PARALLELISM IN BEHAVIOR OF A COUPLED VAN DER POLE OSCILLATORS AND COUPLED MHD PERTURBATIONS IN TOKAMAK

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MHD meeting, PPPL, Nov. 21-23, 2004

KURCHATOV

MOTIVATION

Our Plasma experiments show that the coupled resonant surfaces in tokamaks have many features of nonlinear active media

Features such as:

- Amplification of external perturbations (error or feedback fields)
- Self-amplification of internal perturbations (perturbations from other active modes in the plasma column)
- Amplitude Saturation;
- Locking to external perturbations (error or feedback fields)
- Locking to internal perturbations (dominant mode)
- Beat frequency behavior far from locking point
- Noncoherent phase rotation near locking point

→

A coupled, nonlinear, system of Van der Pole generators has many Similar Features such as:

- Amplification of injected (external) perturbations;
- Amplitude saturation;
- Locking to injected perturbation;
- Beat frequency behavior far from the locking point;
- Noncoherent phase rotation near the locking point;

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Well known, well studied system

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Predictions

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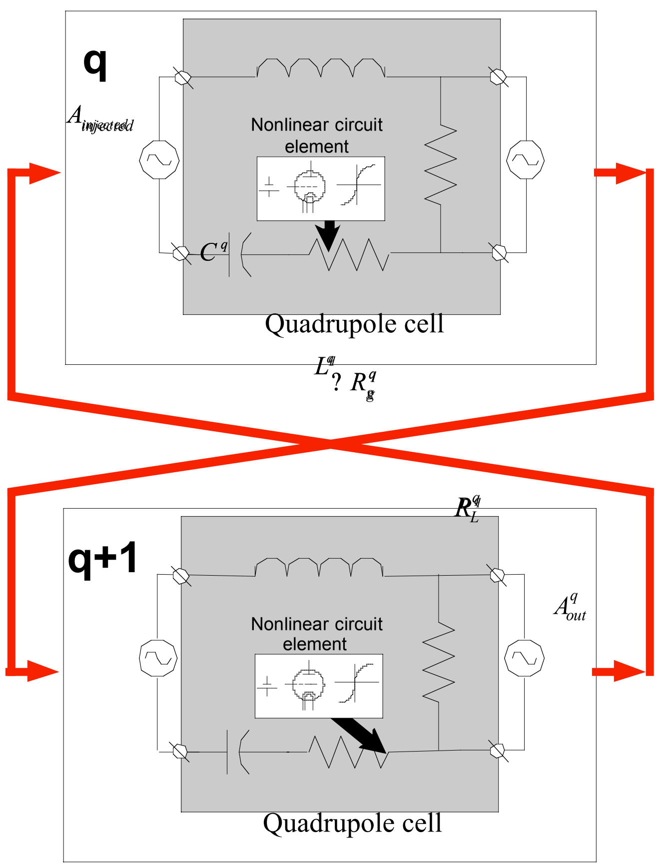
RWM feedback nonlinear regulator

Coupled Van der Pole system

(shown for only two oscillators)



Van der Pole equation set

$$\ddot{A}_{out}^q - \alpha^q (1 - \beta^q {A_{out}^q}^2) \dot{A}_{out}^q + {\omega_0^q}^2 A_{out}^q = A_{injected}^{sum-q}$$

ω_0^q - Free running frequency

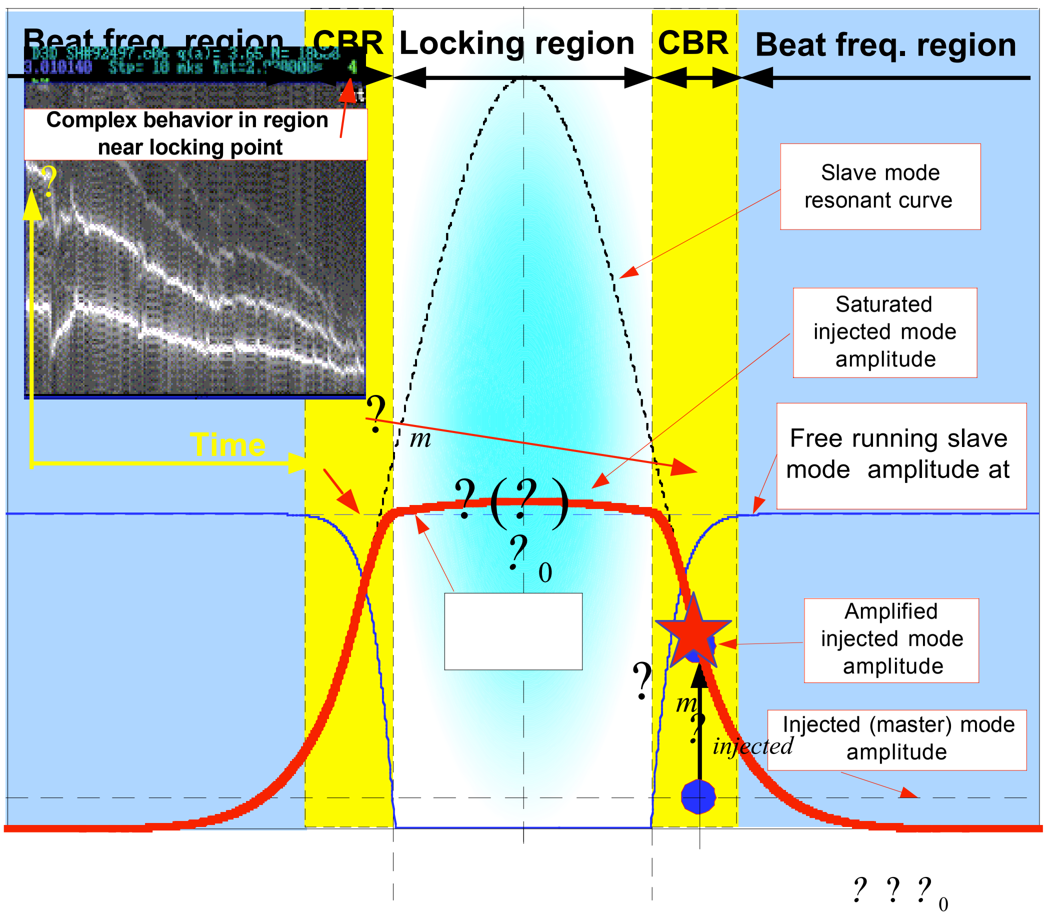
α^q, β^q - Van der Pole generator parameters

Well known and well studied system in:

- Mechanics (Huygens 1629-1695,Coupled pendulum clocks);
- Radio engineering;
- Lasers;
- Astronomy (Solar system);
- And in many other applications.

