

$$\frac{\Omega(0)}{\dot{N}} = v^2 I^* \qquad I^* = \frac{1}{v}\int_0^{\Phi_{max}} \frac{d\Phi}{\Phi} T \qquad T = T_1 + T_2$$

$$v = \left(2E / M\right)^{1/2} \left(R_a \omega_{c,a}\right)^{-1}$$

7. Analytic considerations motivate the introduction of v so that I* is insenstive to physics parameters

ROTATION IN PHYSICAL UNITS

1. Select baseline initial particle values used in computing I^* to be representative of Alcator C-Mod. Employ ICRF Model 1

- E=48 keV, pitch = 0.25, rho = 0.165, low-field midplane, and N=2000.
- Result:

$$I^* = 22.5$$

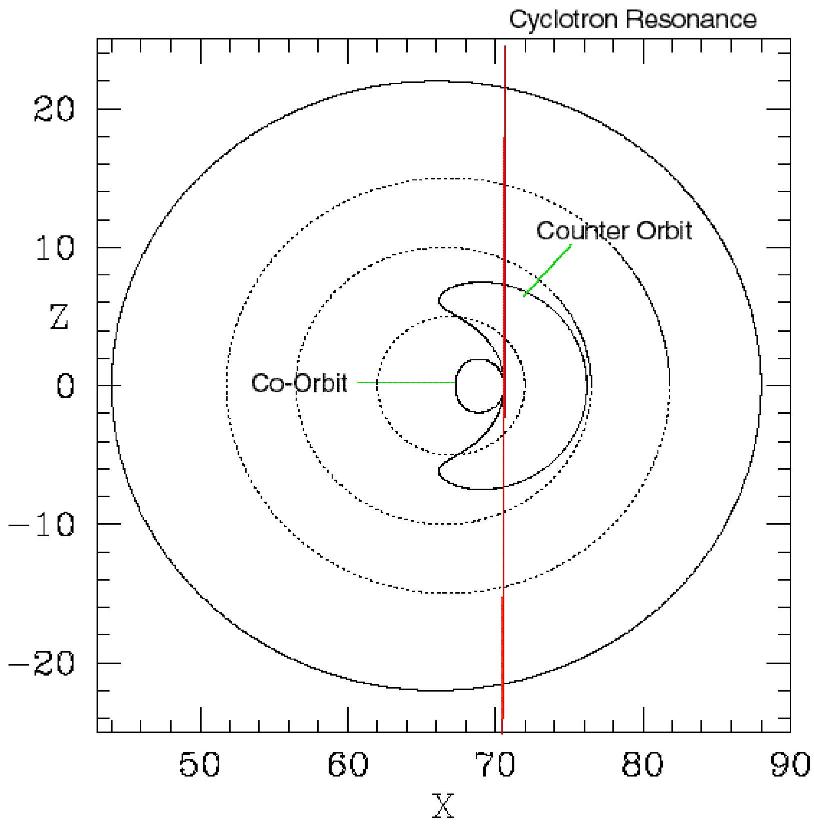
2. In physical variables the rotation rate is

$$\Omega(0) = \left\{ \frac{6\,W}{e\,B_a\,R_a^3\,a^2\,\bar{n}\,(2\pi)^2} \left(\frac{\tau_M}{\tau_E} \right) \right\} I^*$$

For the shot on sheet 4, this gives $v_{tor} = \Omega(0)\,R_a = 7 \cdot 10^4$ m/s, in good accord with the reported value. Results insensitive to E, pitch, and N.

