Robust Wall Mode Feedback

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Definition of Robust Feedback

Feedback systems amplify the flux through sensors by a factor K_p and the voltage on sensors by a factor K_d to specify a voltage or current on feedback coils.

A feedback system is robust if stable feedback occurs with fixed K_p and K_d over the full range of plasma stability for which feedback is possible.

If a feedback system is not designed carefully the required K_p and K_d can change sign by passing through infinity as the plasma stability changes.

Critical Coefficients

The success and robustness of a feedback system is determined by four dimensionless collections of mutual inductance coefficients.

$$(1)\ c_w \equiv \frac{M_{wp}^2}{L_p L_w}; \qquad (2)\ c_{wf} = c_f \equiv 1 - \frac{M_{pw} M_{wf}}{L_w M_{pf}};$$

$$(3)\ c_{sw} \equiv 1 - \frac{M_{sw} M_{wp}}{L_w M_{sp}}; \qquad (4)\ c_{sf} \equiv 1 - \frac{M_{sf}}{M_{sp}} \frac{M_{wp}^2}{L_w M_{pf}}.$$

The inductance coefficients involve the current or flux pattern on the conducting wall (w), the plasma surface (p), the feedback coils (f), or the sensors (s). Feedback only possible if $s < c_w/(1 - c_w)$.

Critical Values of Coefficients

Robustness for all $s < c_w/(1 - c_w)$ requires:

(1) $c_{wf} > 0$, feedback/plasma interaction sufficiently strong compared to feedback/wall interaction.

(2) $c_{sw} > 0$, sensor/plasma interaction sufficiently strong compared to sensor/wall interaction. (Proven below)

(3) $c_{sf} > 0$ sensor/plasma interaction sufficiently strong compared to sensor/feedback, but $c_{sw} + c_{wf} > c_{sf}$.

Origin of Inequalities

Flux on plasma surface defined by $\square\vec{B}\cdot\hat{n} = \dfrac{\square}{A} f_\ell(\square,\square)$.

Total flux $\square = \square\dfrac{\square_x}{s}$ with $\square_x$ due to external currents.

s is stability parameter;

s | | | |	rigid perfect conductor
s=-1	in absence of a plasma
s>0	plasma unstable without a wall

Flux due perturbed plasma currents $\square\ \square\square_x = L_p I_p$.

Equivalent current potential on plasma surface $\square = I_p f_\ell(\square,\square)$

Equivalent perturbed plasma current $I_p \equiv \frac{-\delta\Psi_x}{L_p} = -\left(1+\frac{1}{s}\right)\frac{\Psi_x}{L_p}$.

If only a wall: $\Psi_x = M_{pw} I_w$ so $I_p \equiv = -\left(1+\frac{1}{s}\right)\frac{M_{pw}}{L_p} I_w$

Flux through wall $\Psi_w = L_w I_w + M_{wp} I_p = -L_w D(s) I_w$

$$D(s) \equiv \left(1+\frac{1}{s}\right)c_w - 1, \text{ where } c_w \equiv \frac{M_{wp} M_{pw}}{L_p L_w}$$

$D=-(1-c_w)$	for rigid perfect conductor.
$D=-\infty$ or $+\infty$	for $s=0$.
$D>0$	for $0 < s < c_w/(1- c_w)$, wall mode unstable.

Perturbed plasma current $I_p = -\left(1 + D(s)\right)\frac{L_w}{M_{wp}} I_w$

D=+∞ for barely unstable wall mode; $D \to 0$ as ideal mode goes unstable.

Feedback desired through full *D>0* region.

Flux through a sensor

$$\Phi_s = M_{sw} I_w + M_{sp} I_p = -\frac{M_{sp} L_w}{M_{wp}} \left[D + 1 - \frac{M_{sw} M_{wp}}{M_{sp} L_w} \right] I_w .$$

Unless $c_{sw} \equiv 1 - \frac{M_{sw} M_{wp}}{M_{sp} L_w} > 0$ sensor flux vanishes for some s.

Multimode Analysis (Dmitry Maslovsky)

Dmitry Maslovsky has developed a post processor for the DCON code that sets up the required matrices for multimode ananlysis in VALEN.

Let $\square\vec{B}\cdot\hat{n} = \frac{1}{A}\sum_i \square_i(t) f_i^*(\square,\square)$

$\square = \sum_j J_j(t) f_j^*(\square,\square)$ be a surface just outside plasma

With plasma present $\square_i = \sum_j \square_{ij} J_j$ or $\vec{\square} = \overleftrightarrow{\square}\cdot\vec{J}$.