Locking by injected perturbation

(simulation, one Van der Pole generator connected with fixed amplitude of ext. generator)



Transition time into locking condition~ $1/\Omega_m$

Phase slippage is given by Adler equation

$$\frac{d\Phi(t)}{dt} = -\delta\omega - \mu\ \Gamma(\delta\omega)\frac{A_{inj}}{A_{free-running}} Sin(\delta\Phi)$$

Where μ is coupling coefficient

$$\delta\omega = \omega_{free-run\_slave} - \omega_{inj}$$

$$\delta\Phi(t) = \Phi(t) - \Phi_{inj}(t)$$

Lock mode frequency range

$$2\Omega_m = 2\mu\Gamma(\delta\omega)\frac{A_{inj}}{A_{free-run}}$$

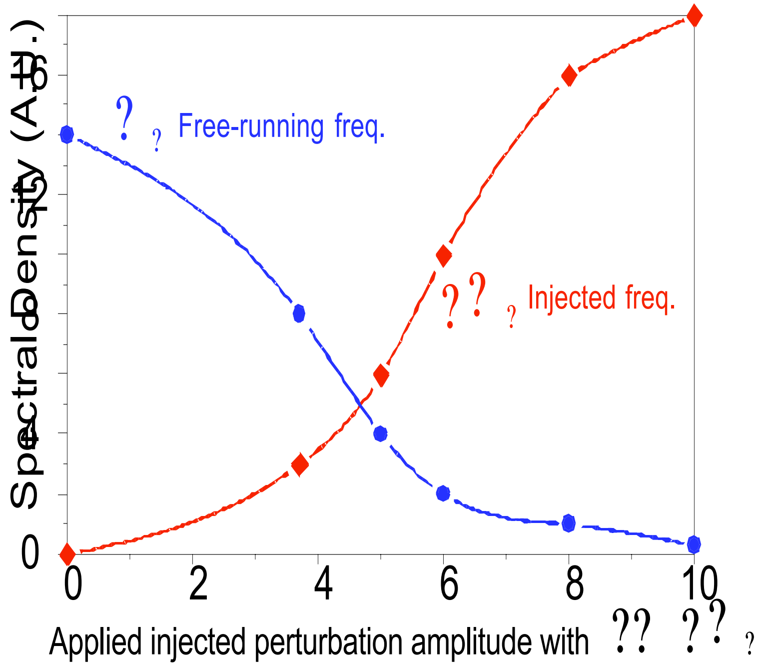
Solution of Adler equation in transient region

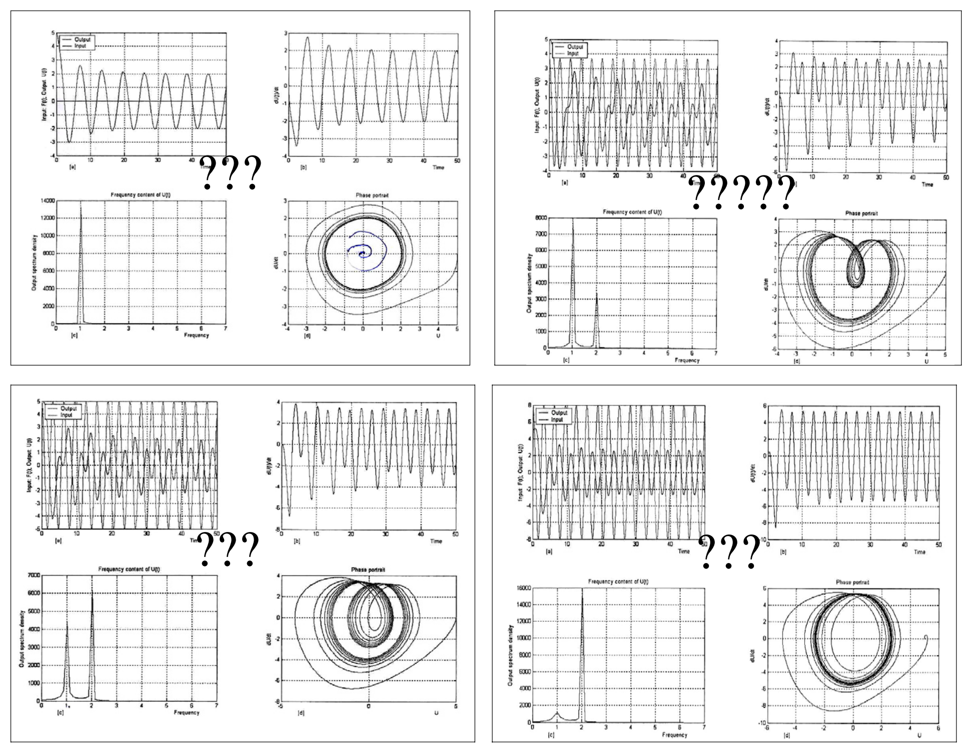
$$\delta\Phi(t) = \Gamma_{saturated}\Phi_{ss} + (\delta\Phi(0) - \delta\Phi_{ss})\exp(-\Omega_m t)$$

Where $\delta\Phi_{ss} = \frac{-\delta\omega}{\Omega_m}$

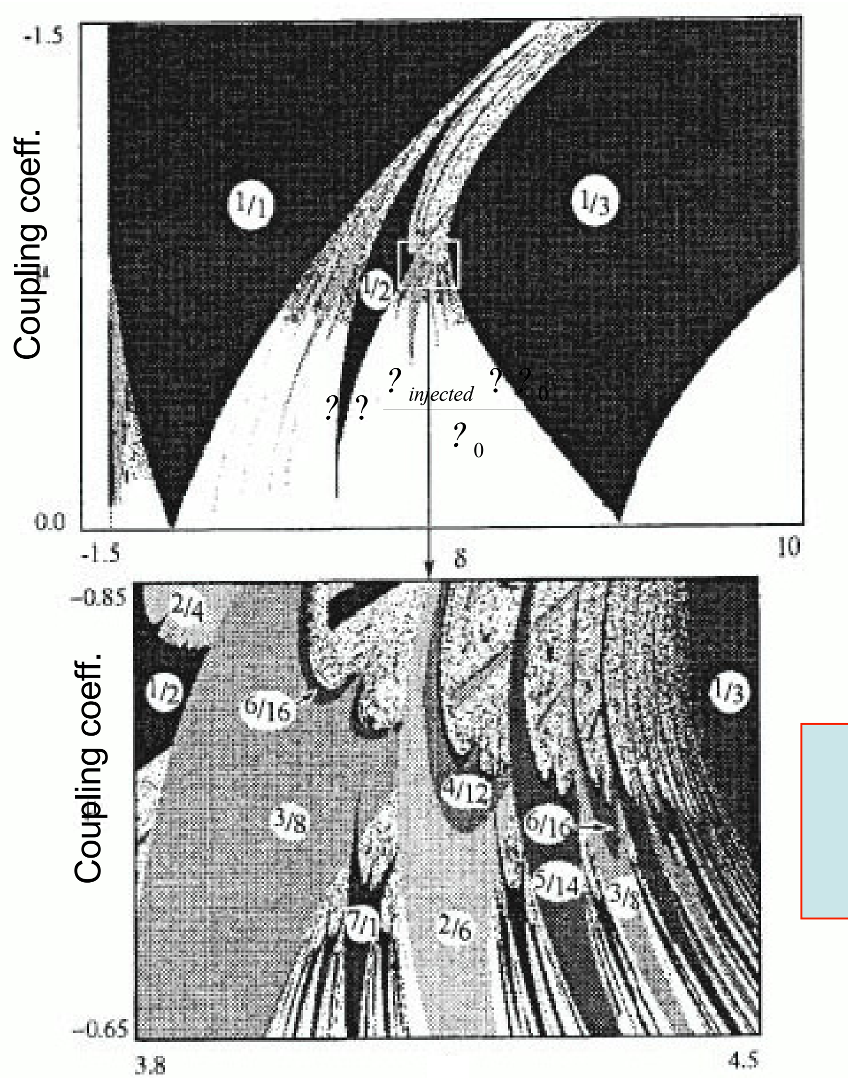
Locking by external perturbation

(simulation, fixed freq. ext. perturbation with $\omega=2\omega_o$, Yu. Mitrishkin)





Simulations. Two coupled Van der Pole Generators.



