Basic Plasma Physics with Trapped Nonneutral Plasmas

Dan Dubin *84

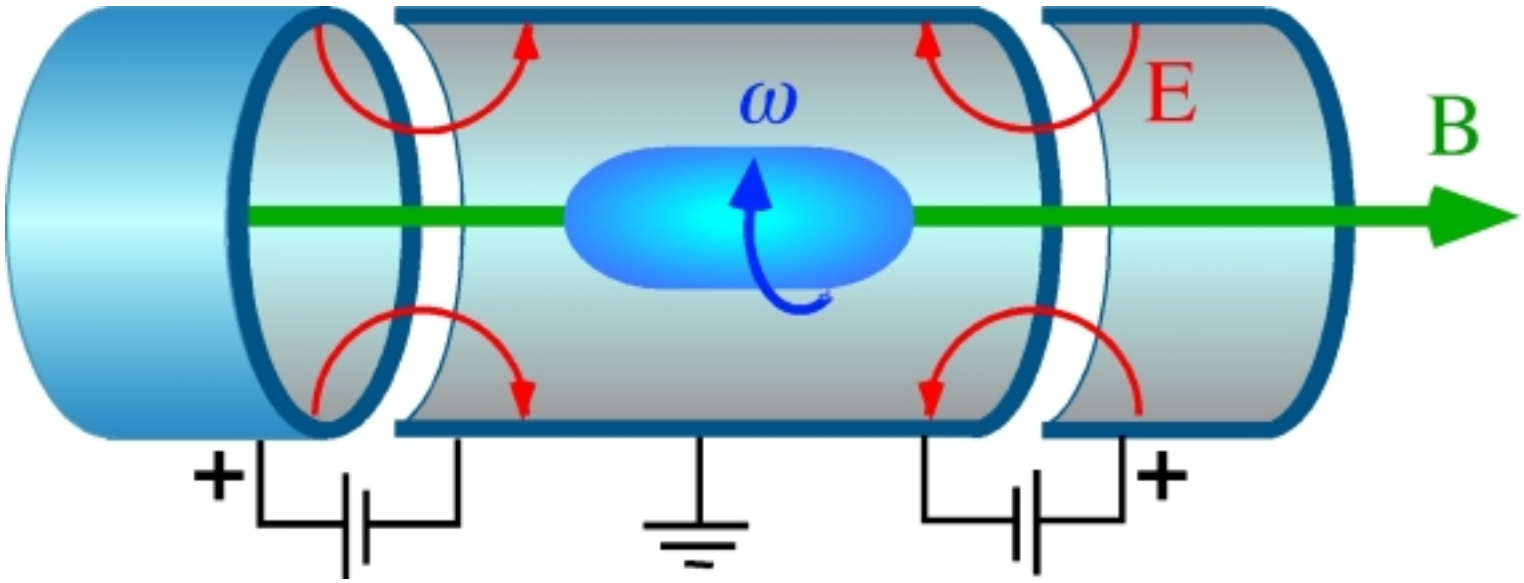
UCSD

Collaborators:

Experiment	Theory
Fred Driscoll	Tom O'Neil
Francois Anderegg	David Schecter
Kevin Fine	Dezhe Jin
John Bollinger	
Wayne Itano,	
Travis Mitchell	

Supported by the NSF and the ONR

Penning-Malmberg Trap



Cylindrical symmetry, single species => long confinement time

Confined thermal equilibrium: T uniform, ω uniform (rigid rotor)

$$P_\theta = \sum_{i=1}^{N} [m v_{\theta i} r_i + \frac{eB}{2c} r_i^2] = const.$$

Davidson and Krall, Phys. Fluids 1970
Dubin and O'Neil, R.M.P. 1999

Advantages of using Nonneutral plasmas

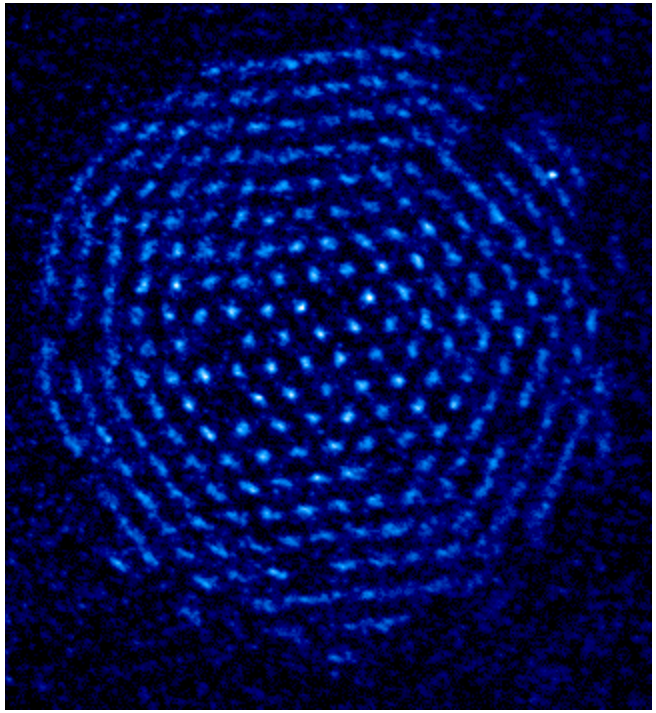
- *Thermal equilibrium → plasmas are highly reproducible, low noise*
- *experiments are relatively cheap*
- *Typical densities, temperatures:*
  $n = 10^6 - 10^8$ cm^{-3}, $T \sim 0.01 - 10^4$ *K*
- *A large range of phenomena can be studied:*
  atomic physics: fundamental constants, entangled quantum states, trapped antimatter studies,…
  fluid dynamics: 2D vortex motion, turbulence,...
  condensed matter physics: strong-correlation, phase transitions,...
  plasma physics: waves, transport, …..
- *Applications : frequency standards, ion mass spectrometry, microwave sources,...*

