The basic question of wall times

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Thanks to B. Davis (pppl IDL master) !!!!!!

Outline:

1) Equations for wall currents & eigenvalue analysis
2) Examine simplest problem a rectangular plate
3) Examine standard DIII-D vacuum vessel
4) Field penetrating a wall, compare models with many vs. 1 time constant.
5) Conclusions & recommendations

Columbia University

$$\nabla \cdot \vec{B} = 0, \quad \vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{E} = -\dot{\vec{B}}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{E} = \eta \vec{J}, \quad \nabla \cdot \vec{J} = 0$$

Equations for wall times

Many people stop at $\frac{\eta}{\mu_0} \nabla^2 \vec{B} = -\frac{\partial \vec{B}}{\partial t}$

Write equations in terms of currents,
Use the standard definitions & assumption

$$\int_{volume} \vec{w} \cdot \left(\eta \vec{J} + \dot{\vec{A}} + \nabla \phi \right) dv = -\int_{volume} \vec{w} \cdot \dot{\vec{A}}_{external} \, dv$$

$$\vec{J}(\vec{r},t) = \sum_{\substack{\text{all} \\ \text{elements}}} I_k(t) \vec{w}_k(\vec{r})$$

Where the w_k are shape functions (closed loops of current)
This gives the standard set of familiar circuit equations

Circuit equations are a set of simultaneous o.d.e.

$$[L]_{NxN}\{\dot{I}\}_{Nx1} + [R]_{NxN}\{I\}_{Nx1} = \{V\}_{Nx1}$$

With eigenvalues (time constants τ_k) & eigenvectors$\{\xi_k\}$
We may express any answer in terms of the eigenvectors

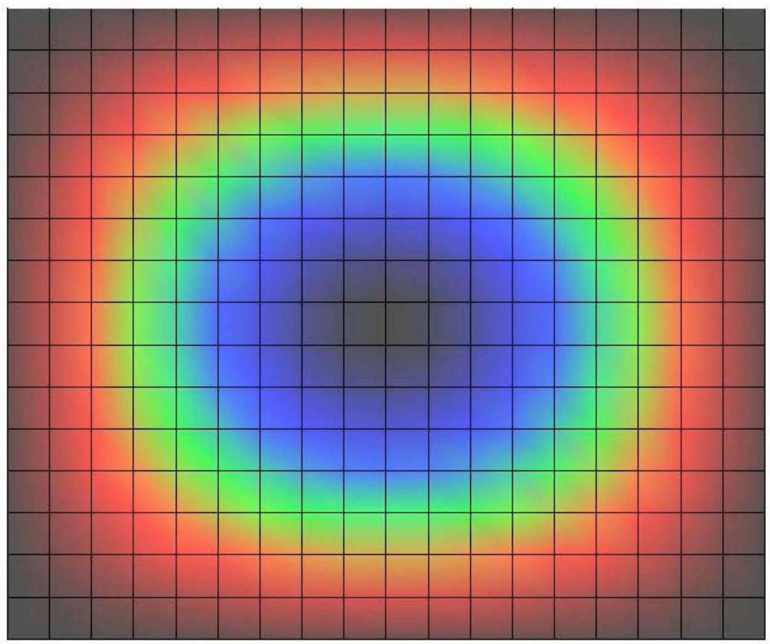
$$\{I(t)\}_{Nx1} = \sum_{k=1}^{N}\{\xi_k\}_{Nx1}c_k(t) = \left[\{\xi_1\}_{Nx1} \quad \ldots \quad \{\xi_N\}_{Nx1}\right]_{NxN}\{c(t)\}_{Nx1}$$

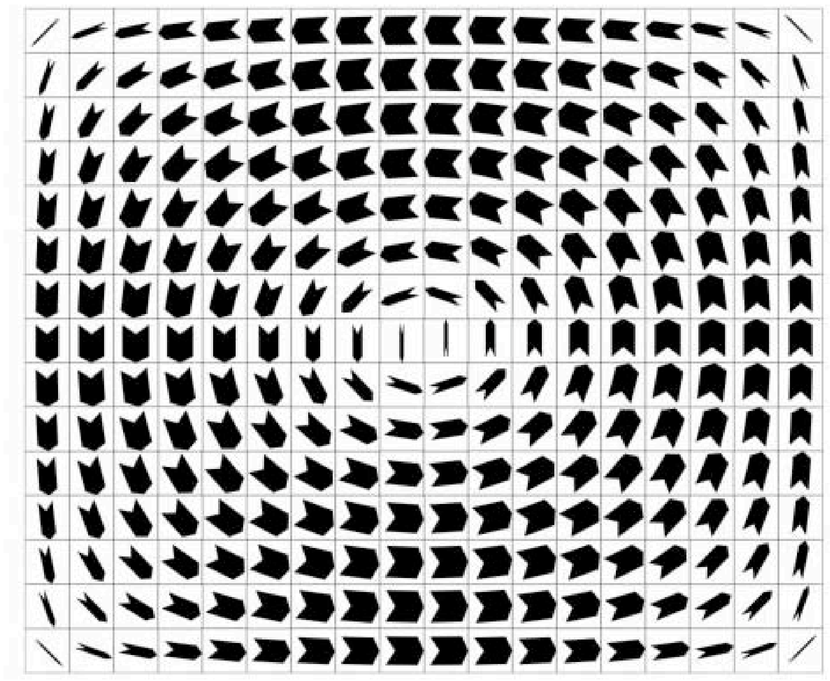
$$\{I(t)\} = [\Psi]\{c(t)\}, \quad \{c(t)\} = [\Psi]^{-1}\{I(t)\}$$

We can find the most important modes by looking at the largest values of the vector {c(t)}. We may reconstruct the result with a subset of modes

We examine the modes & time constants of a thin plate
1.8 x 1.5 x 0.01 [m] with resistivity = 130.e-08 [ohm m]
In this model we have 270 equations / modes

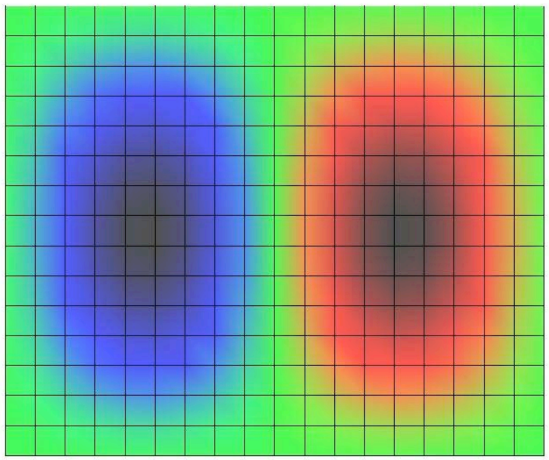
Stream function graphics & Eddy current plots

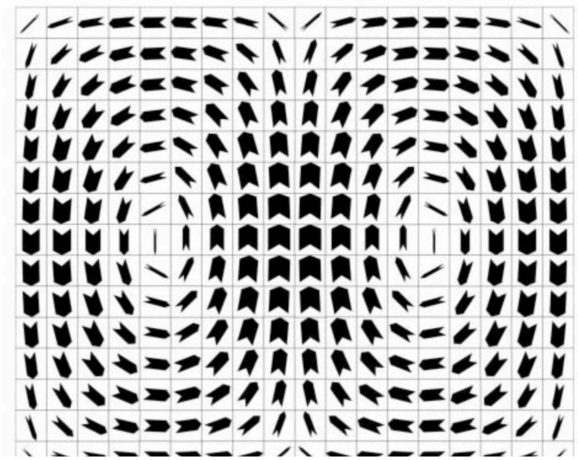




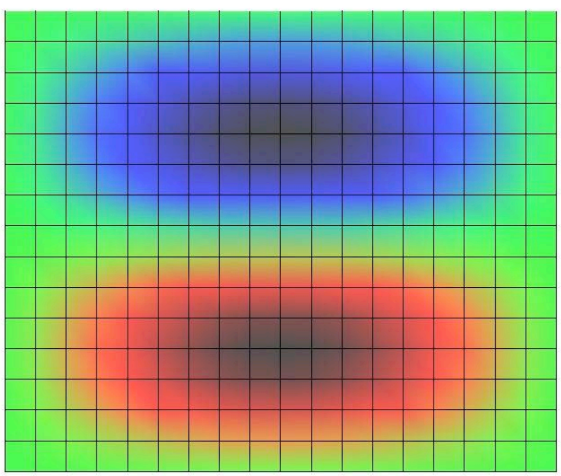
The slowest mode (shown above) is #270
Time constant = τ_{270}=1.521e-3 s

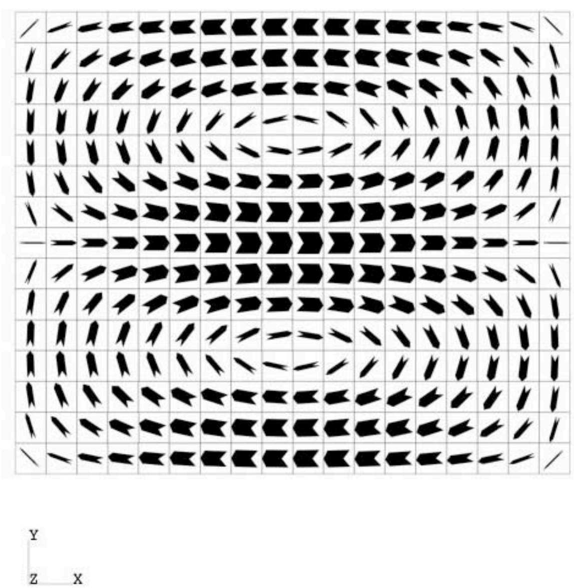
The following illustrates the slowest modes (in order)