EXAMPLE

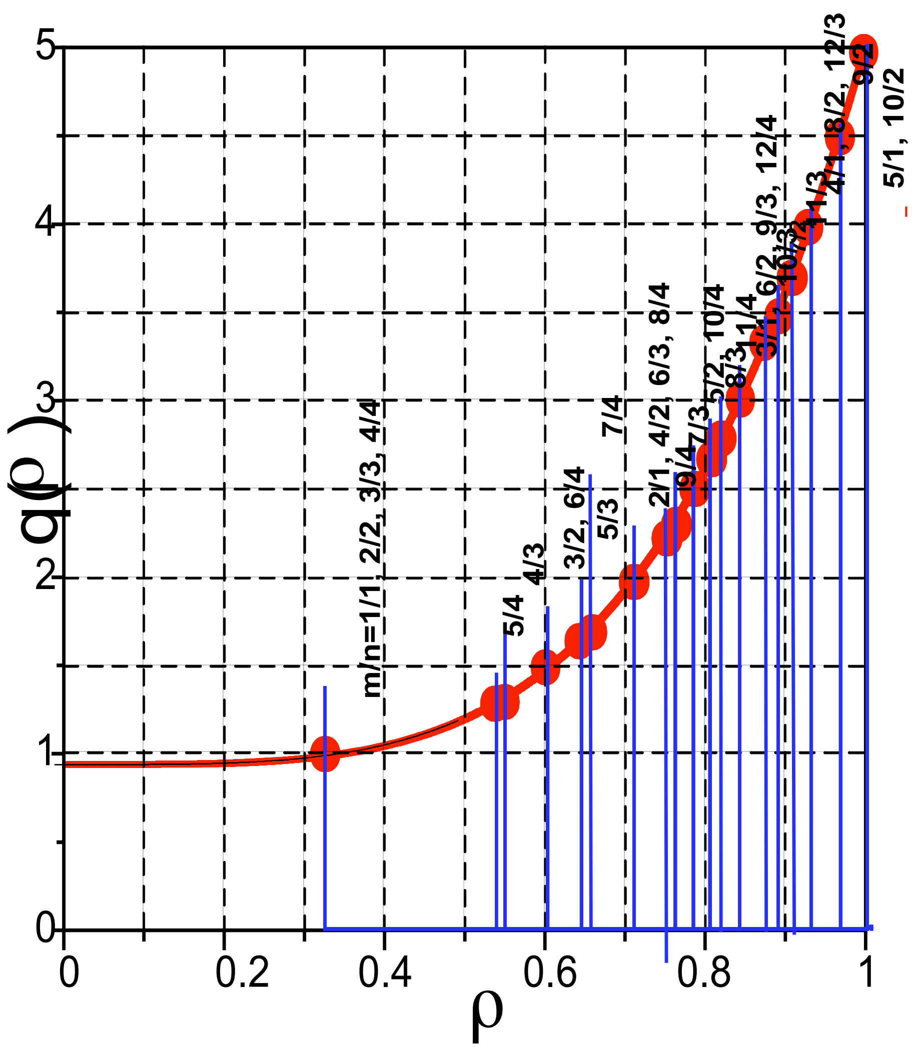
MODE STRUCTURE IN CASE OF NEGATIVE COUPLING COEFFICIENT

(It could be in some cases if oscillators controlled by active feedback)

Varying α, β, ω, μ it is possible to generate very complicated spectrum up to peak of spectrum

Picture from
A.P. Kuznetsov&V.I.Paksutov, Rus. Izvesia Vuzov, v. 11, n. 6, p. 48, 2003.)

RESONANCE SURFACES IN PLASMA COLUMN (THEORETICAL PREDICTION)



If

q=m/n<5

m<12

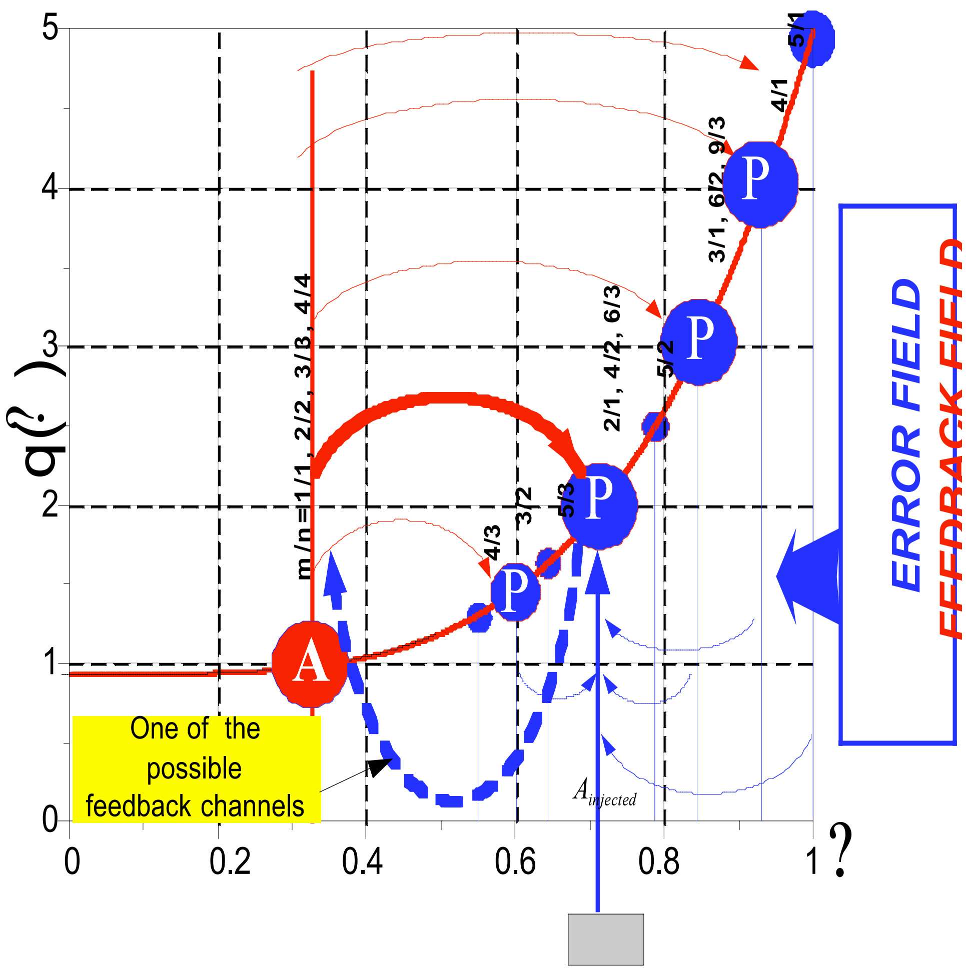
n<4

then

theoretically 33 coupled modes could be excited at 19 resonance surfaces

How do these modes interact with each other?