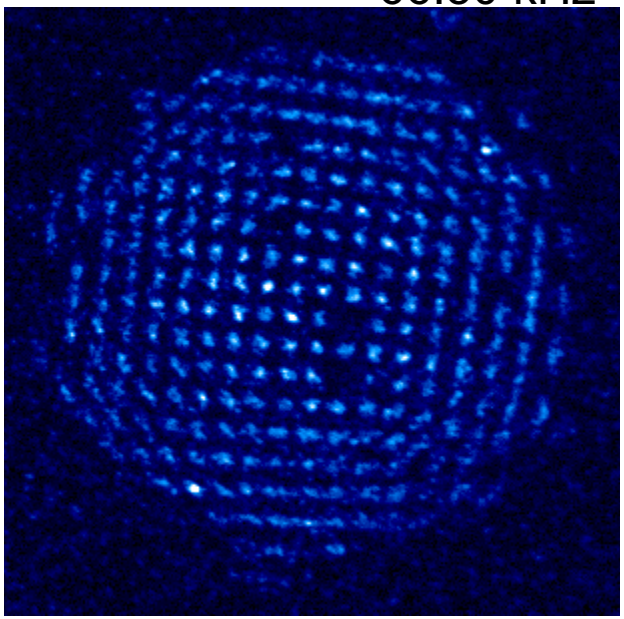
STRUCTUAL PHASE TRANSITIONS by changing **w**