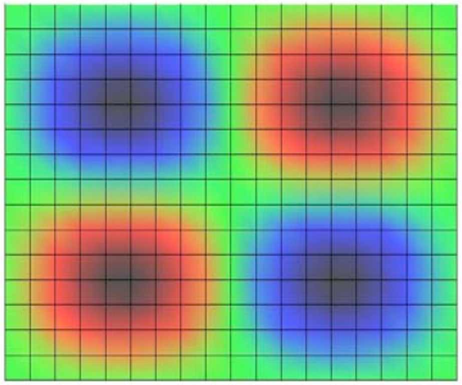
τ_{260}=1.082e-3 s
mode #269



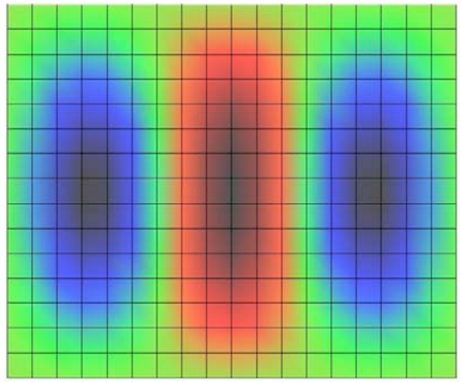


τ_{268}=9.675e-4s
mode #268

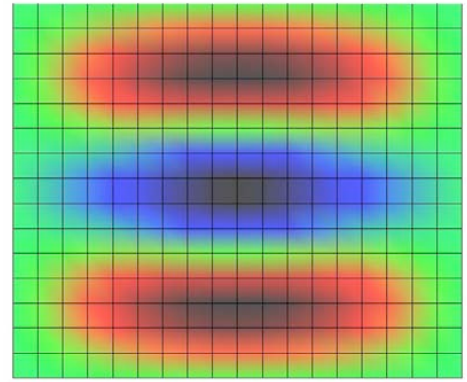
The following illustrates the slowest modes (in order)



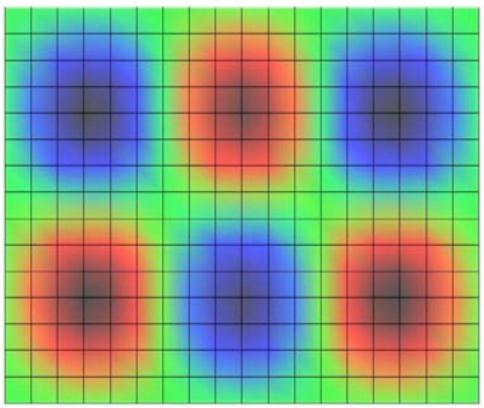
8.185e-4 s
mode #267



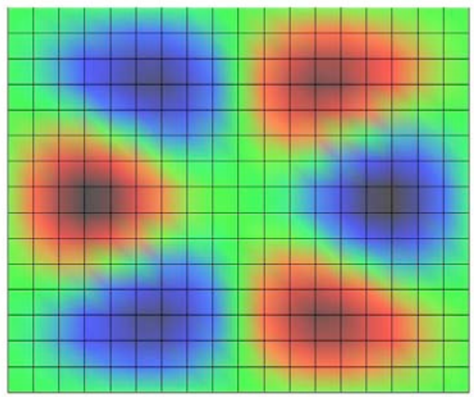
7.904e-4 s
mode #266



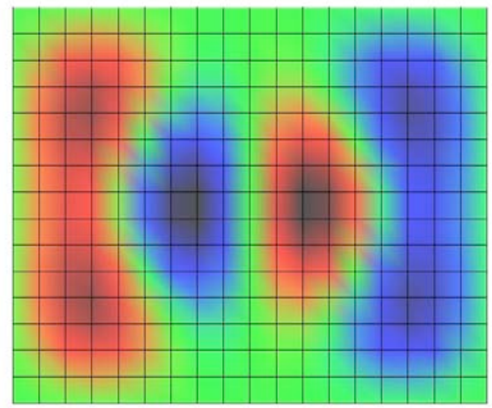
6.765e-4 s
mode #265



6.689e-4
mode #264

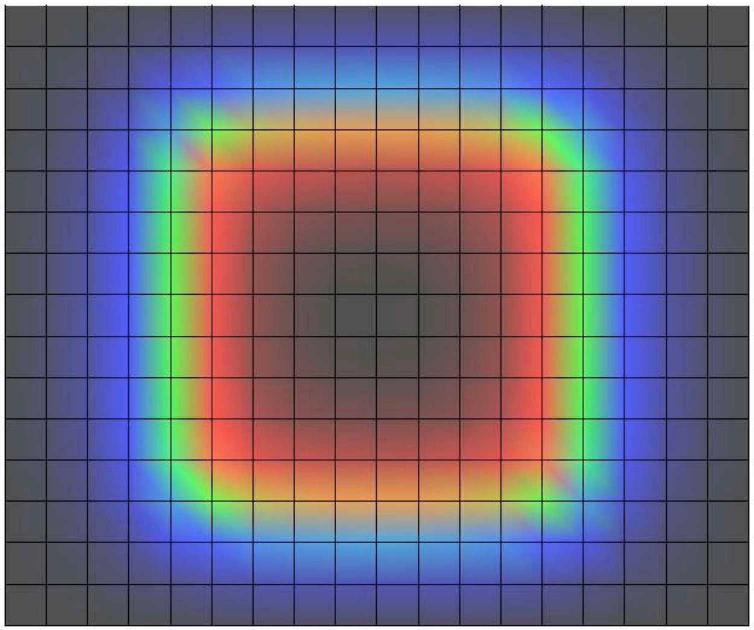


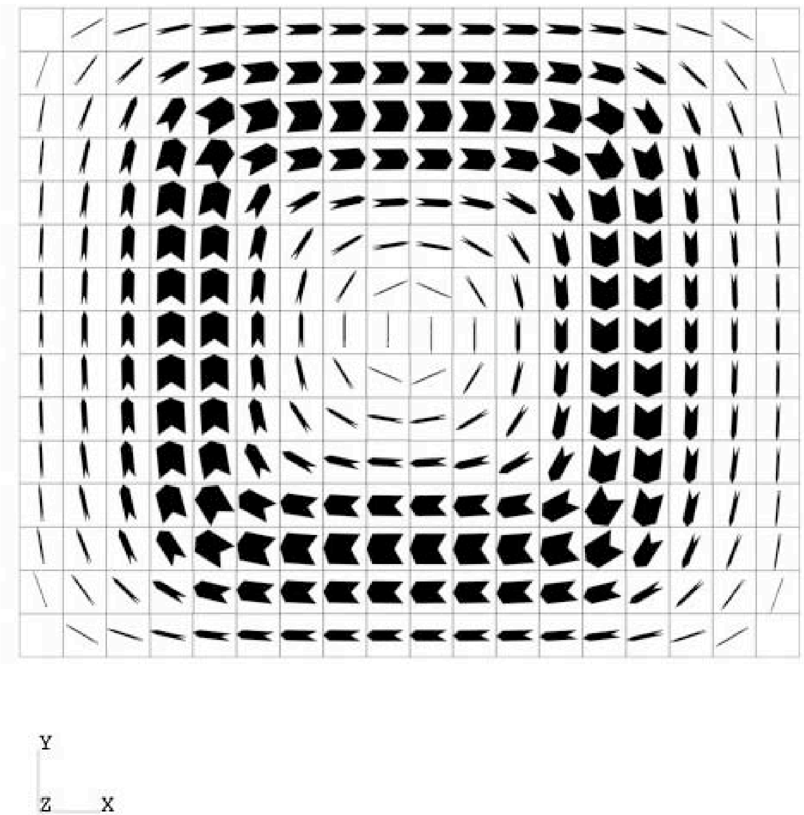
6.227e-4 s
mode #263



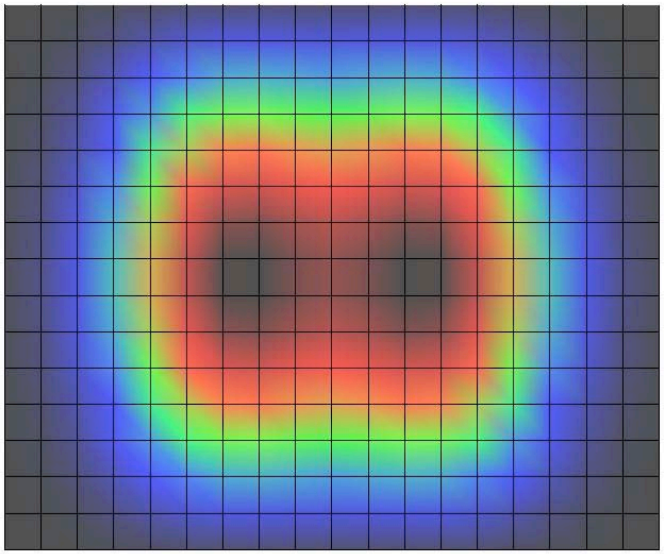
6.136e-4 s
mode #262

Examine fast time scale current response in plate produced by current step in square (1x1 [m]) coil. Coil is 0.1 [m] from plate