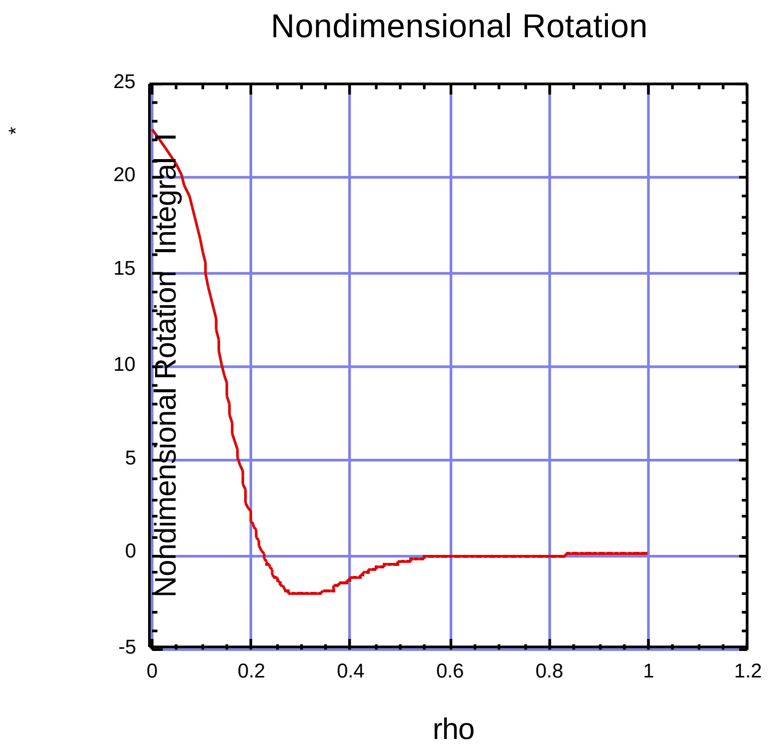
INITIAL ORBITS

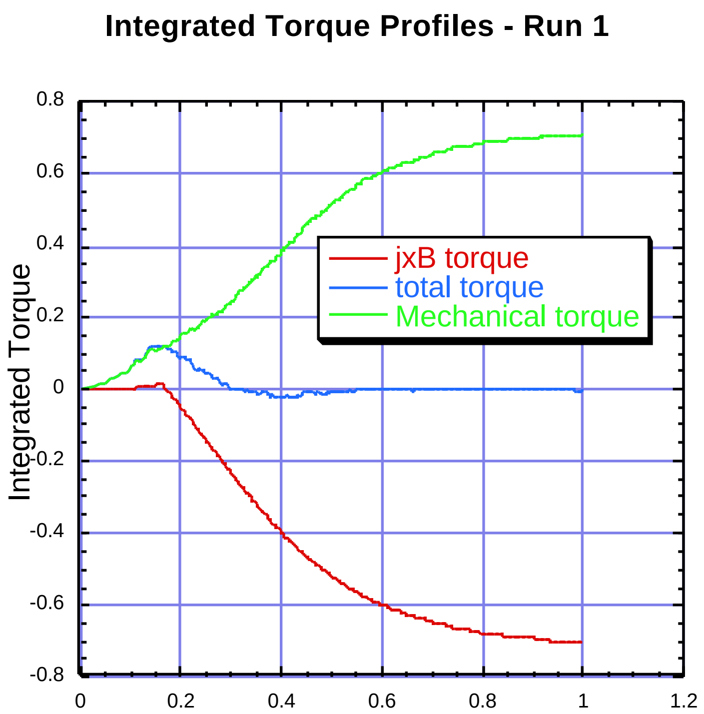
1. Initial orbits are characteristic of orbits near the magnetic axis