65.70 kHz

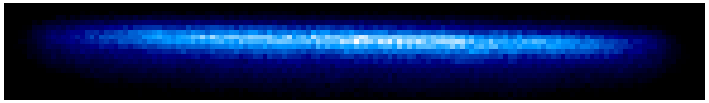
66.50 kHz

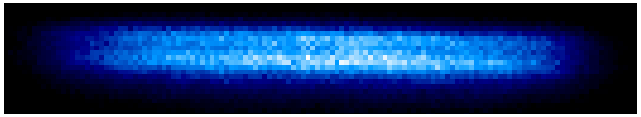
top-views





side-views



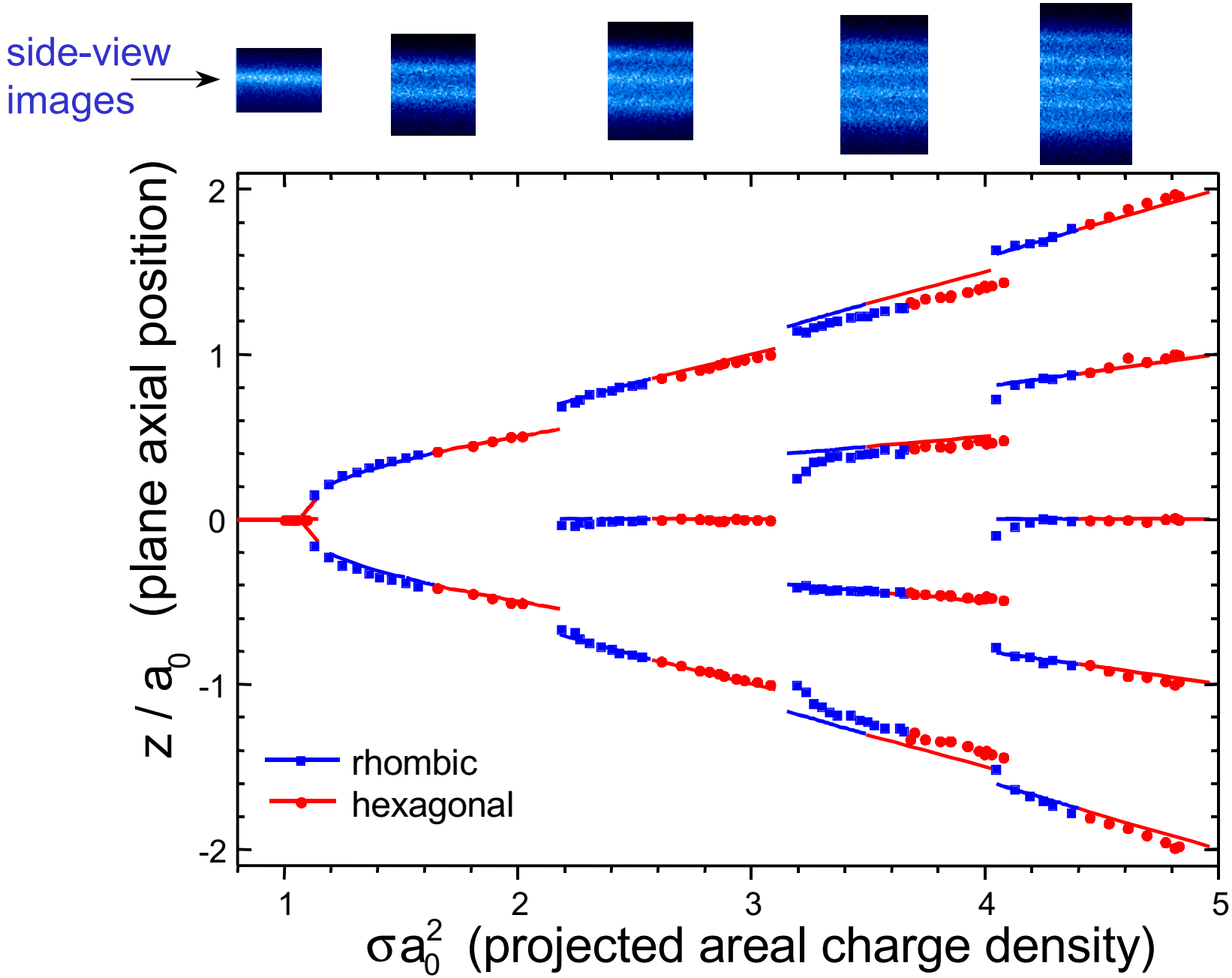


1 lattice plane, hexagonal order

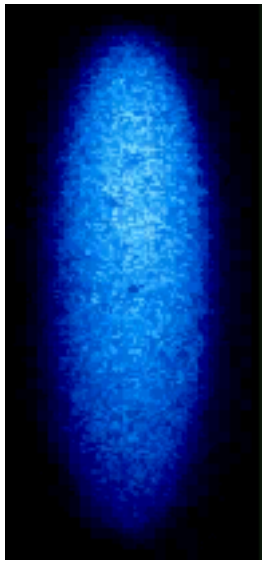
2 planes, cubic order

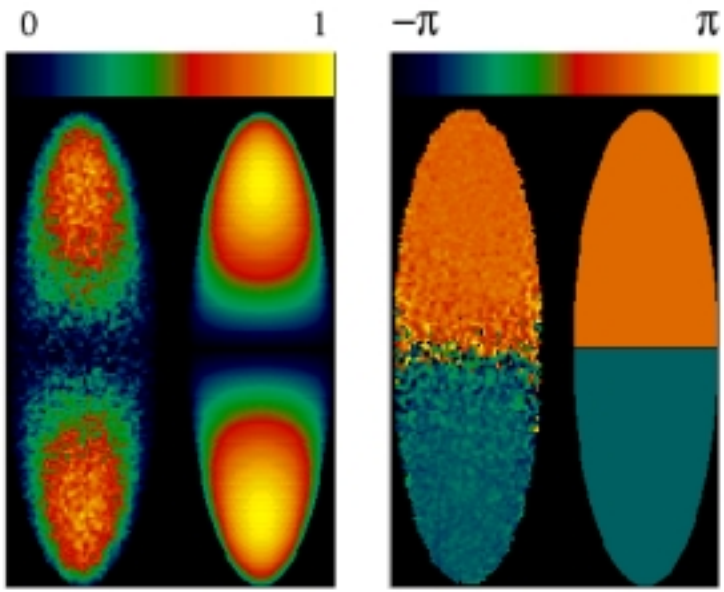
Mitchell, Bollinger, Dubin, Huang, Itano and Baughman, Science 1998

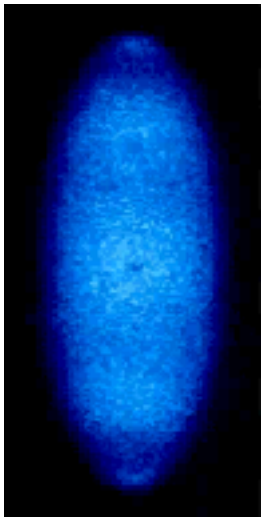
Structural Phase Transitions: experiment and theory