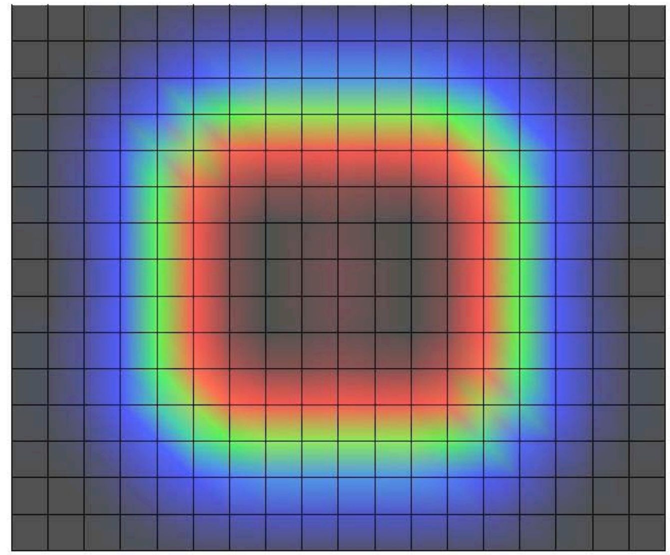


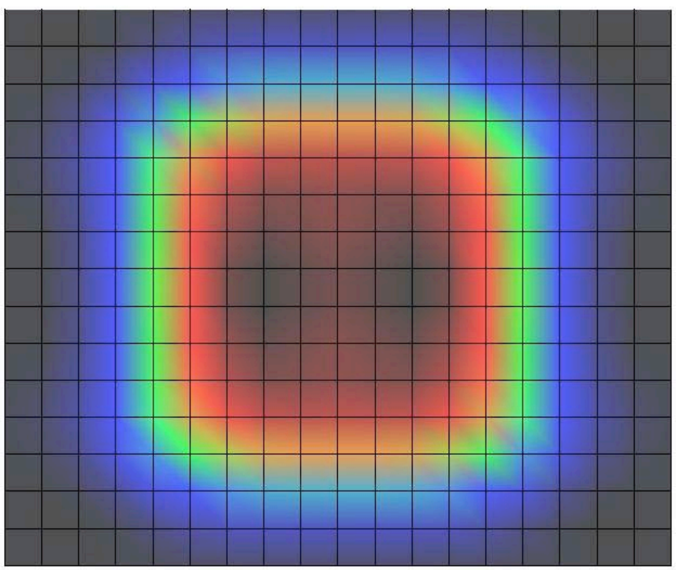


best 2 mode approximation

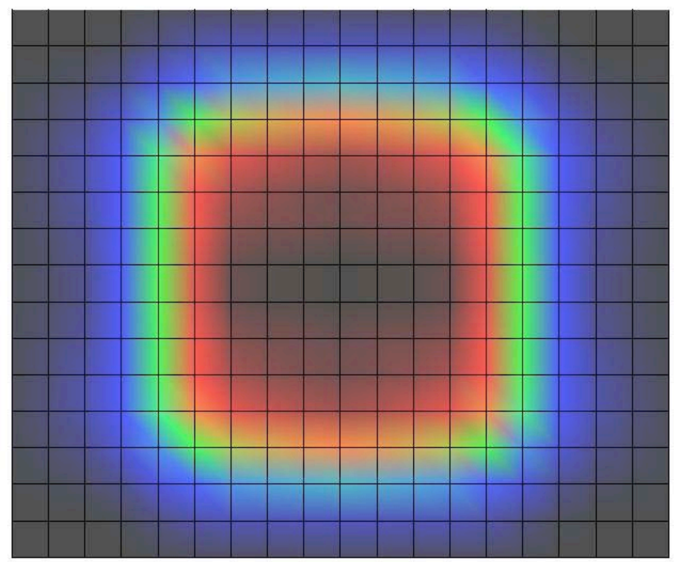


best 4 modes



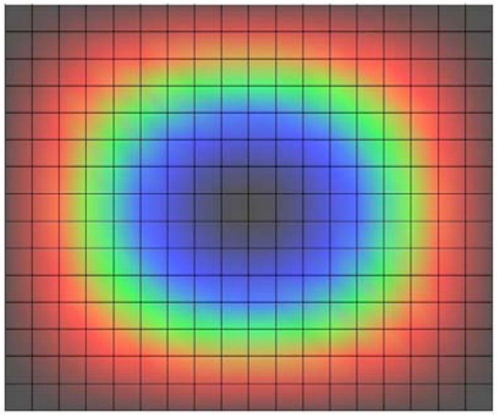


best 6 modes

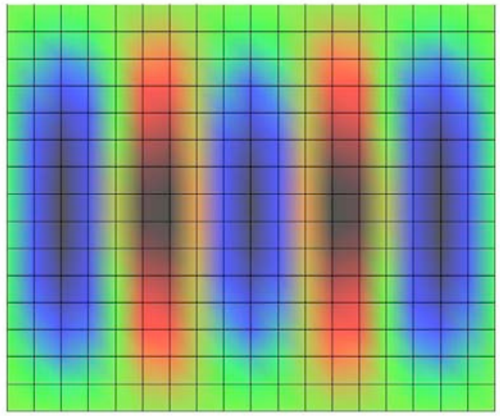


best 8 mode approximation

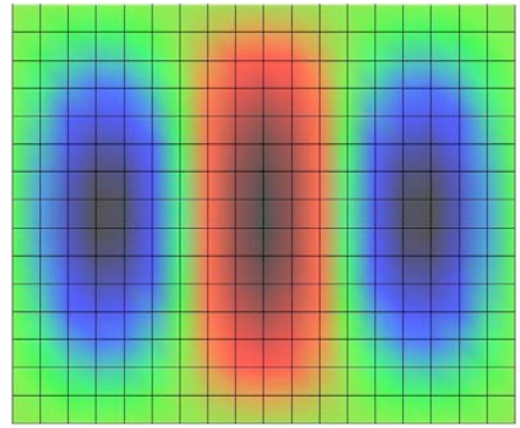
Most important (highest c_k or ‘weight’) shown below