ROTATION AND TORQUE PROFILES - RUN 1

• **Rotation Profile is peaked.**



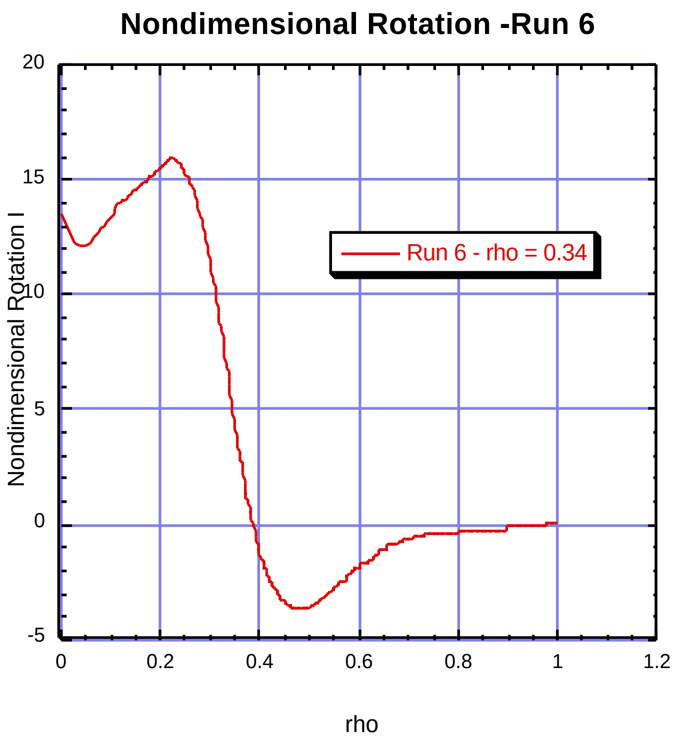


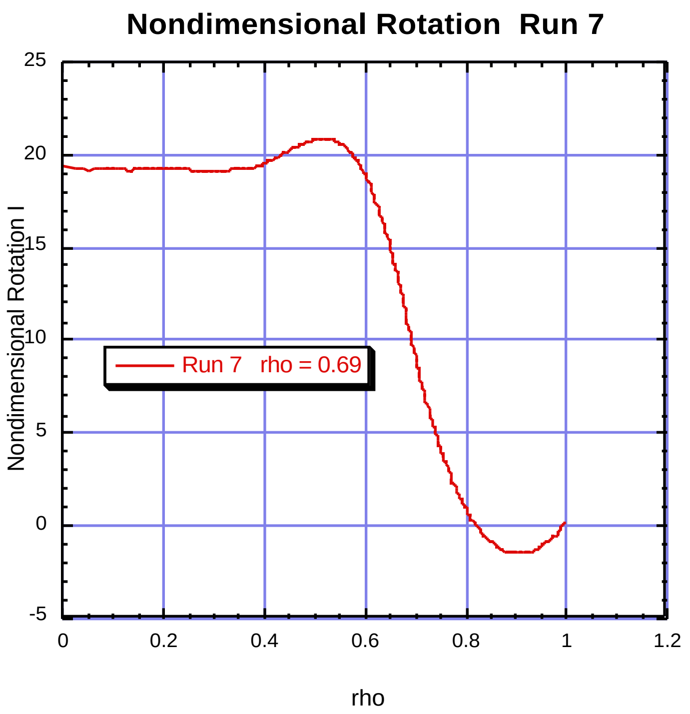
SENSIVITY STUDIES

• **How much does the central rotation change as E, rho, pitch, q-profile, and initial surface (ICRF resonance surface) vary ? Results expressed as I*.**

Run	Objective (N=500)	I*	rho	E (keV)	pitch	q_{max}	resonance
1	Baseline (N=2000)	22.5	0.165	48	0.25	4.0	LFS
1.1	Baseline (N=200)	24.9	0.165	48	0.25	4.0	LFS
2	Pitch variation	28.3	0.165	48	0.35	4.0	LFS
3	Energy dependence	24.6	0.165	24	0.34	4.0	LFS
4	HFS vs LFS (run 1)	-18.6	0.165	48	0.25	4.0	HFS
5	q_{max}	17.5	0.165	48	0.25	8.0	LFS
6	initial rho	13.5	0.34	48	0.5	4.0	LFS
7	initial rho	19.3	0.69	48	0.64	4.0	LFS
8	Banana vs Circulating (run 6)	11.4	0.34	48	0.32	4.0	LFS
9	HFS vs LFS (run 7)	-22	0.69	48	0.64	4.0	HFS
10	On axis	7.3	0.0	48	0.35	4.0	On-axis
11	On axis - pitch	-1.9	0.0	48	0.25	4.0	On-axis