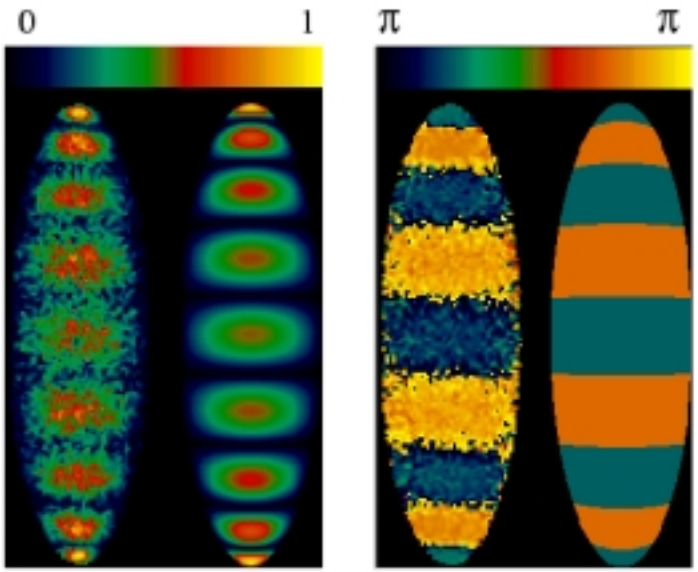


MAGNETIZED PLASMA MODES in a cold spheroidal plasma





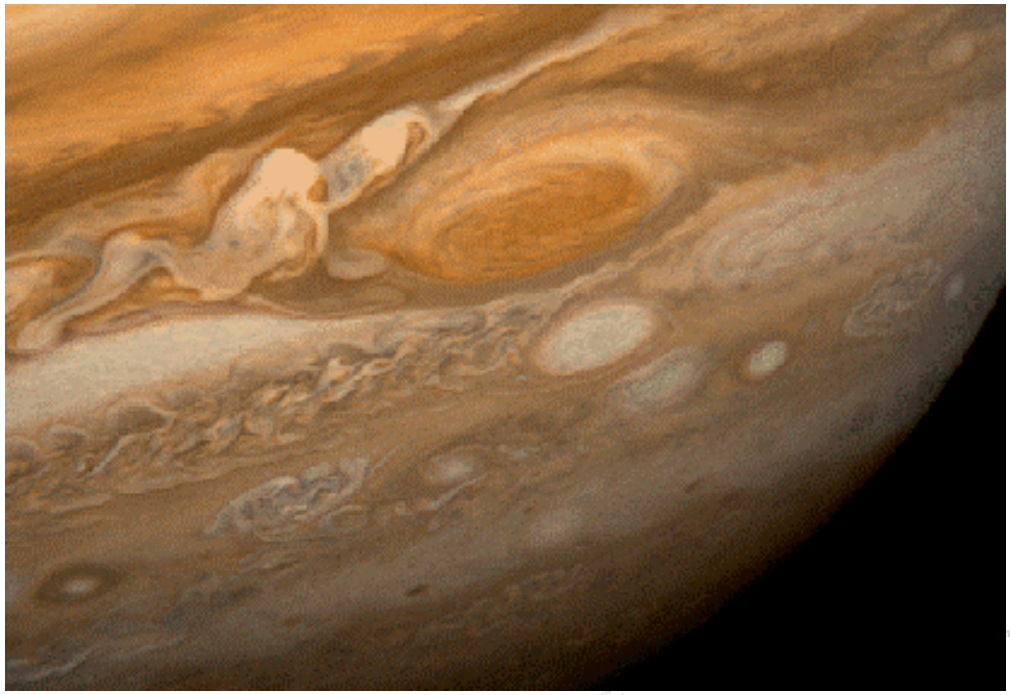


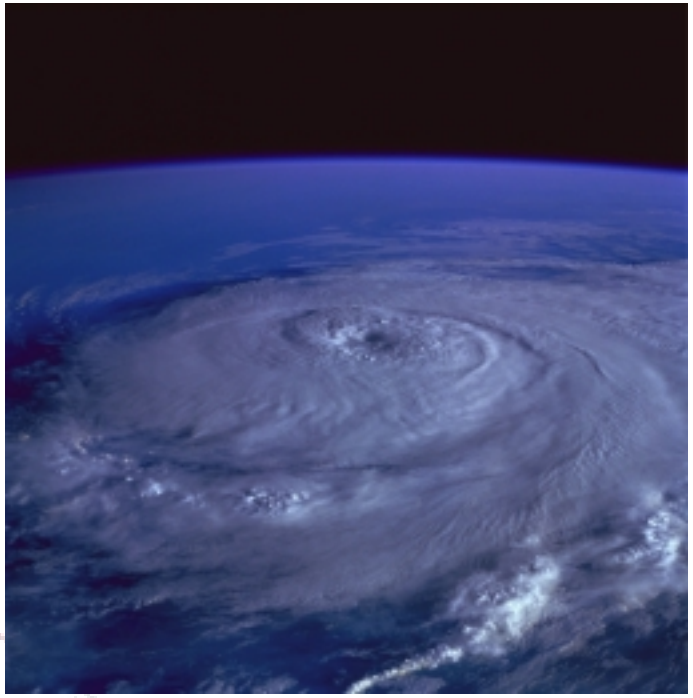


Mitchell, Bollinger, Huang and Itano, Optics Express 1998

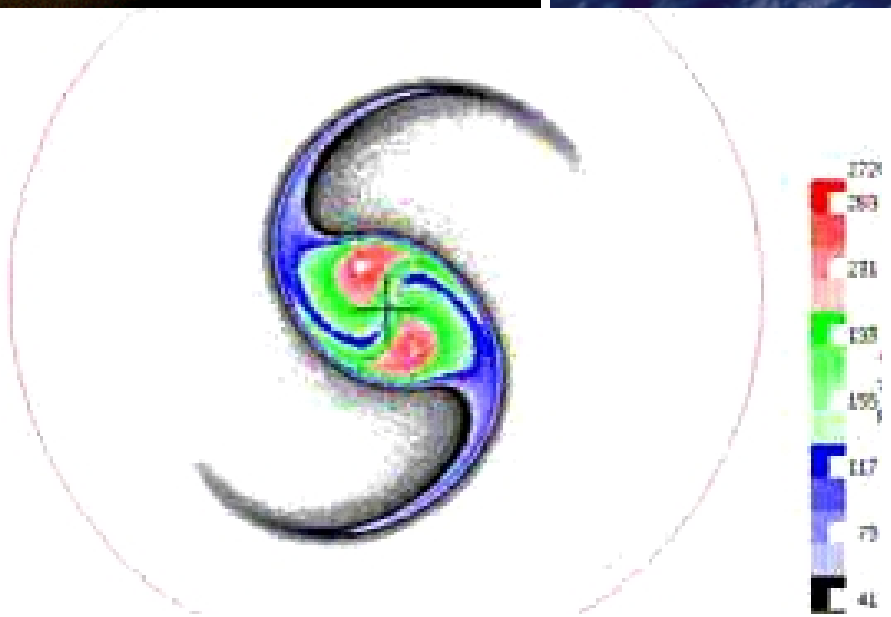
Dubin, PRL 1991