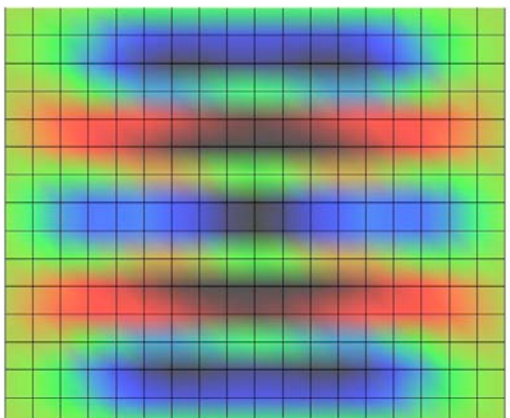
mode #270
wt=-0.852e-2
0.152e-2 s



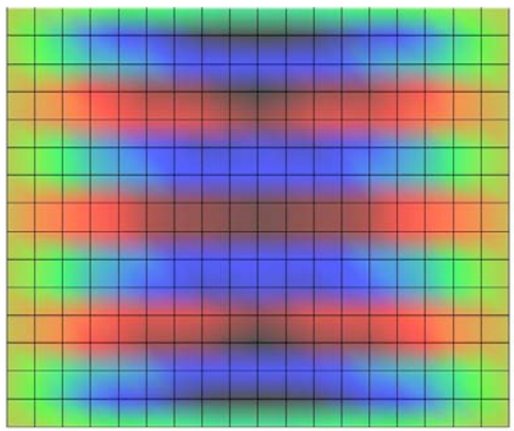
mode #258
wt = 0.2058e-2
0.498e-3 s



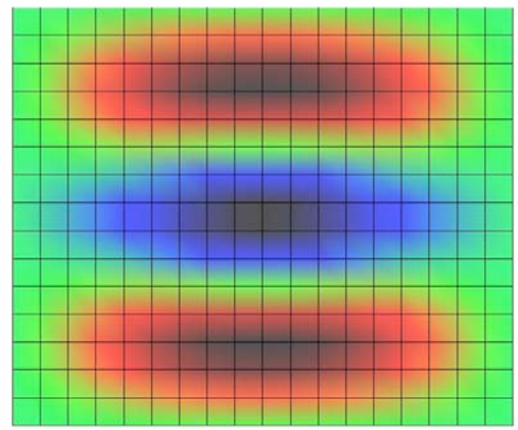
mode #266
wt = 0.130e-2
0.790e-3 s



mode #252
wt = 0.122e-2
0.417e-3 s

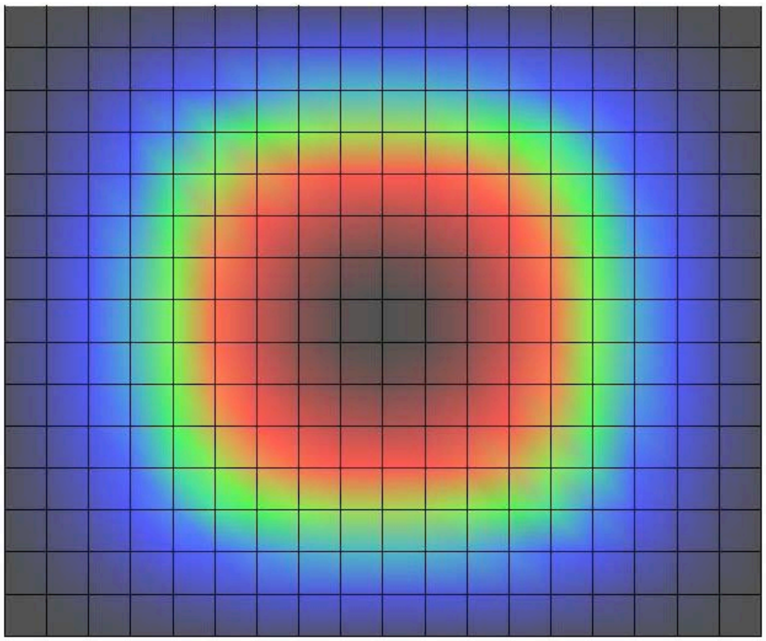


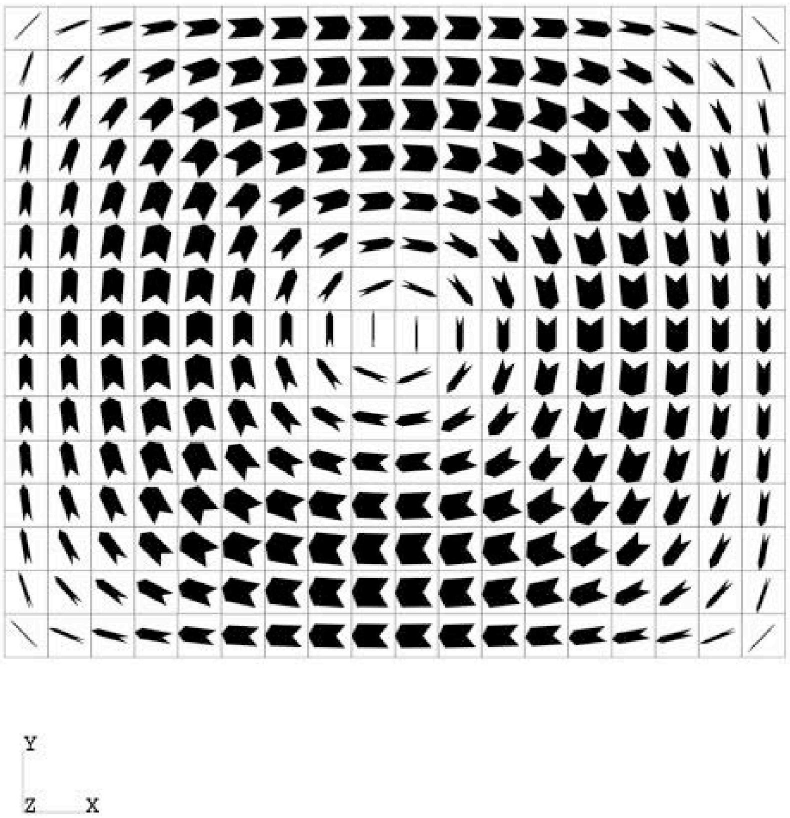
mode #232
wt = 0.978e-3
0.298e-3 s



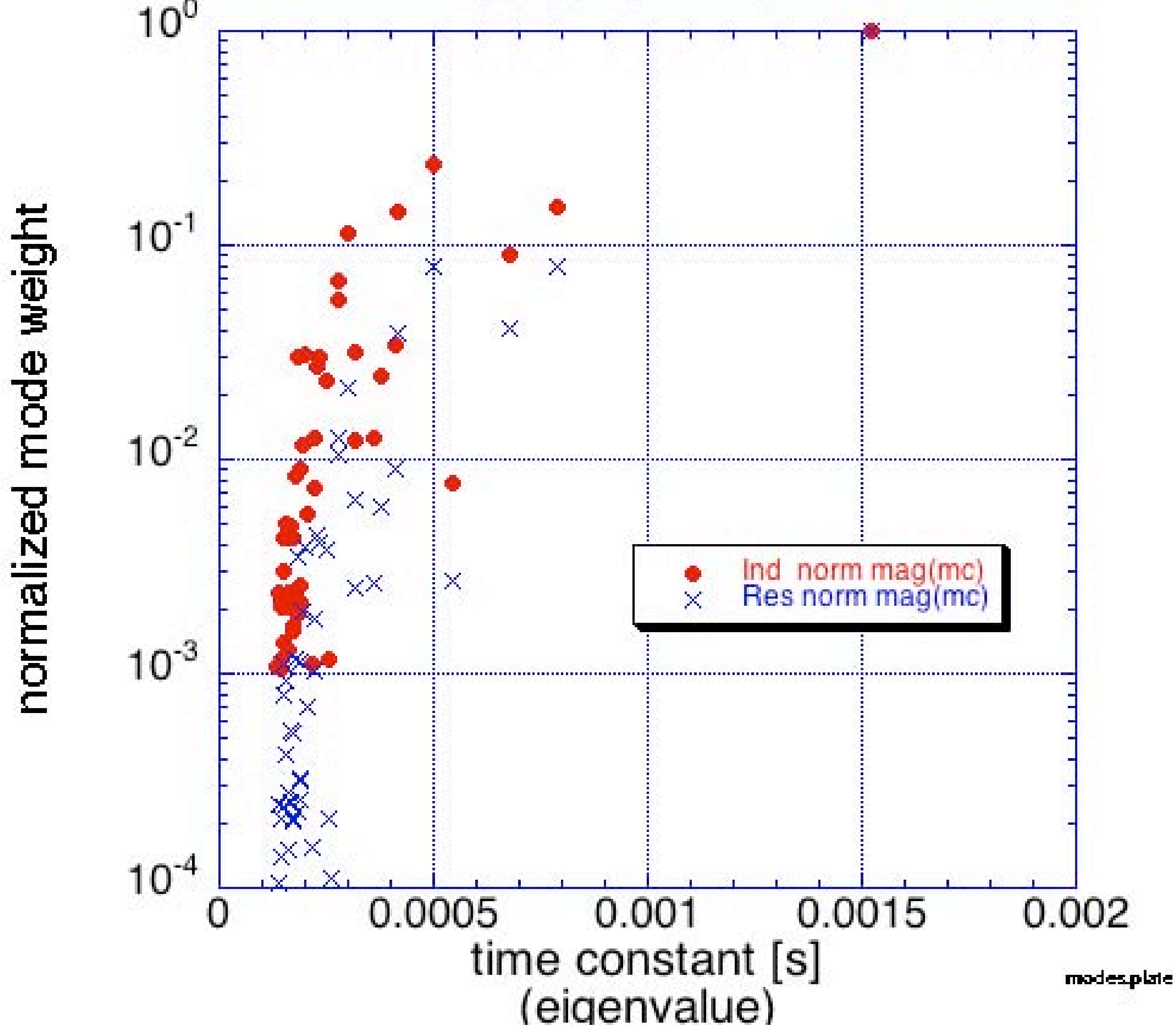
mode #265
wt = 0.765e-3
0.676e-3 s

Examine steady state (resistive response) in same plate



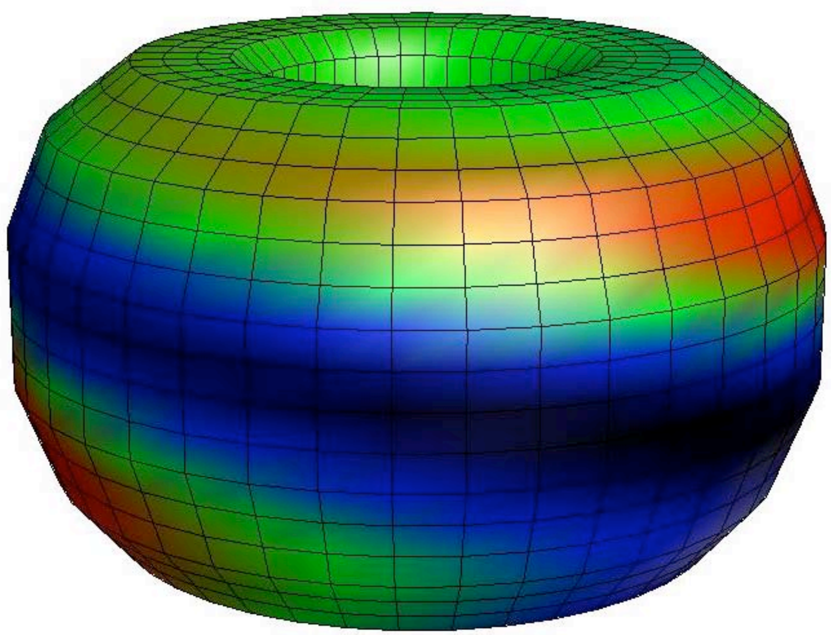


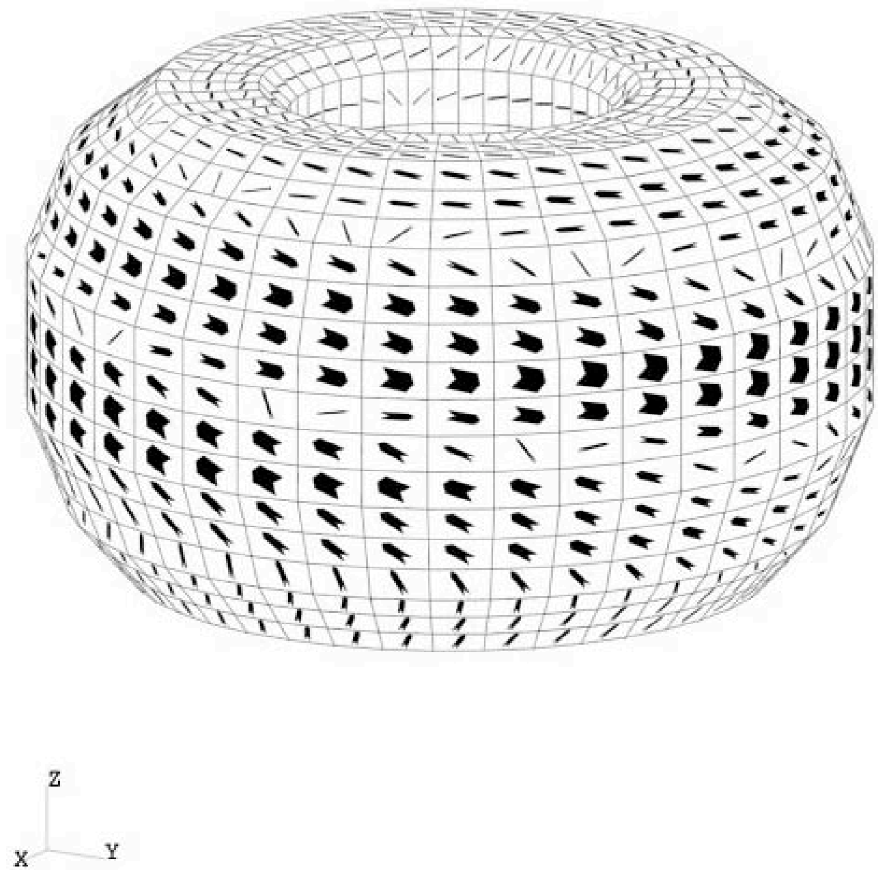
50 largest mode weights
for flat rectangular plate
inductive limit
resistive limit