RESONANCE SURFACES IN PLASMA COLUMN (experiment)



If

q=m/n<5

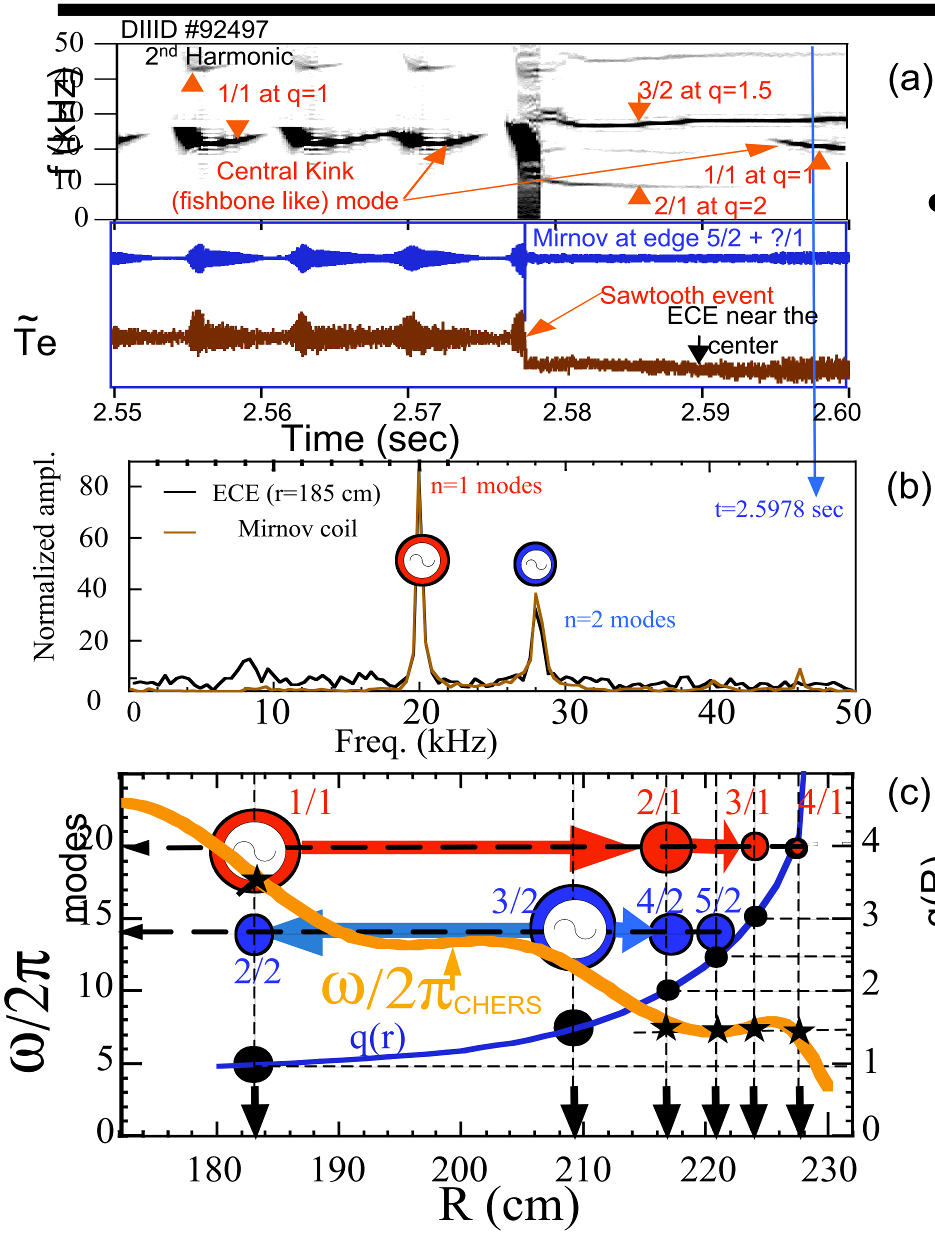
Experimentally ~16 Active or Passive coupled modes were observed in tokamak

- In one shot ~ 6

we use the following definitions:

- *Active mode* is the tearing or kink mode which has positive or close to 0 positive growth. The behavior of this mode depends on main plasma parameters such as shear, pressure profile, etc
- *Passive mode* is the mode which has negative growth or close to 0 negative growth. The behavior of this mode depends on amplitude and phase of the external perturbations.
- *Dominant mode* is the mode, which determines the global phase synchronization of the secondary active or passive modes.

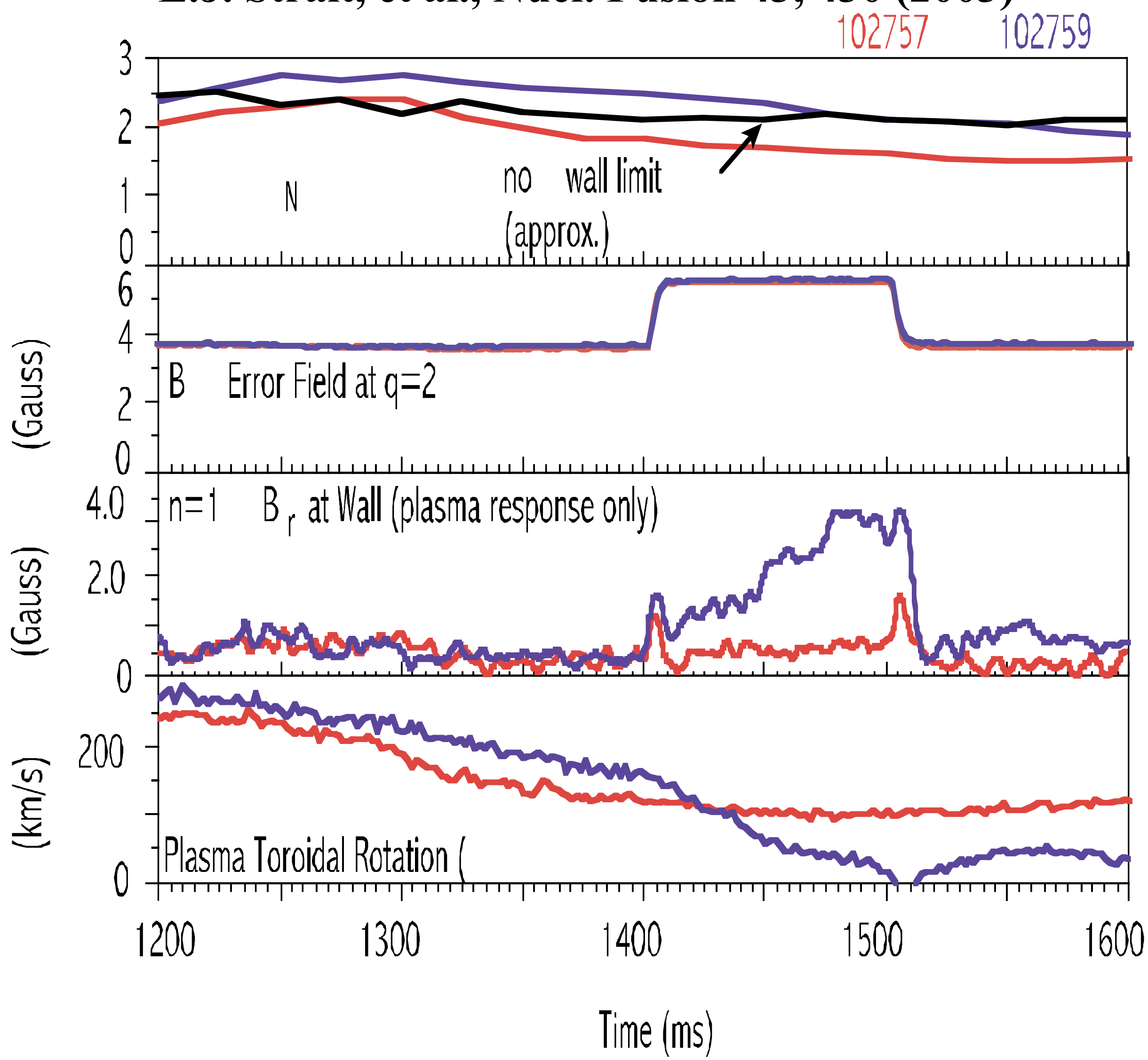
MODE SELF INTERACTION (small amplitude) (DIIID)