Nonneutral plasmas as 2D inviscid fluids

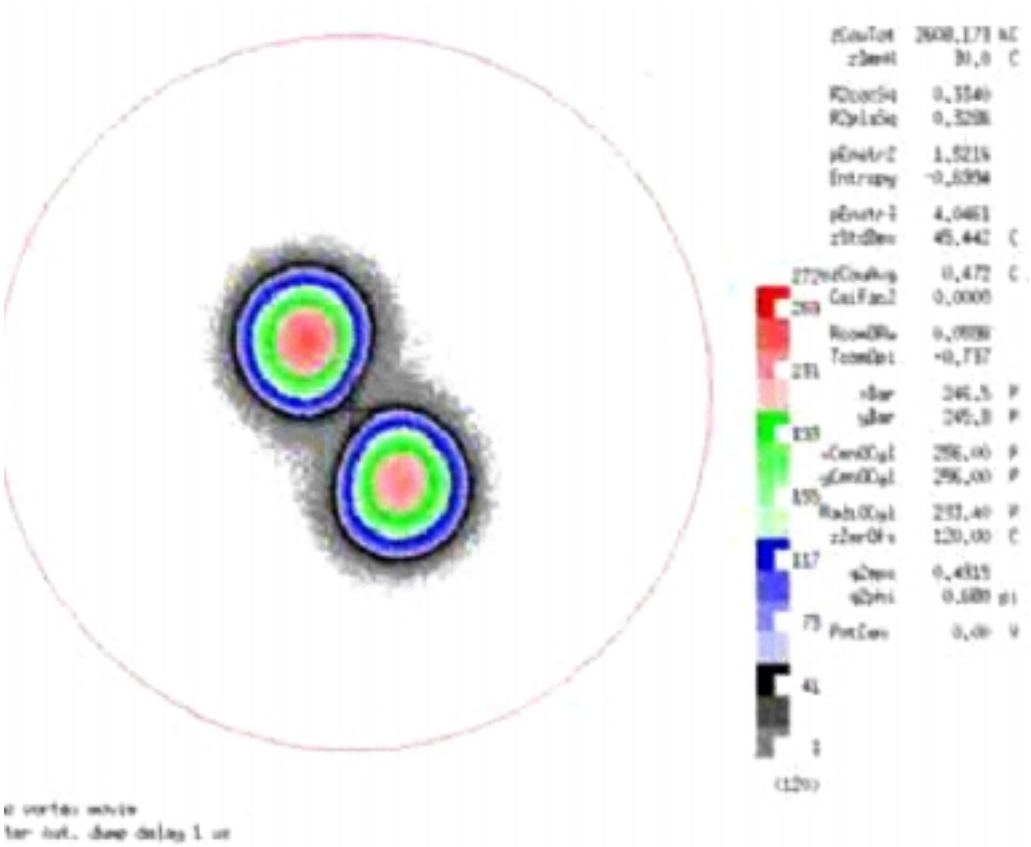


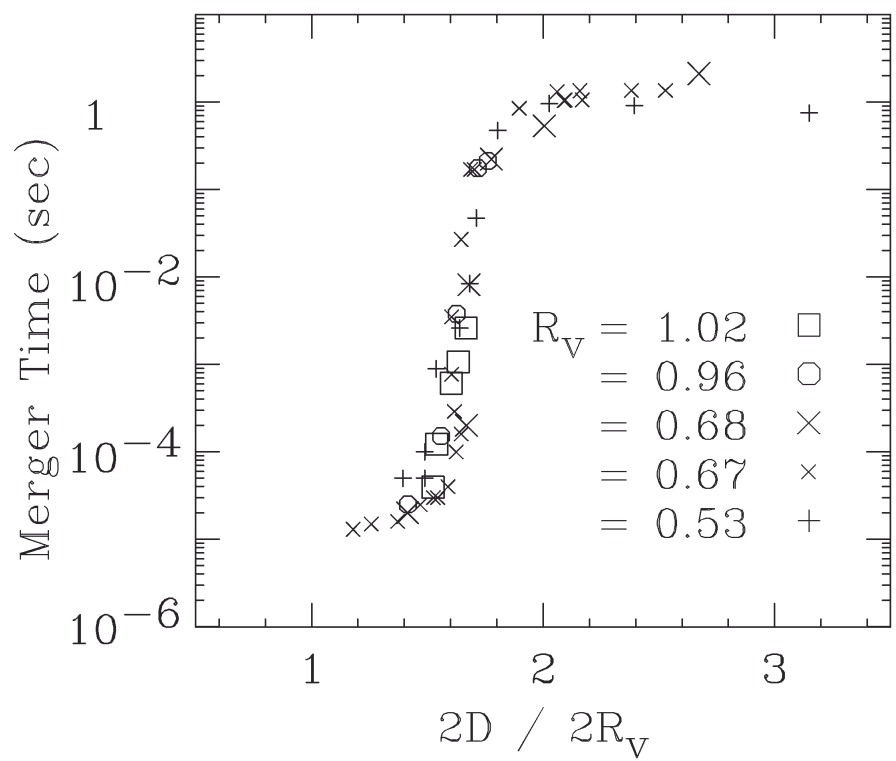


$Re \sim 10^5$



Merger of like-sign vortices





High T plasma Topic:

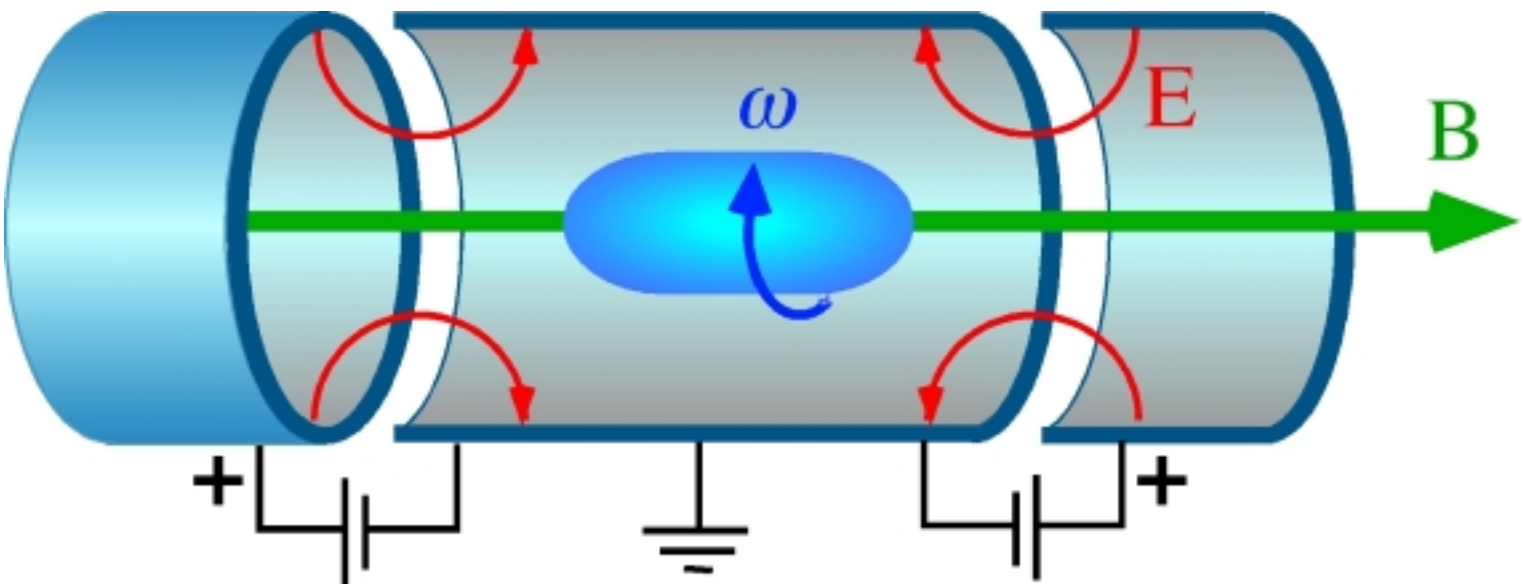
Cross-magnetic-field collisional transport of particles and heat

Collisional Transport

Confined thermal equilibrium:

Uniform ω from shear viscosity η

Uniform T from thermal conductivity κ



test particle diffusion D

Fick's law:
radial flux of test particles

$$\Gamma_{test} = -D n_{total} \frac{\partial}{\partial r}\left(\frac{n_{test}}{n_{total}}\right)$$