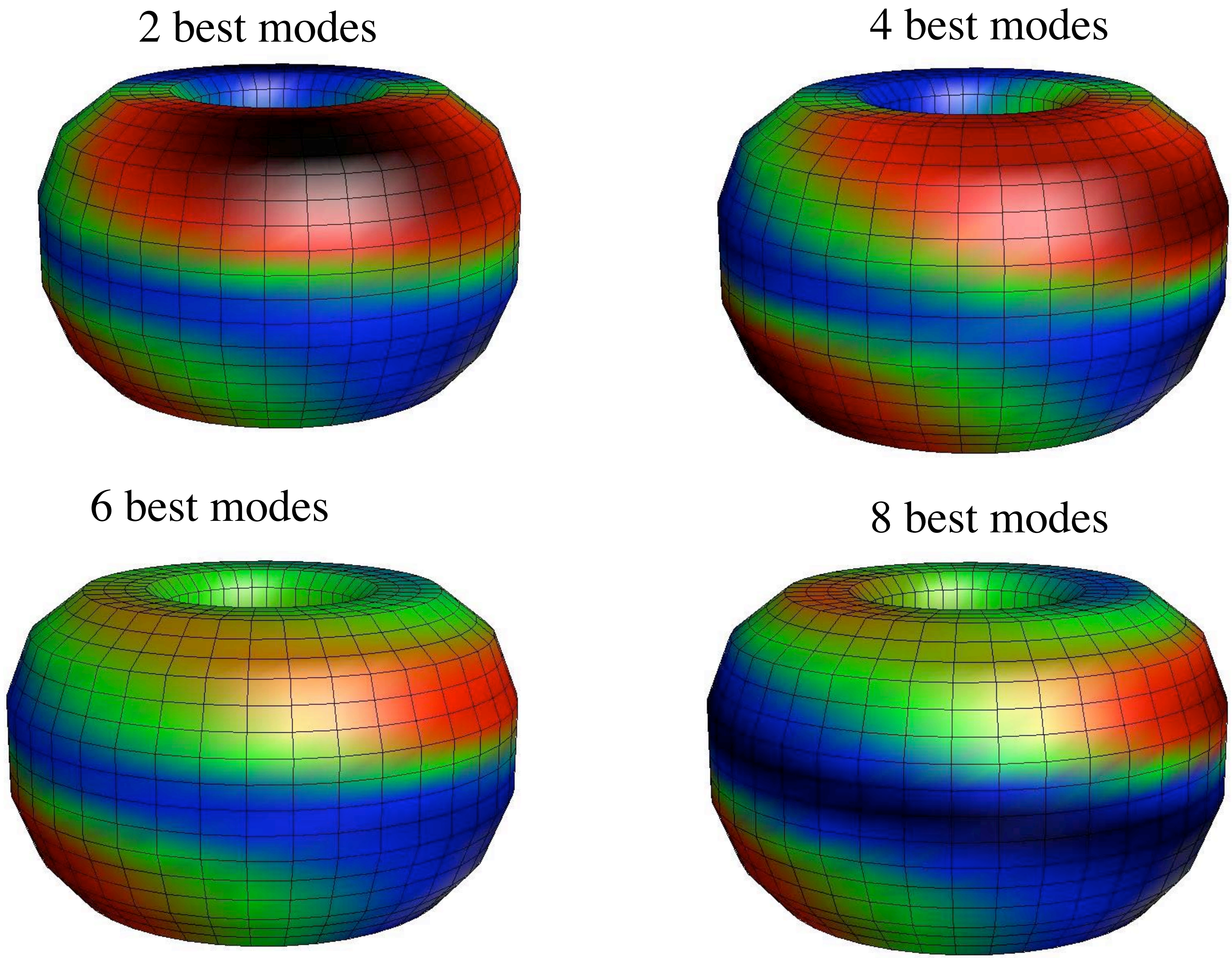
Examine fast time scale response in standard DIII-D model

(thick ‘belly band’, remainder thin, constant resistivity)
Using B_n from A.Turnbull (GATO) analysis of shot #92544



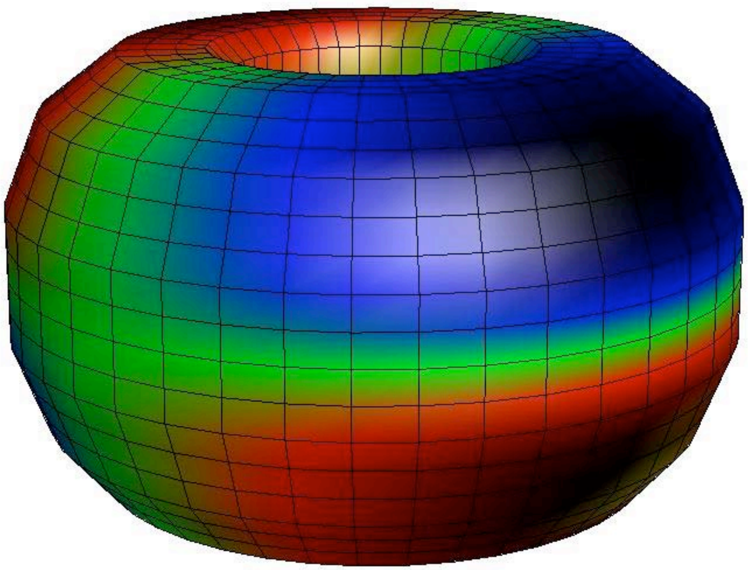


1281 equations/modes

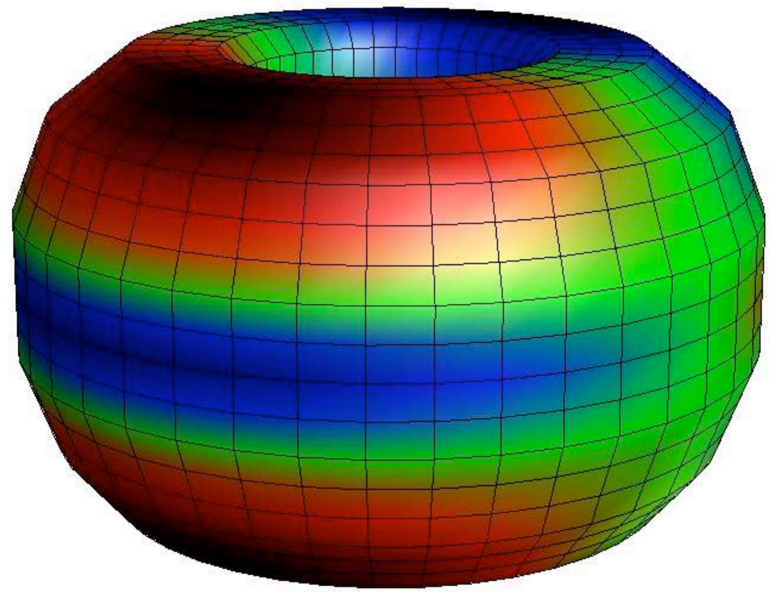

2 best modes
4 best modes
6 best modes
8 best modes

Most important modes (greatest weights) follow:

We never see a helical mode !!!!!

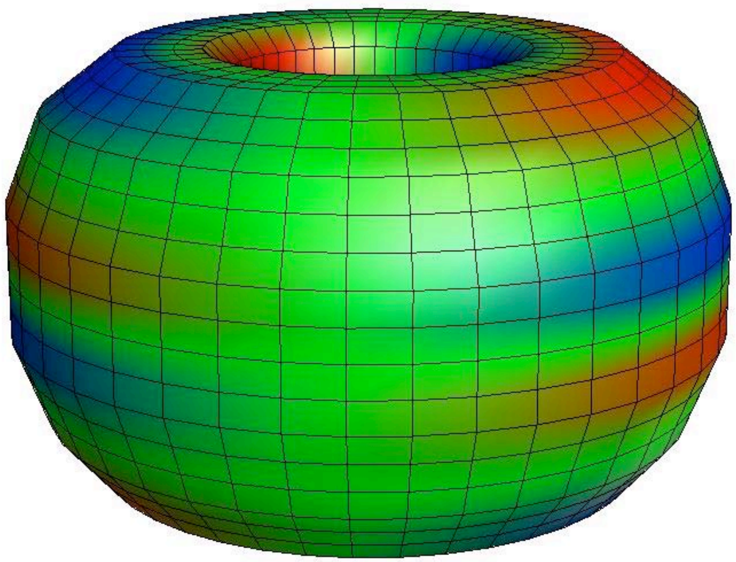


mode # 1278
wt = -0.108e-1
0.555e-2 s
greatest contribution

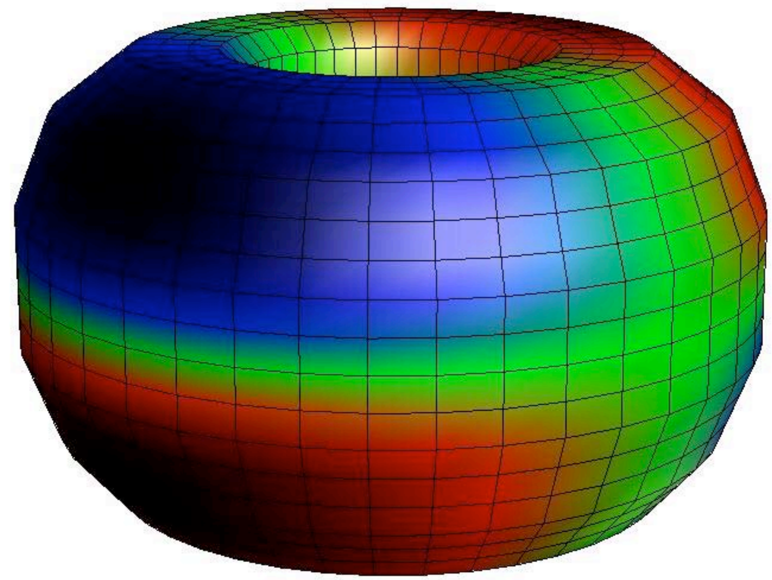


mode #1260
wt = 0.899e-2
0.3068e-2 s

Most important modes (greatest weight) follow:

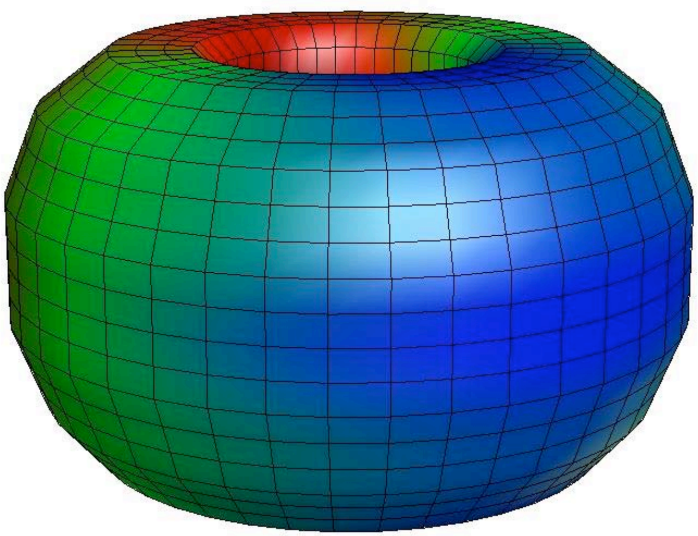


mode #1231
wt = -0.669e-2
0.2043e-2 s

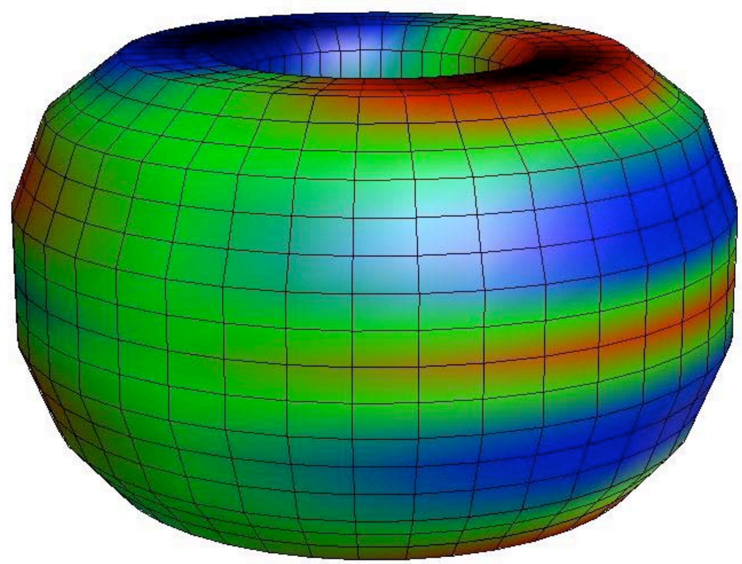


mode #1277
wt = 0.600e-2
0.555e-2 s

Most important modes (greatest weight) follow:

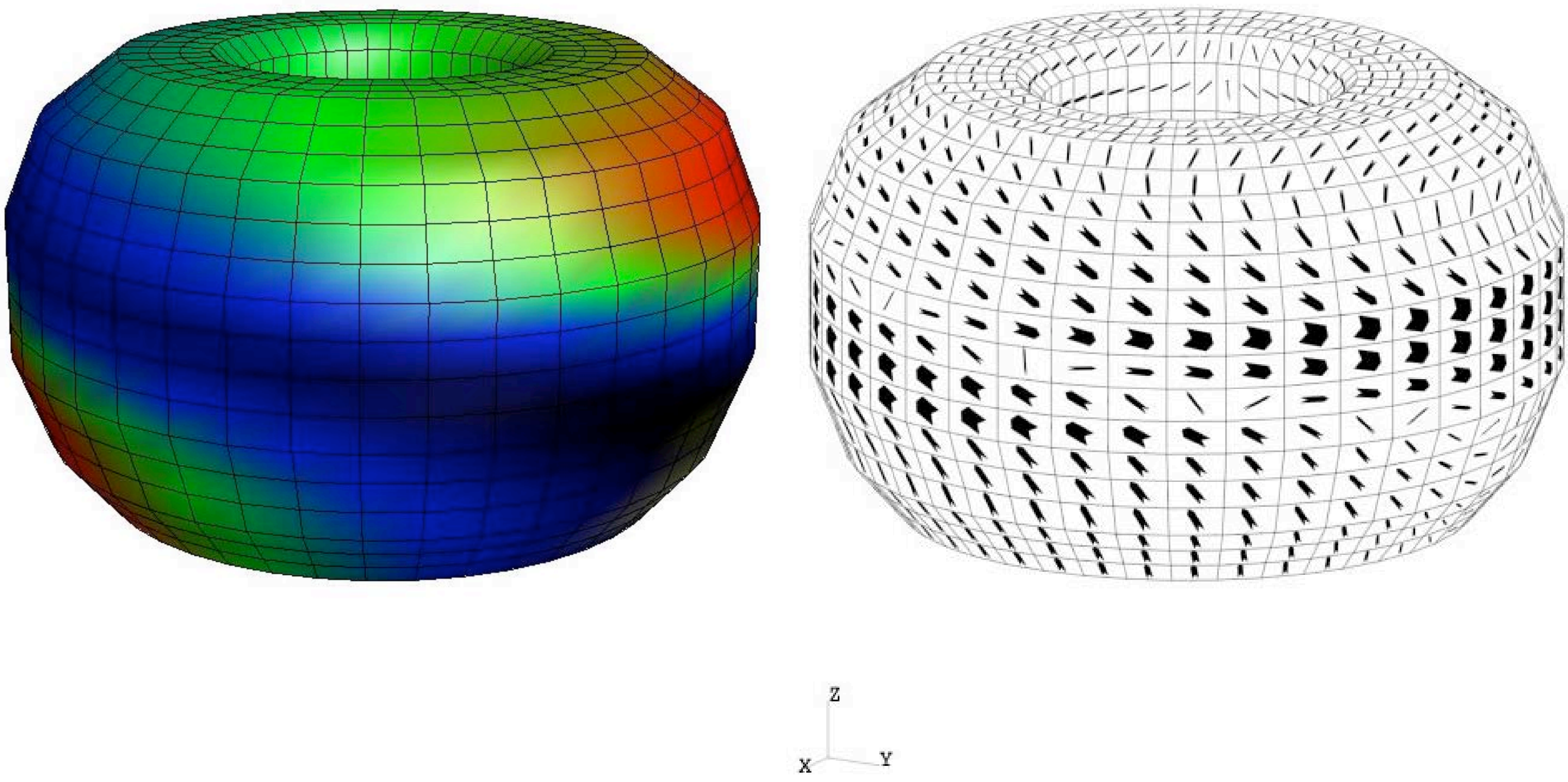


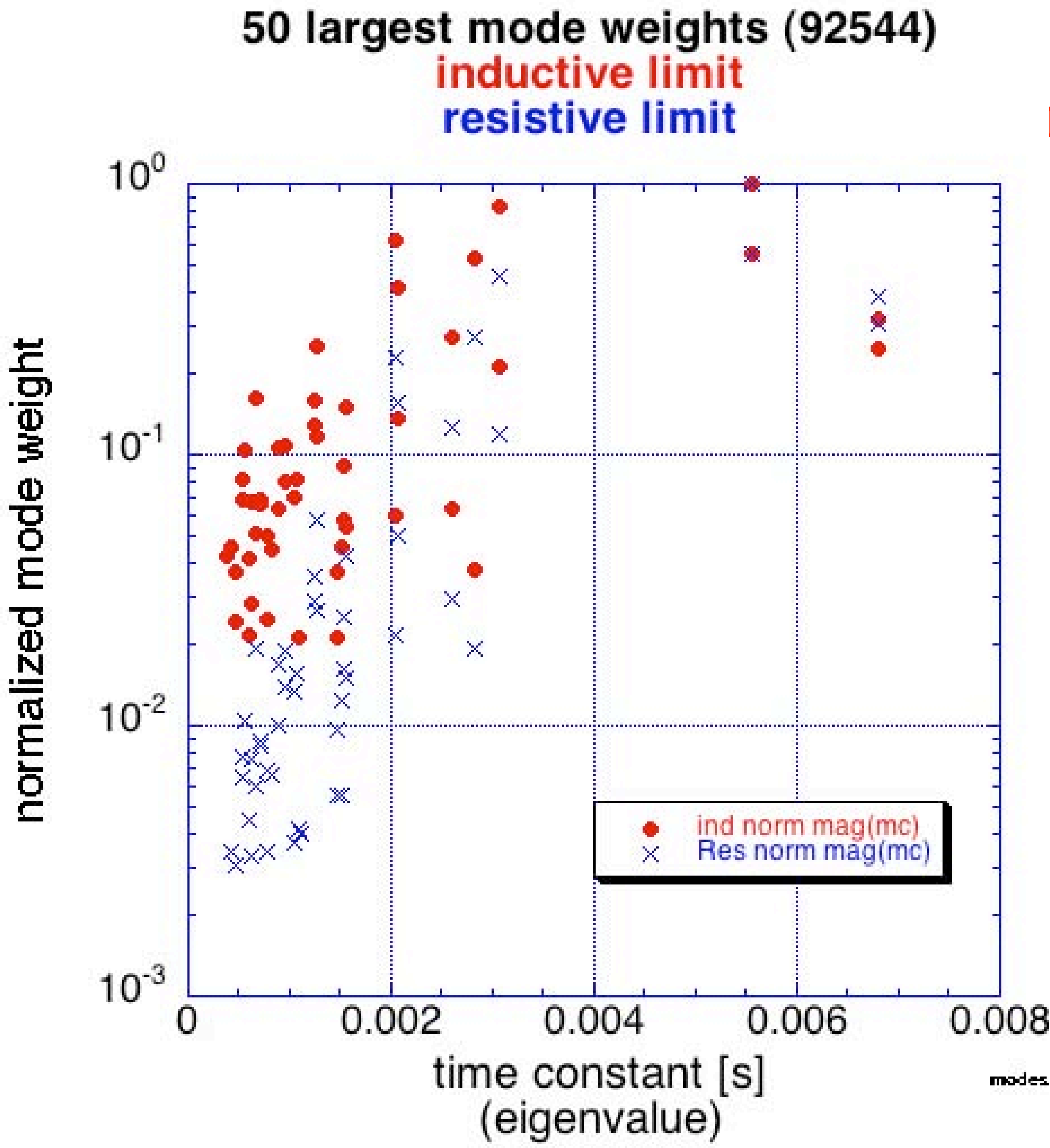
mode#1255
wt = 0.573e-2
0.282e-2 s



mode#1233
wt = -0.451e-2
0.207e-2 s

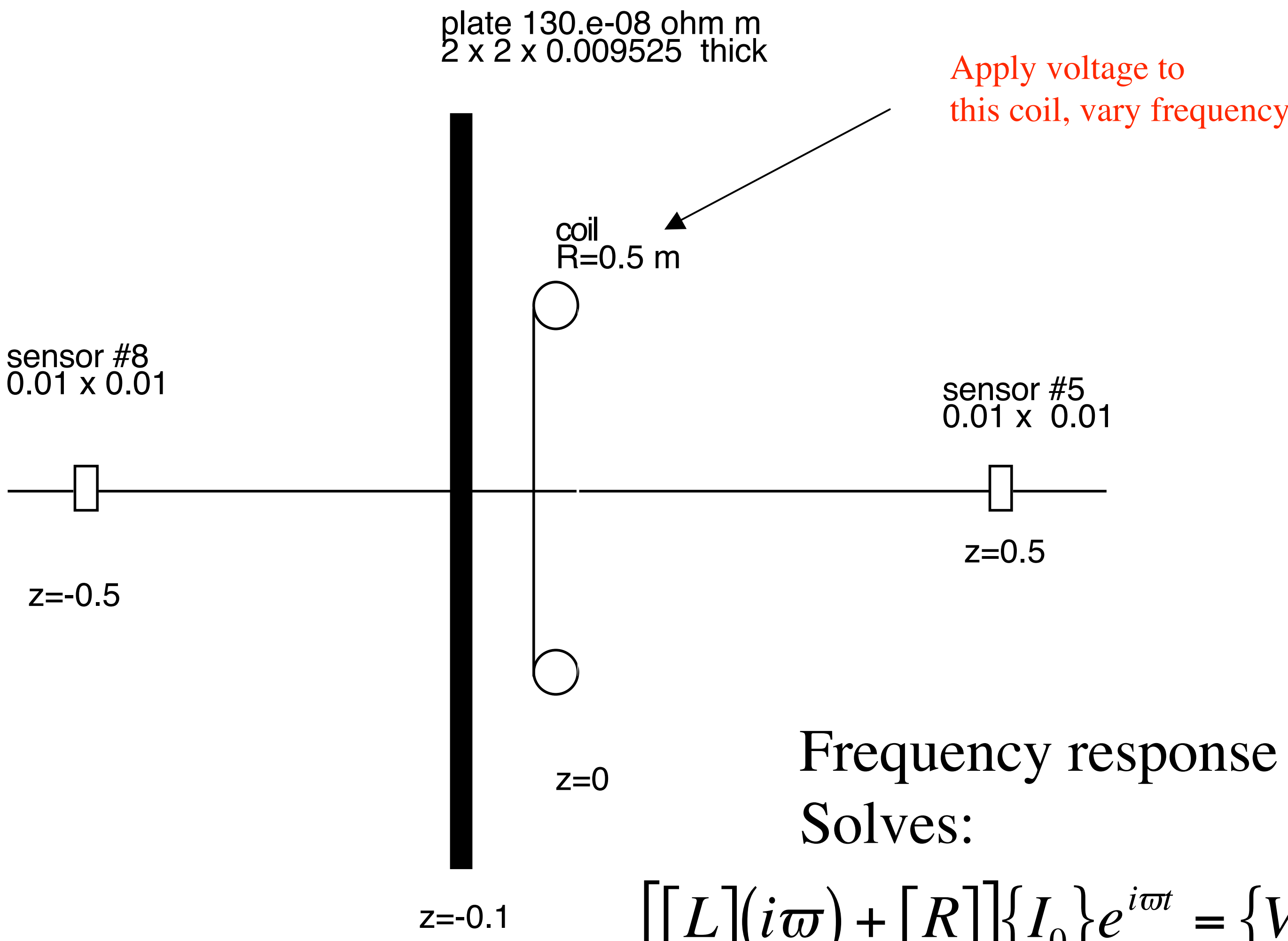
Examine steady state (resistive) response in standard DIII-D model





Many modes are important !!

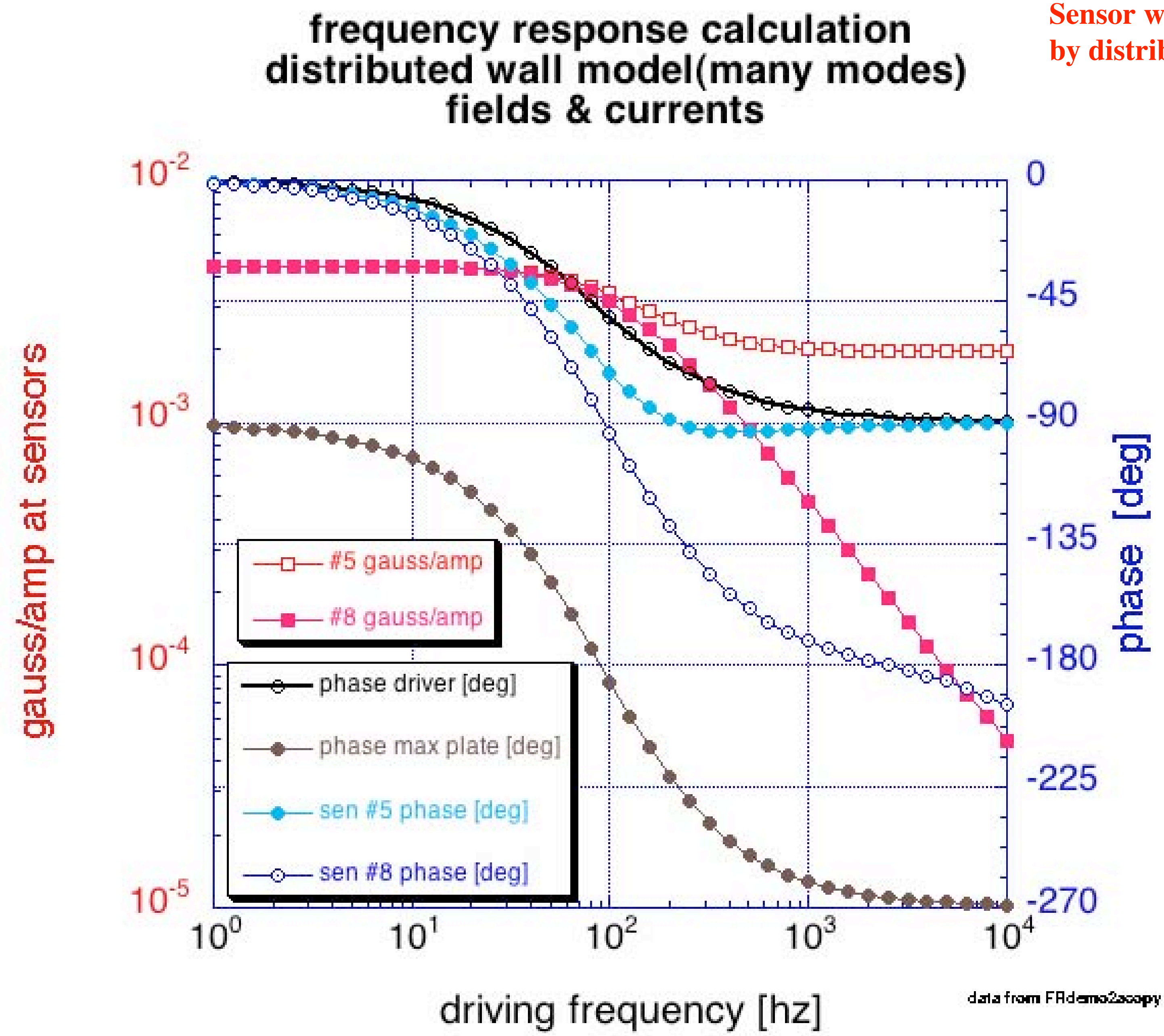
<u>Examine magnetic field penetrating a wall</u> via frequency response of a driving coil, distributed wall model, sensors measure net axial field



Frequency response Solves:

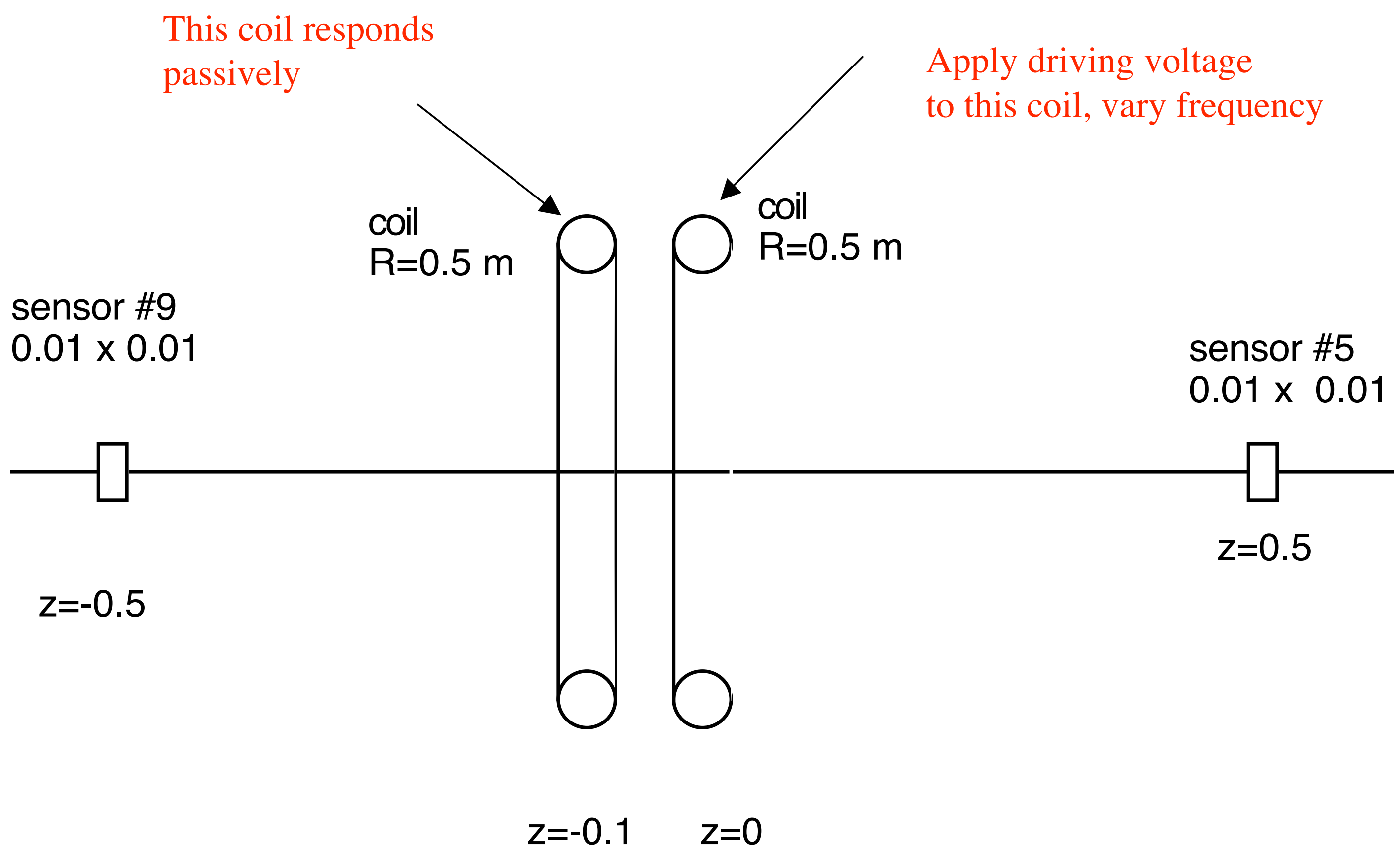
$$\left[\left[L\right]\left(i\varpi\right)+\left[R\right]\right]\left\{I_0\right\}e^{i\varpi t}=\left\{V_0\right\}e^{i\varpi t}$$

Sensor well shielded by distributed wall !