- **two active modes (fishbone like m=1/n=1and tearing m=3/n=2) excite a set of passive modes at several singular surfaces with frequencies equal to the frequencies of the active modes**

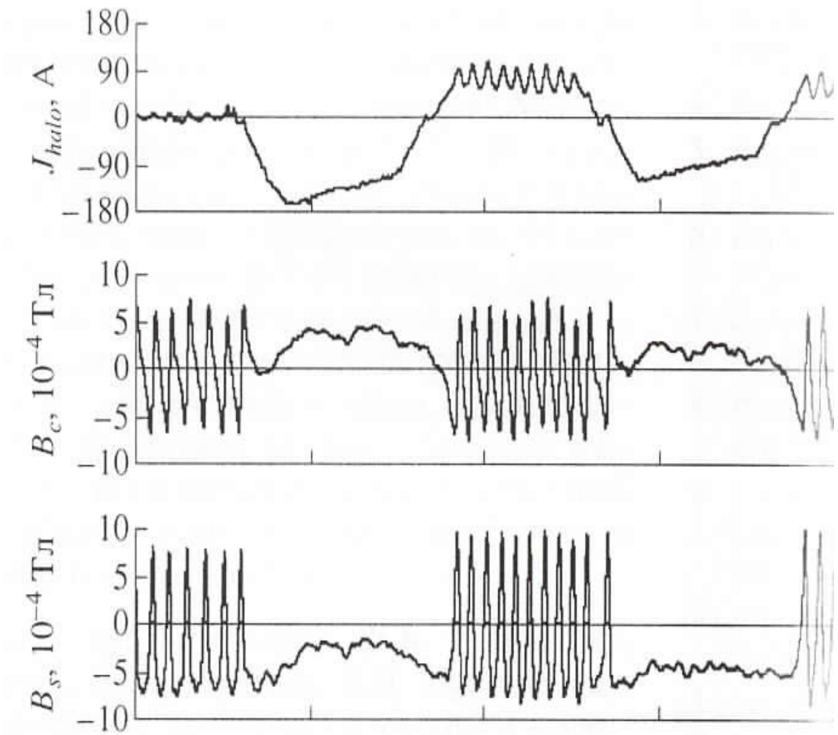
INTERACTION WITH EXTERNALY APPLIED PERTURBATION (DIIID)

E.J. Strait, et al., Nucl. Fusion 43, 430 (2003)



- Plasma – active media, amplifies the spacial harmonics of the feedback control system

T-10 - Phase lock by external perturbation (Perturbation applied as halo current through a rail limiter A.Kakurin, N. Ivanov et al.)



Plasma – active media,

- When external perturbation is rather high it is possible to excite phase locked condition. (Plasma in this case continues rotation)

Amplitude dependence (TFTR)

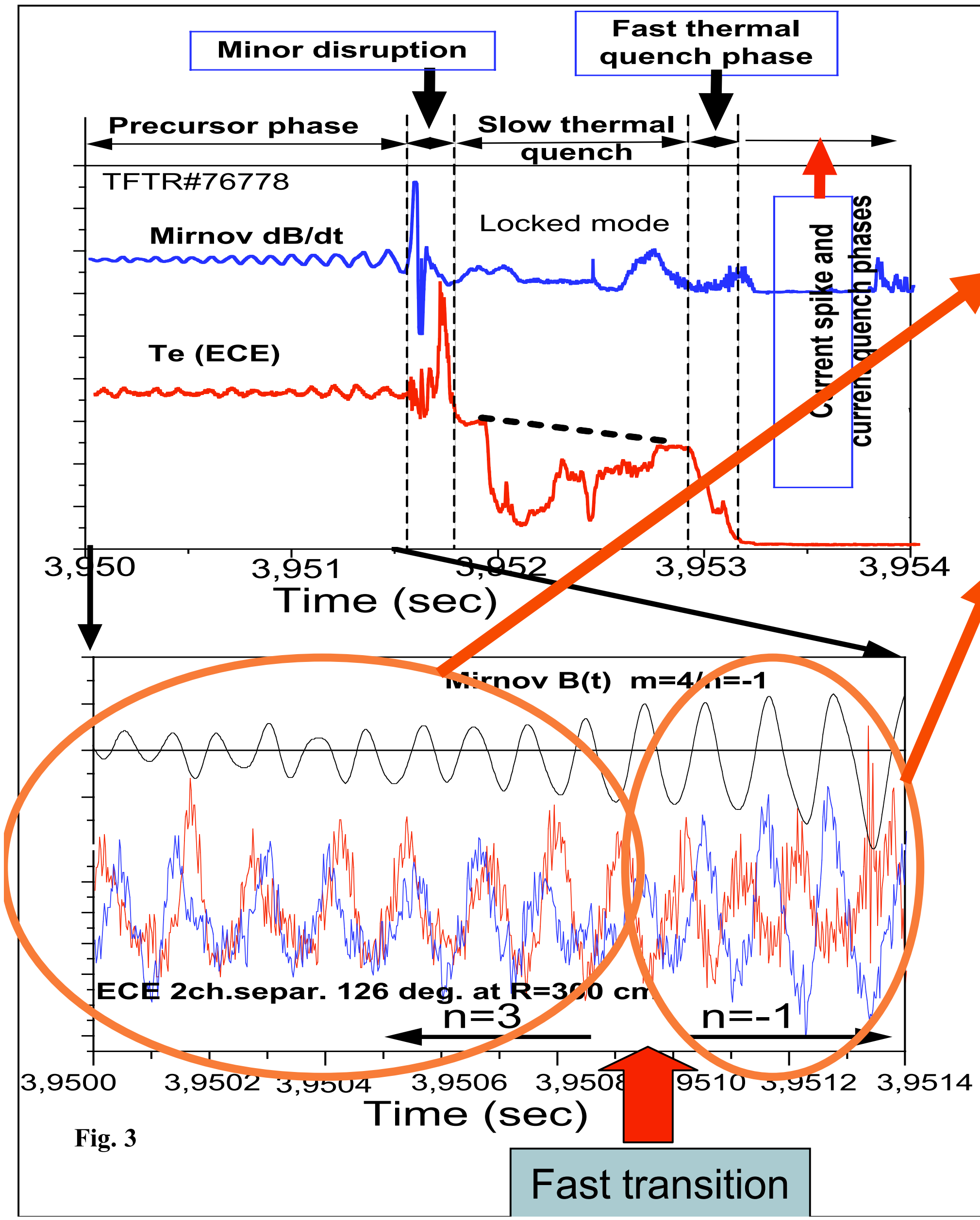


Fig. 3

Interaction has two stages:

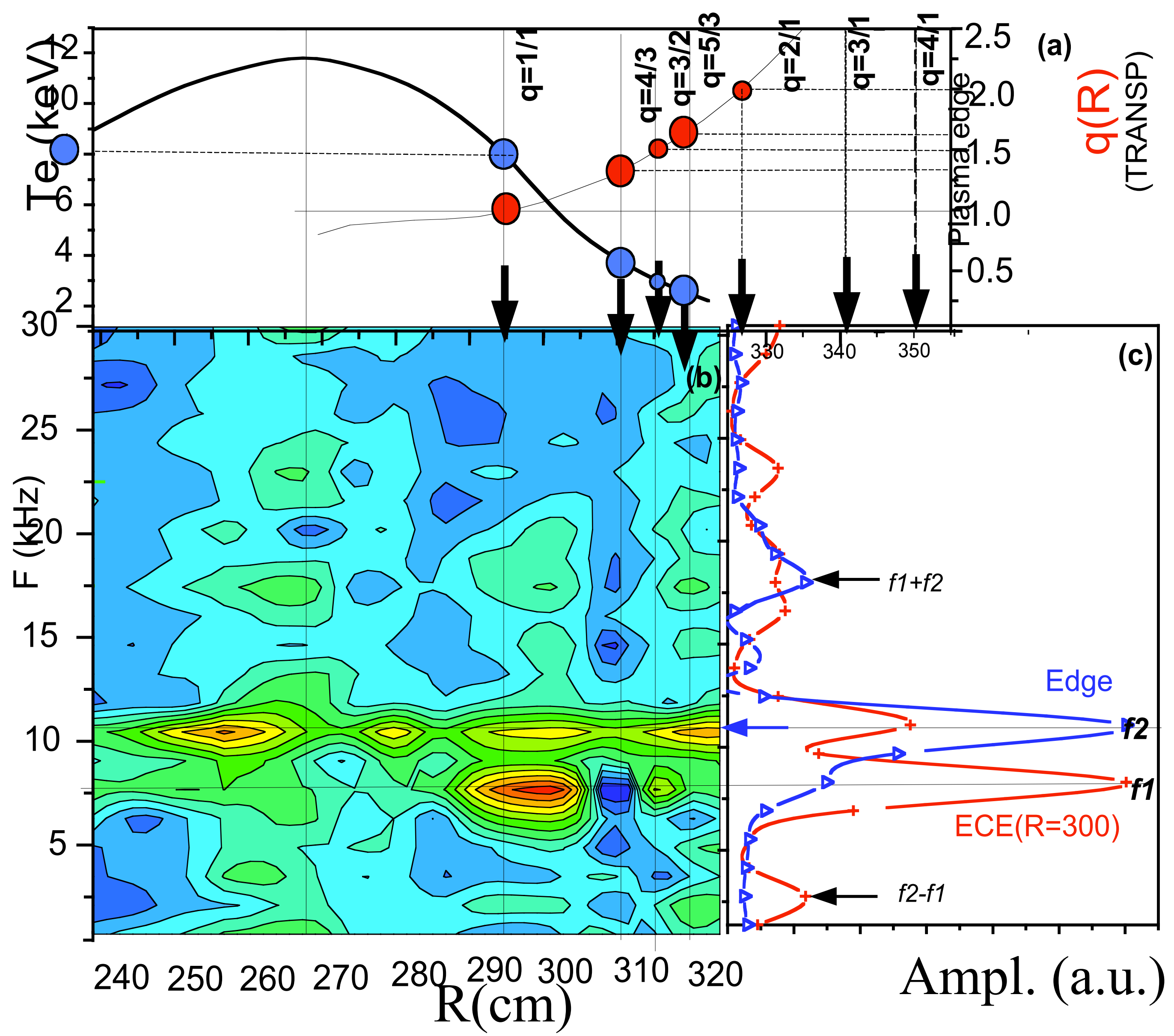
- Stage I. (Small amplitude)

  Several active modes have independent behavior (or have partial locking)

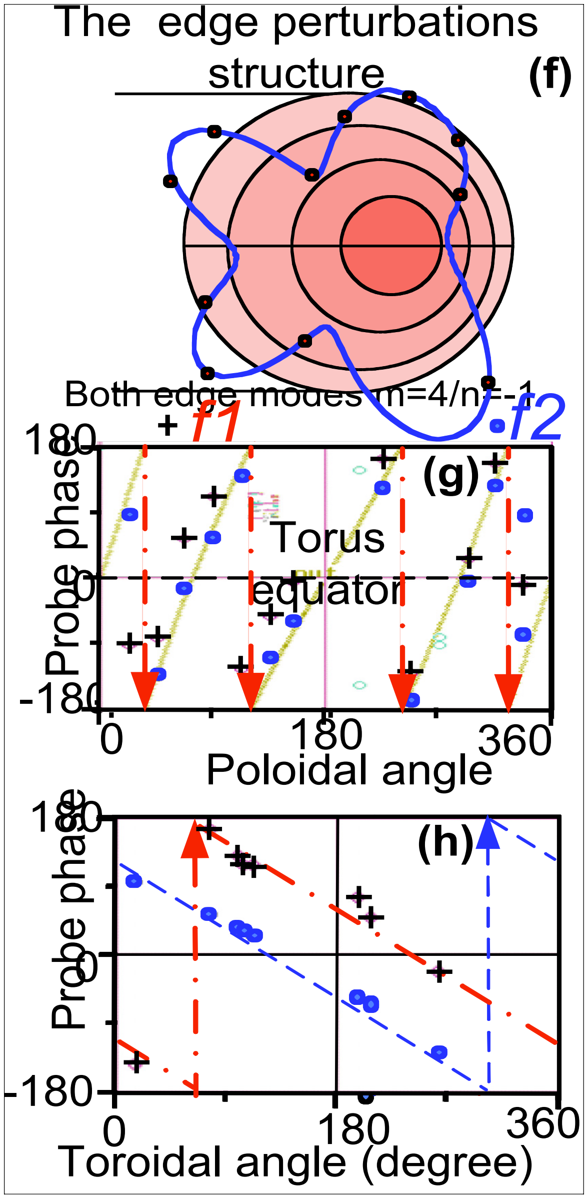
- Stage II. (High amplitude)

  As amplitude of active modes increases one mode may become *the Dominant mode.* That is, the mode which determines the phase synchronization of all internal modes (This is Phase Locked stage, plasma continues rotation)

STAGE I. Small ampl. coupling (TFTR)