$D \sim \mathrm{cm}^2/\mathrm{sec}$

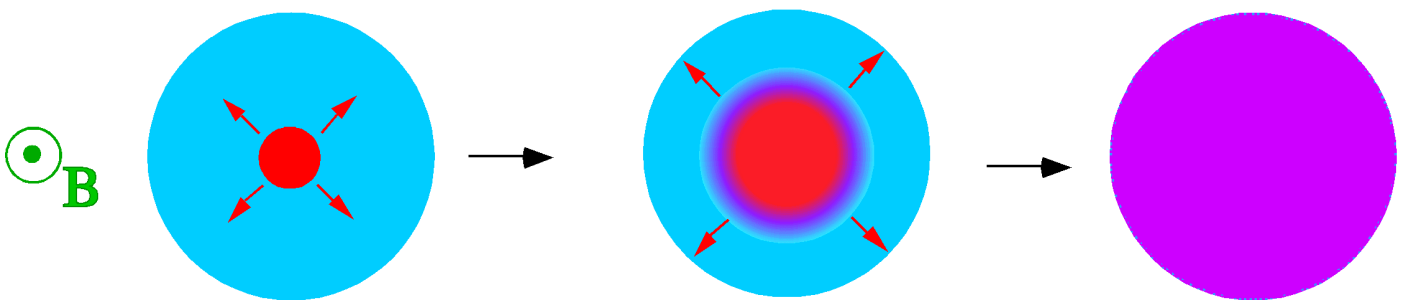
Thermal conductivity κ

radial flux of heat

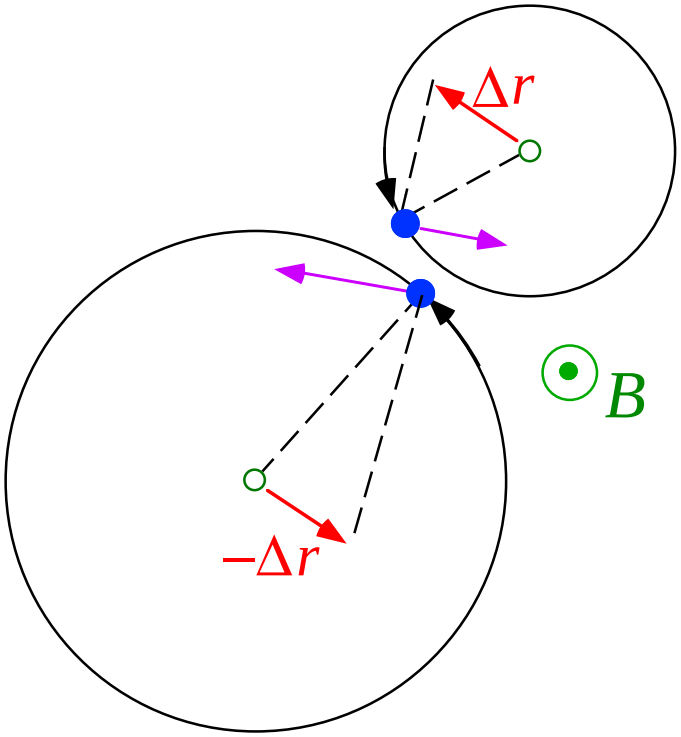
$$\Gamma_Q = -\kappa \frac{\partial T}{\partial r}$$

$\kappa/n \sim \mathrm{cm}^2/\mathrm{sec}$

thermal diffusivity χ



Classical Theory of Collisional Transport implicitly assumes $r_c >> \lambda_D$



Step size $\Delta r \sim r_c$

Frequency of steps ~ collision frequency ν_c

$$D \sim \frac{<\Delta r^2>}{\Delta t} \sim \nu_c r_c^2$$

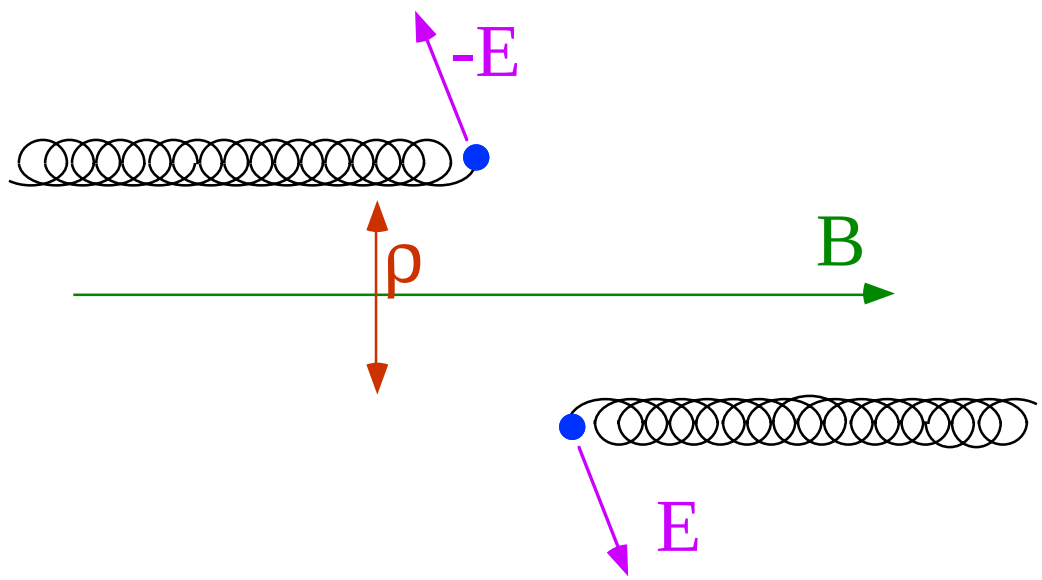
$$\frac{\kappa}{n} \sim D \sim \nu_c r_c^2$$