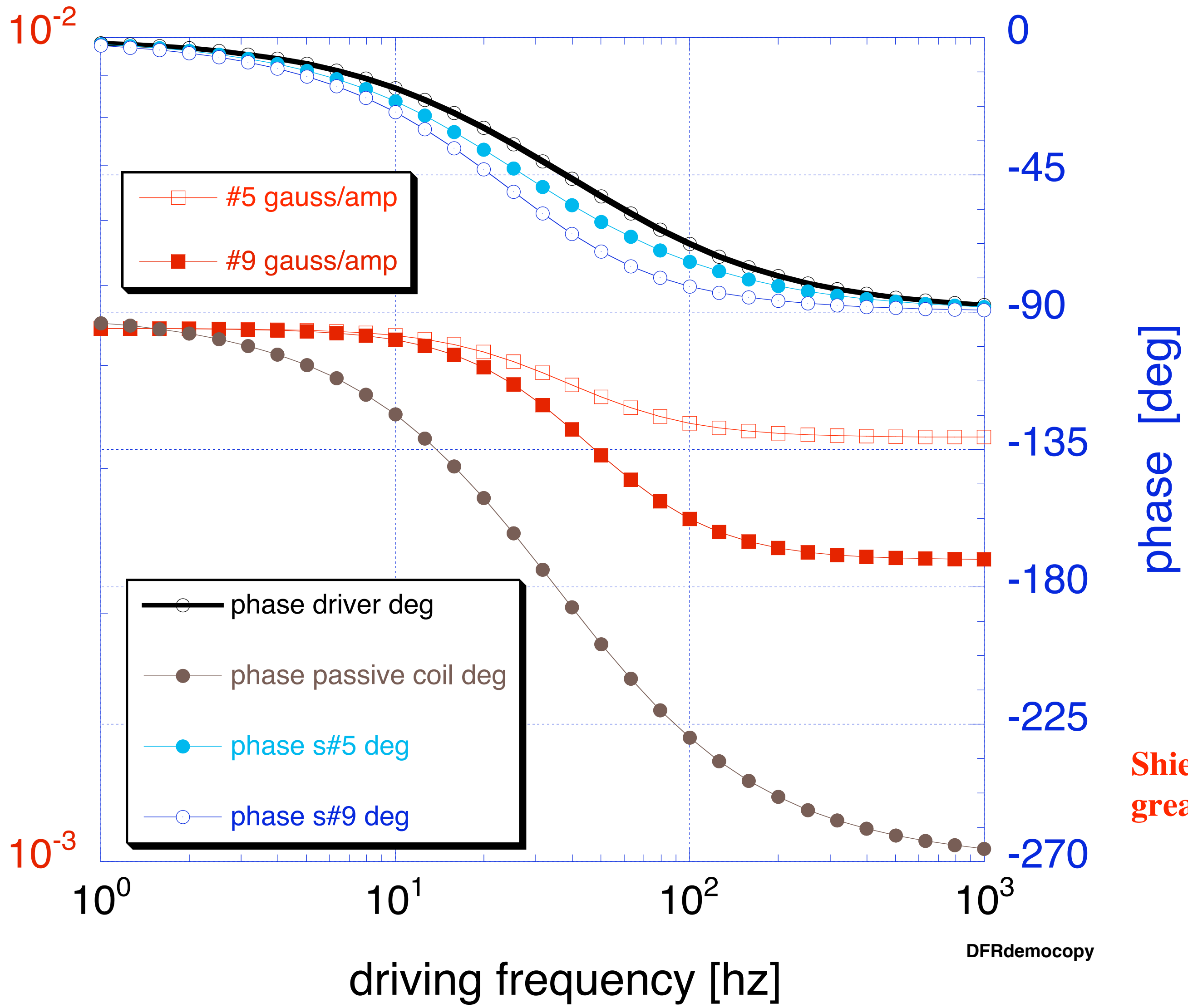
Skin depth
=.009525 [m]
~ 3.6 k hz

Examine magnetic field penetrating a wall via frequency response of a driving coil, the simplest wall model (a passive coil), sensors measure net axial field

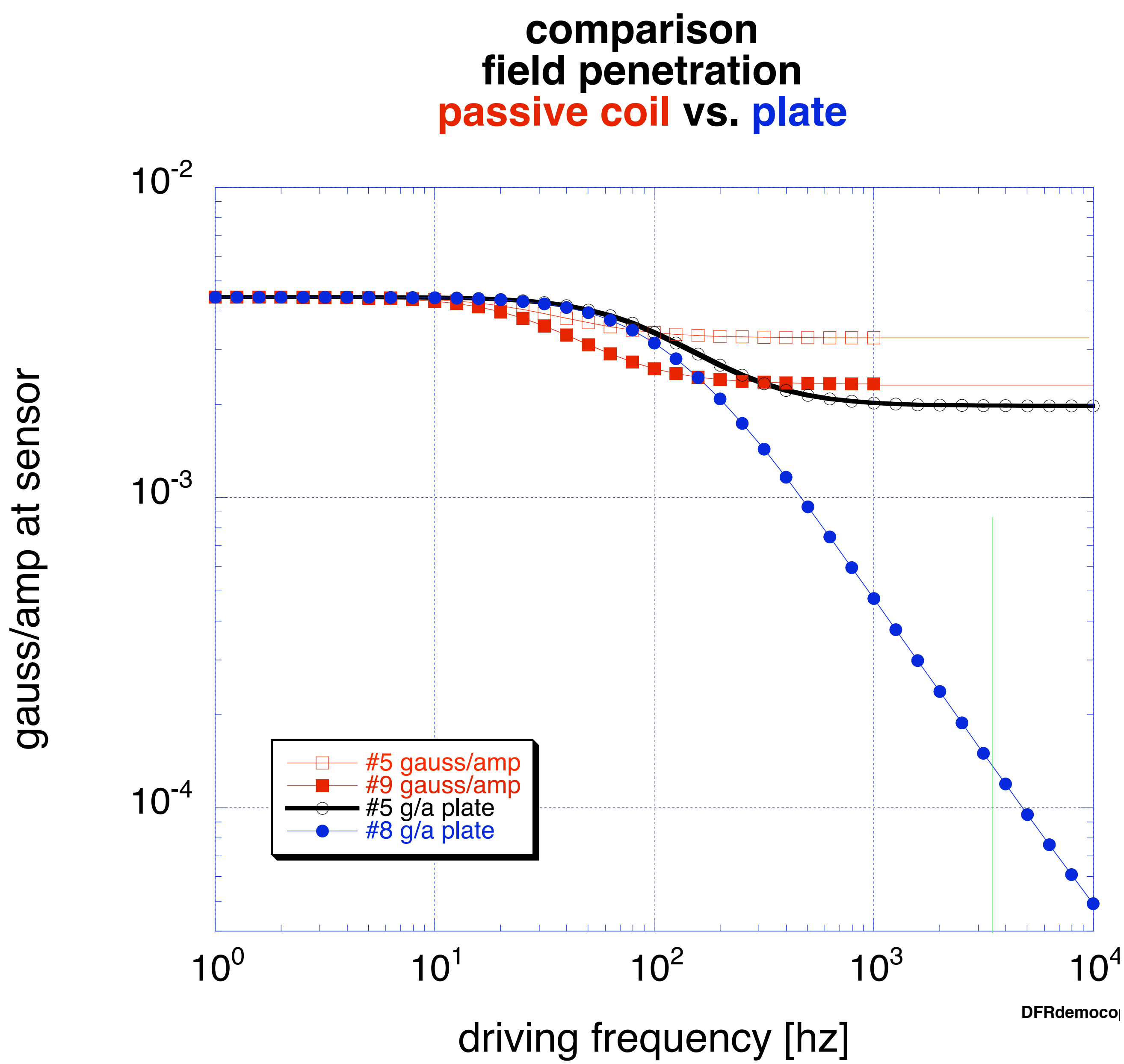


Frequency response calc solves $\left[\left[L\right]\left(i\varpi\right)+\left[R\right]\right]\left\{I_0\right\}e^{i\varpi t}=\left\{V_0\right\}e^{i\varpi t}$

frequency response calculations
wall modeled by 1 time constant coil
fields and currents



Shielding by passive coil greatly different !



Distributed wall model shields much more field at interesting frequencies !

Conclusions & Recommendations

1) All walls have many time constants
2) Current distributions may be described by sum of weighted eigenvectors, we may identify mode with the greatest contribution to the total answer
3) In toroidal geometry we never see an eigenvector with a helical pattern and we need many modes to well represent a helical pattern typical of a plasma mode.
4) Penetration of a magnetic field through a wall is not well modeled with a single wall time constant.

When using a single wall time constant proceed with caution.
Can we specify the best way to make this approximation ?