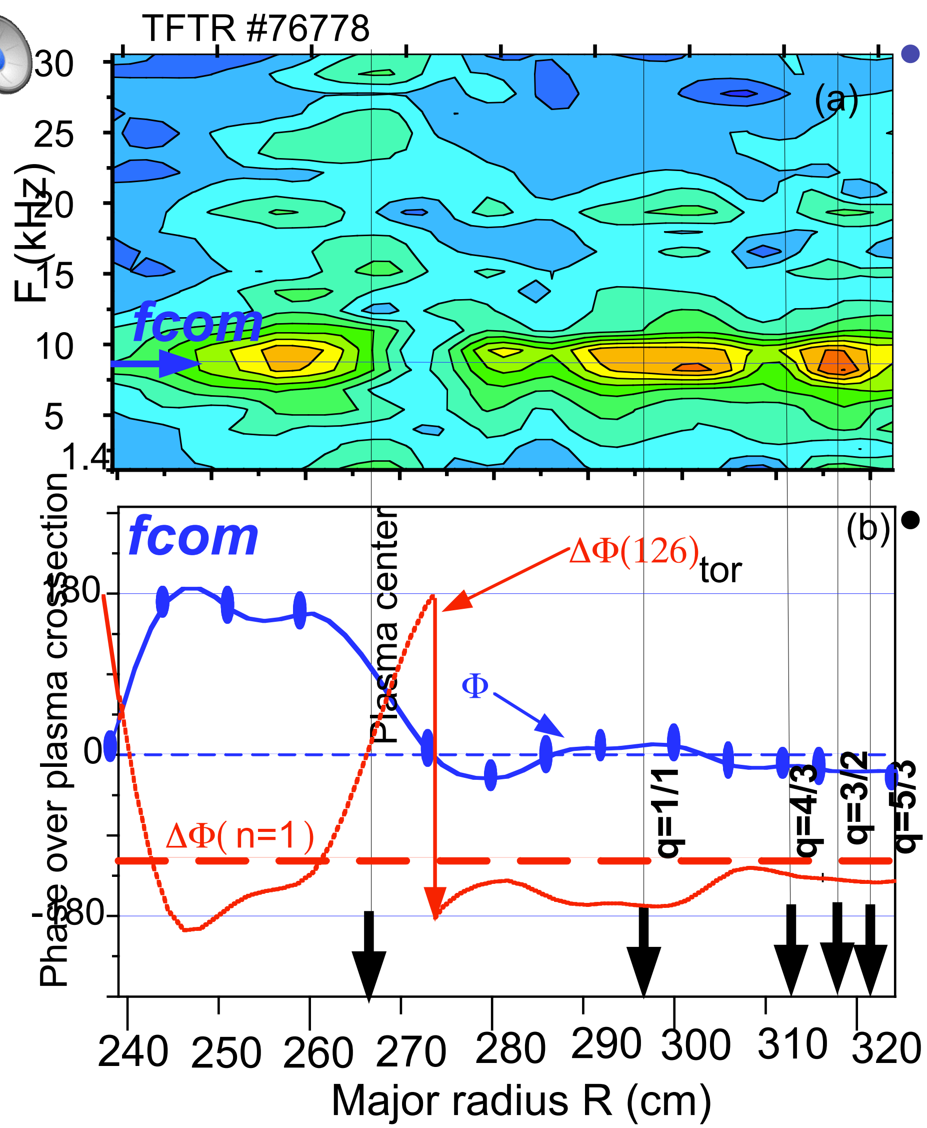


STAGE I. Small ampl. Interaction (TFTR)



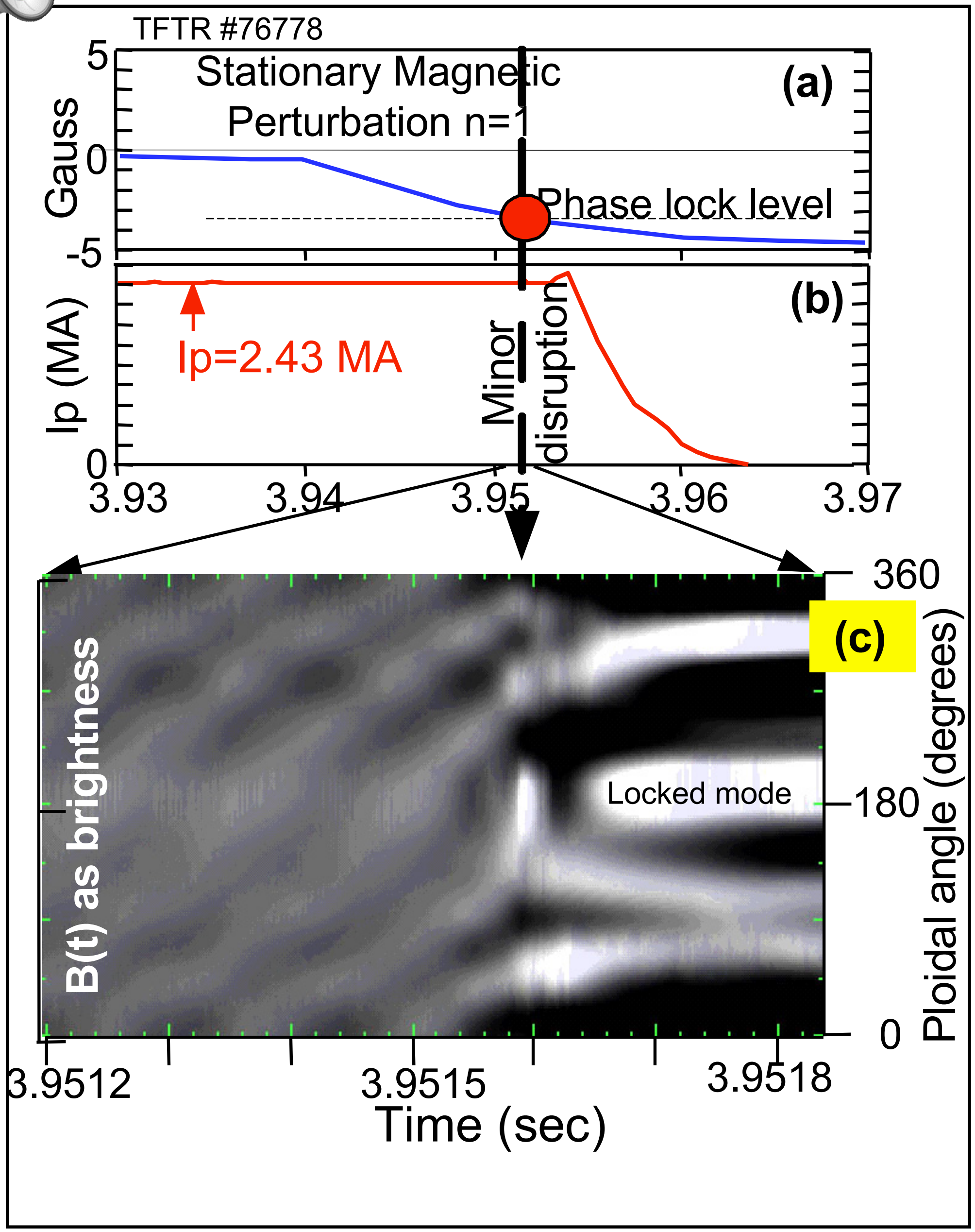
- It is interesting to note. If we have in plasma column two independent active modes. At the edge it is possible to observe *two m=4/n=1 modes.* It looks like as one rotates through another. *One mode* is free running edge mode and *Other mode* is amplified injected harmonic from internal mode.