ROTATION CURVES vs INITIAL RHO





- **As resonance layer is moved outward, rotation profile broadens.**

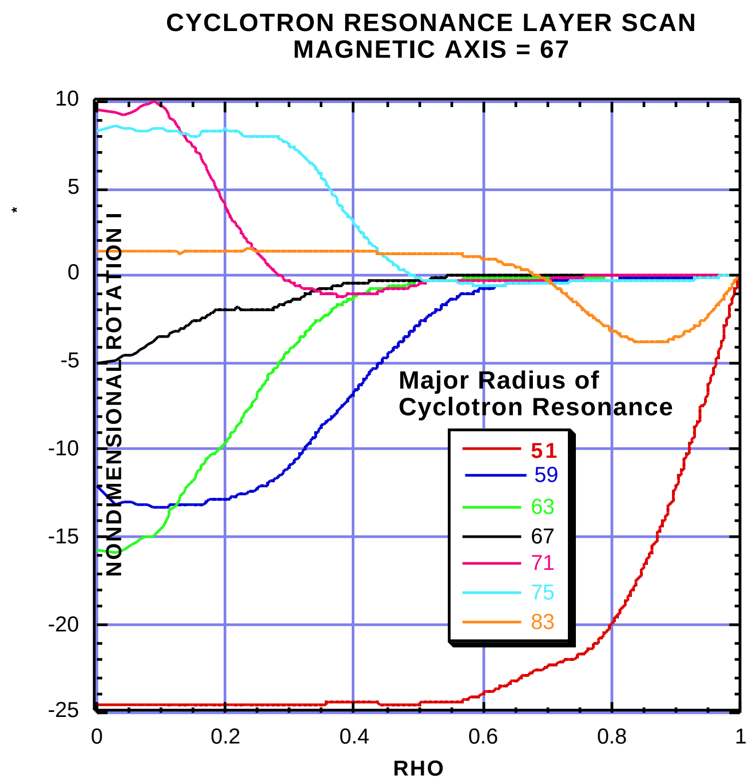
- **Possibilty of positional control of velocity shear layer via ICRF frequency**

  - If shear layer width scales with ρ_θ, layer will affect microinstabilities

  - For C-Mod: $\omega \approx 2.5 \quad 10^6 \text{ rad/s} \approx \left(2T/M\right)^{1/2} a^{-1}$