Without plasma present $\vec{\square} = \overleftrightarrow{L}\cdot\vec{J}$.

Energy required to turn on current $\vec{J}$ is:

$$\square W = \tfrac{1}{2}\vec{J}^{\dagger}\cdot\overleftrightarrow{\square}\cdot\vec{J} = \tfrac{1}{2}\vec{\square}^{\dagger}\cdot\overleftrightarrow{\square}^{\square 1}\cdot\vec{\square}$$

Alan Glasser's DCON code gives a set of $\square\vec{B}\cdot\hat{n}$ distributions on plasma surface and their associated energies. From these one can calculate $\overleftrightarrow{\square}$, inductance with plasma.

Inductance without plasma $\overleftrightarrow{L}$ calculated using either VALEN or DCON keeping a fixed set of Fourier modes with $p=0$ and $q \sqcup$.

Stability matrix $\overleftrightarrow{S} \equiv \overleftrightarrow{L}^{1/2}\cdot\overleftrightarrow{\square}^{\square 1}\cdot\overleftrightarrow{L}^{1/2}$, eigenvalues are $-s_j$.

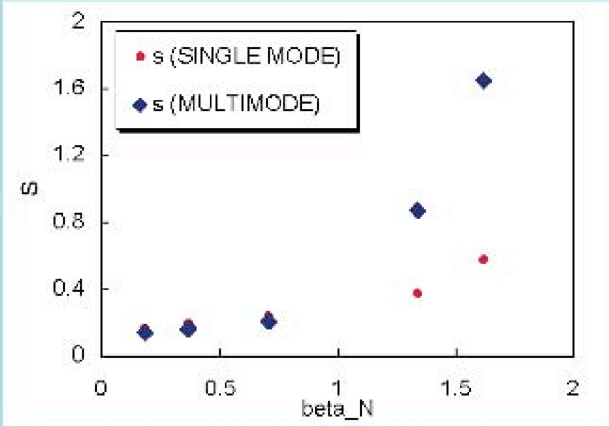
$\vec{\square} = \left(\overleftrightarrow{L}^{1/2}\cdot\overleftrightarrow{S}^{\square 1}\cdot\overleftrightarrow{L}^{\square 1/2}\right)\cdot\vec{\square}_x$ is generalization of $\square = \square\frac{1}{s}\square_x$.

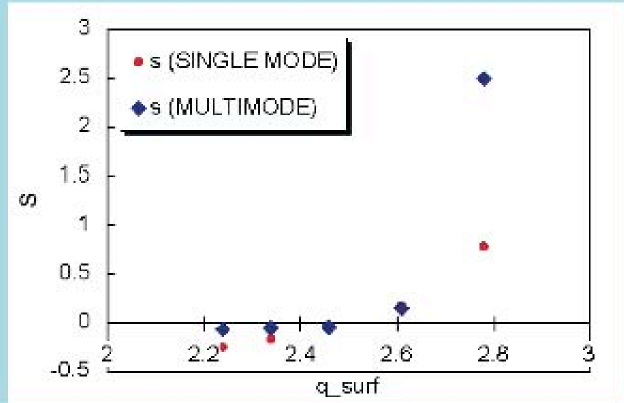
Multimode vs. Single-mode

Plots are generated by computing the effective plasma inductance $\overleftrightarrow{\Lambda}$ and the plasma surface inductance $\overleftrightarrow{L}$ for all DCON modes, and then computing the stability matrix according to:

$$\overleftrightarrow{S} = \overleftrightarrow{L}^{1/2} \cdot \overleftrightarrow{\Lambda}^{-1} \cdot \overleftrightarrow{L}^{1/2}$$

- For low s values, single-mode approximation agrees with the full multimode analysis.
- As the plasma becomes more unstable, mode coupling becomes important, and full multimode description is required.





Summary

The perturbed plasma current produced in response to an external magnetic perturbation is infinite if $s=0$ but drops as the plasma becomes more unstable.

The variable plasma response implies the effect of a changing wall or feedback current can change sign as plasma stability changes.

To have robust feedback, want fixed feedback amplification factors K_p *and* K_d over the full range of s.

Robust feedback implies interactions of feedback coils and sensor loops with plasma must be strong compared to other interactions. Sensor problem eliminated by direct sensing (reflectometry).