Dominant mode (“rotational” locked mode, TFTR)



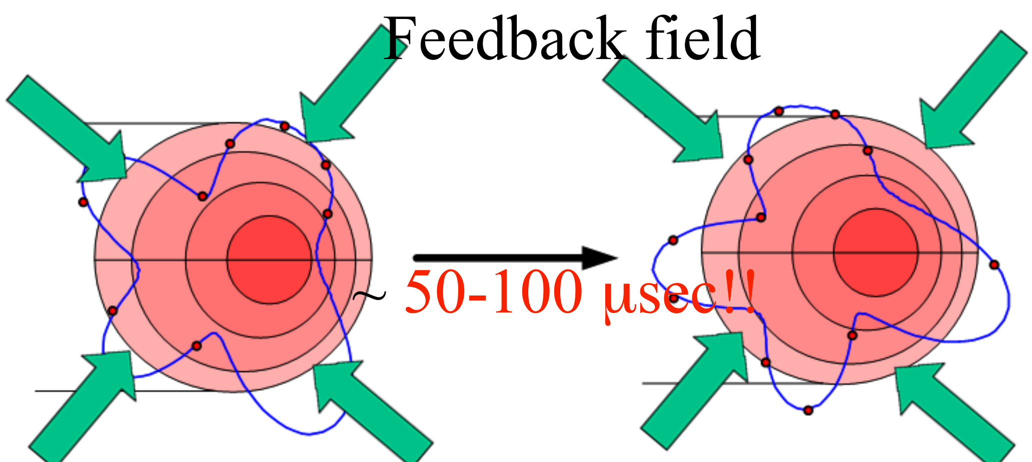
- In the second stage the *dominant* edge mode m=4/n=1 controls the choir of internal modes.
- From this moment the internal and external perturbations are strongly coupled in phase (“rotational” locked mode).

DOMINANT LOCKED MODE (TFTR)



- Amplified external error field may play an important role as dominant mode with ω=0.
- As amplitude of stationary perturbation n=1 increases, abrupt changes in mode structure happen. At this moment (t=3.9516 sec) the stationary dominant mode m=3/n=1 replaces the rotational dominant mode m=4/n=1 or "rotational Locked mode" m=4/n=1 transforms into stationary Locked mode m=3/n=1
- Plasma may continue rotation.
- Again, transition time into locked mode is the same as characteristic time of disruptive instability.

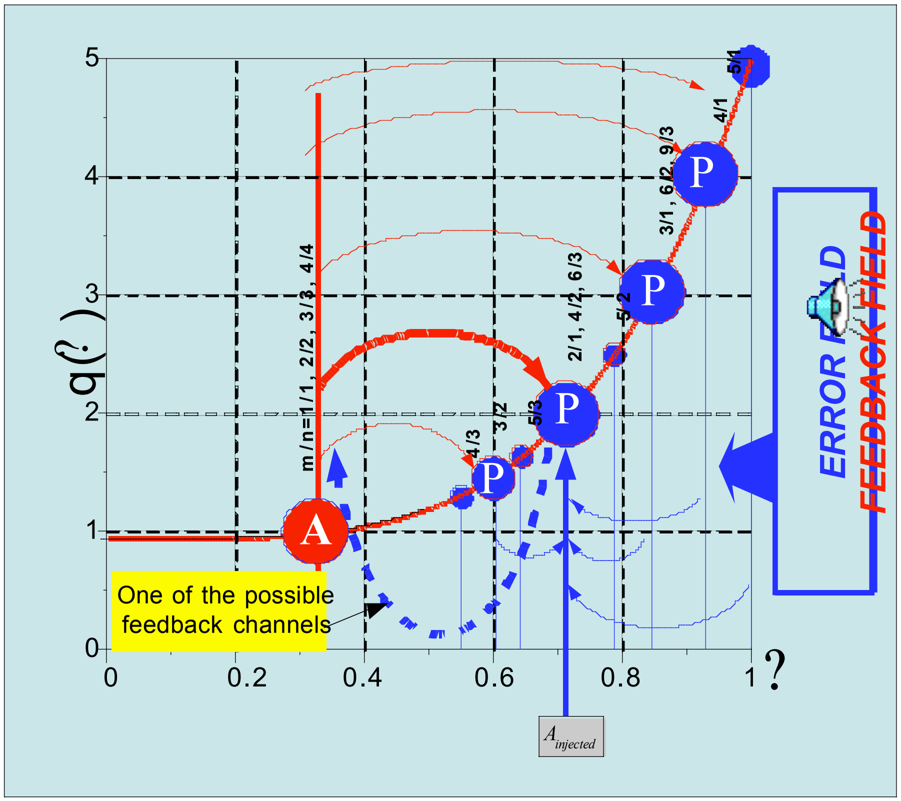
Van der Pole model predicts that feedback field may excite instability



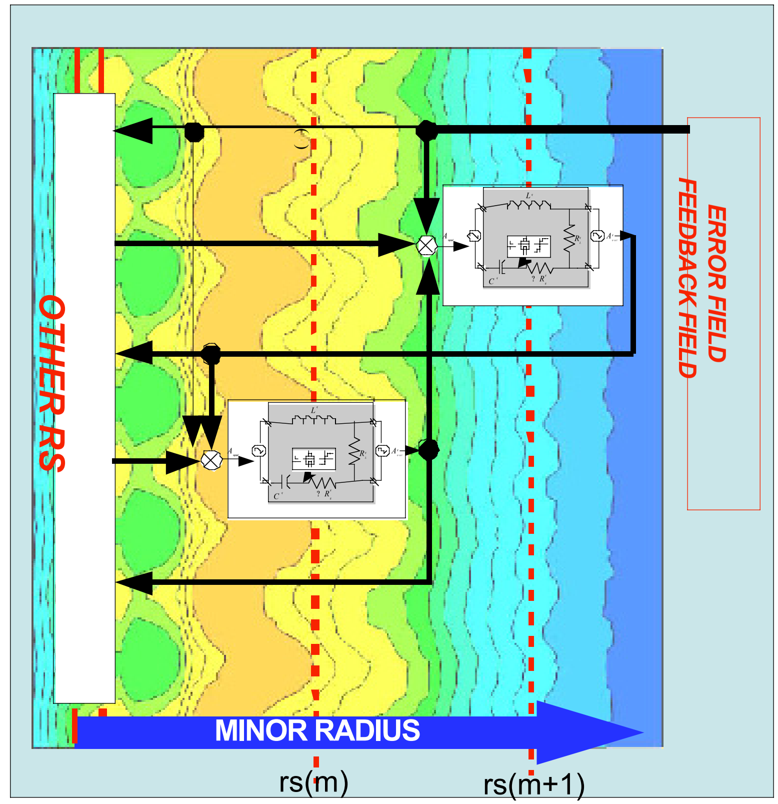
It happens when perturbation from feedback system becomes a dominant mode. In this case negative feedback transforms into positive feedback. Transition time is the same order as disruptive instability characteristic time. It means that using ordinary approach it is practically impossible to create a good feedback system!

Van der Pole model for plasma simulations

PLASMA



"ELECTROMECHANICAL" MODEL



CONCLUSIONS

1. Coupled resonant surfaces as active nonlinear media can be approximated by well known system of coupled Van der Pole oscillators.

2. Effect of frequency locking (or spacial harmonics locking in plasma frame), which is typical for these systems, may create positive feedback between resonant surfaces or between resonant surfaces and external feedback system.

3. Van der Pole model predicts the explosive dynamics of the disruptive instability and shows ways how to develop nonlinear feedback regulator.

Thank you for your attention