ICRF RESONANT LAYER SCAN FOR C-MOD

• **Use ICRF Model 2; calculate nondimensional rotation profile I***

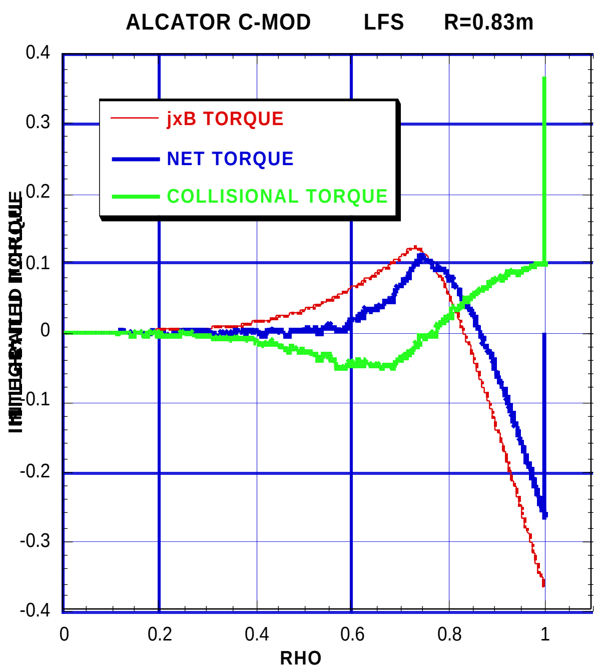


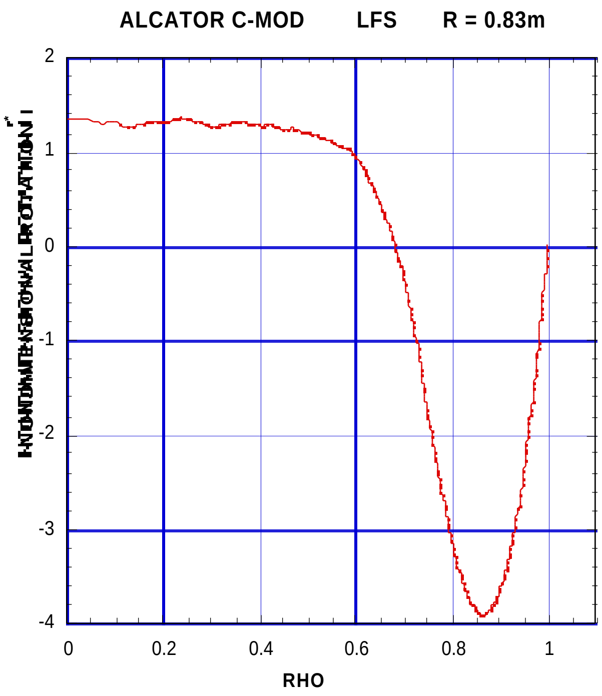
• **When the resonant surface is on the high-field side, Counter-rotation is predicted**

ORBIT LOSS TORQUE

When cyclotron resonance layer is close to the surface, direct orbit loss gives counter rotation

Example: Alcator C-Mod with resonance at 0.83 m and 80 percent loss





SPATIALLY DEPENDENT DIFFUSIVITIES

1. Assume diffusivity scales with q: $\chi = \chi_o q^n$

2. Change relationship between a, χ_o, and τ_E

- **For uniform diffusivity** $\chi_o = a^2 / 6\tau_E$ **(Taken from J_0 Bessel Function)**
- **For diffusivity profile** $\chi_o = a^2 / C_n(q)\ \tau_E$
- **For n=2,**

$$C_n(q) \approx q^2 + 2q + 3$$

$$v_{tor}(0) = \left\{ \frac{\left(q^2 + 2q + 3\right) W}{e\, B_a\, R_a^{\,2}\, a^2\, n\, (2\pi)^2} \left(\frac{\tau_M}{\tau_E}\right) \right\} I^*$$

- **Provides for increase of rotational velocity with decreasing current.**

- **For resonance close to axis, I* depends only weakly on q.**

SUMMARY - 1

1. **Separated regions of positive and negative torque density can generate central rotation**

    - **General property of a diffusion equation**

2. **ICRF generates two types of torque density on bulk plasma which are comparable in magnitude and integrate to zero net torque**

    - $j_r \times B_\theta$ **and mechanical angular momentum transfer by collisions**

3. **ORBIT code follows individual particles and computes the torque densities**

    - **ICRF model (initial condition for ORBIT) replaces a cold particle by an energetic particle.**

    - **Equal numbers of co- and counter energetic particles assure not net momentum injection. Angular momentum check to $2 \cdot 10^{-4}$ level.**

SUMMARY - 2

4. Central rotation arises

- **Co-current sense, magnitude, and scaling in accord with Alcator C-Mod**

  - $v_{exp}(0) = 10.0 \cdot 10^4$ m/s $\quad\quad v_{model}(0) = 7 \cdot 10^4$ m/s

- **Insensitve to particle energy, pitch, q_{max} , N, and initial ρ.**

- **High-field-side initial ρ gives counter-current rotation.**

5. Summary formula

$$v_{tor}(0) = \left\{ \frac{6\,W}{e\,B_a\,R_a^{\,2}\,a^2\,n\,(2\pi)^2} \left(\frac{\tau_M}{\tau_E} \right) \right\} I^*$$

$I^* = 10\text{-}20$

CONCLUSIONS

- **A mechanism to create central rotation in tokamaks with ICRF heating has been indentified.**

- **Toroidal velocity scales diamagnetically**

    - Magnitude and sense in accord with C-Mod data.

- **Precise treatment of angular momentum needed and provided by ORBIT code**

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- A mechanism to create central rotation in tokamaks with ICRF heating has been identified.

- Toroidal velocity scales diamagnetically

  - Magnitude and sense in accord with C-Mod data.

- Precise treatment of angular momentum needed and provided by ORBIT code

- Provides means to control rotational profile.