Spitzer 1955
Longmire and Rosenbluth 1956
Simon 1956
Rosenbluth and Kaufmann 1958
Braginskii 1958

BUT, classical theory **does not apply** if $r_c < \lambda_D$

Eg. Electron-electron collisions in edge of tokamak, nonneutral plasma, …

Now most collisions are long range : $r_c \lesssim \rho \lesssim \lambda_D$

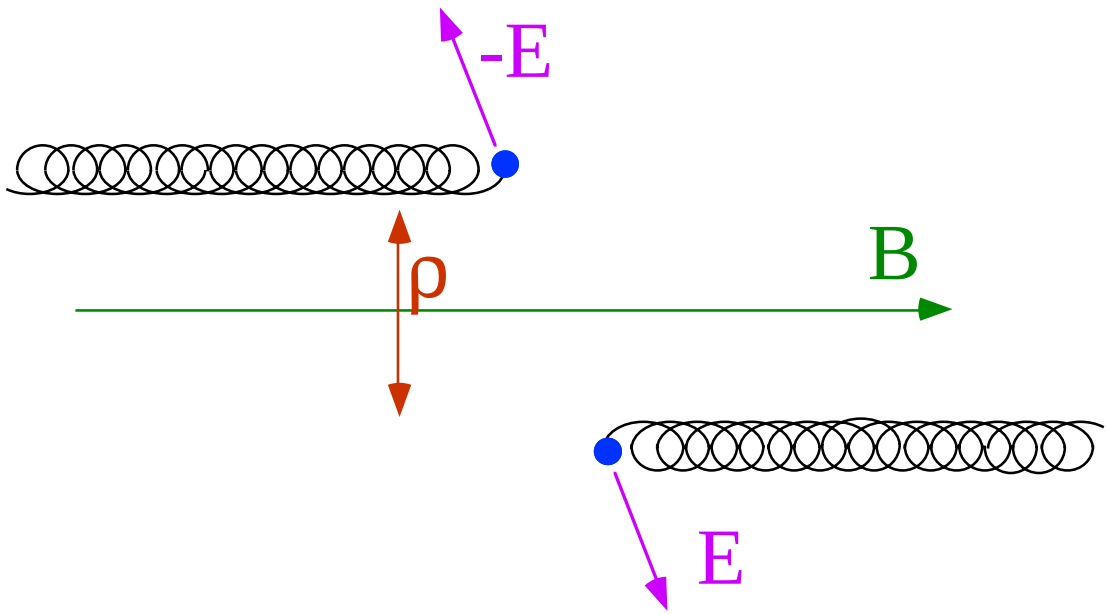


Viscosity, thermal conductivity, diffusion,…:

all need to be recalculated, measured!

have different scalings with n, B, T

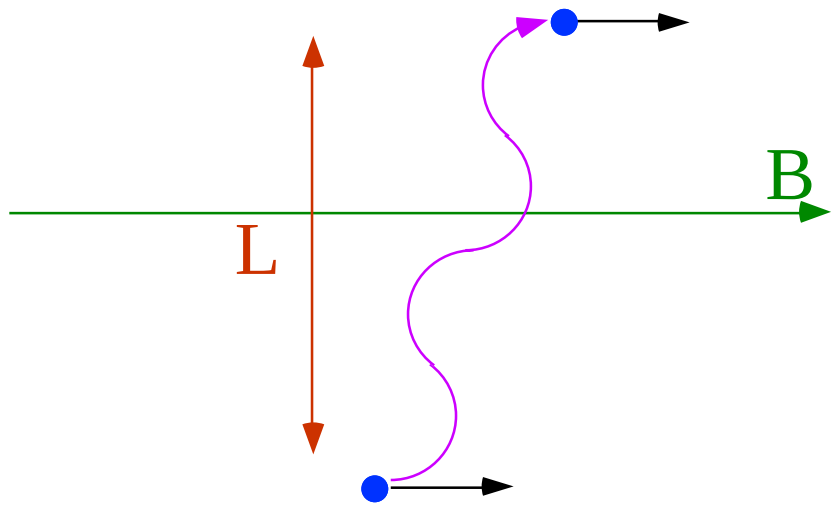
can be **MUCH** larger than classical predictions



$$\kappa / n \sim \nu_c < \rho^2 > \quad \boxed{\Rightarrow \kappa \sim n \nu_c \lambda_D^2 >> \kappa^{class}}$$

Dubin + O'Neil '97, Psimopolis and Li '92

Emission and adsorption of lightly-damped plasma waves enhances transport



$\rho \sim$ plasma width L (but $\nu < \nu_c$)

$$\kappa^{waves} > n\nu_c\lambda_D^2 \quad \text{if L} \gtrsim 10^2\,\lambda_D$$

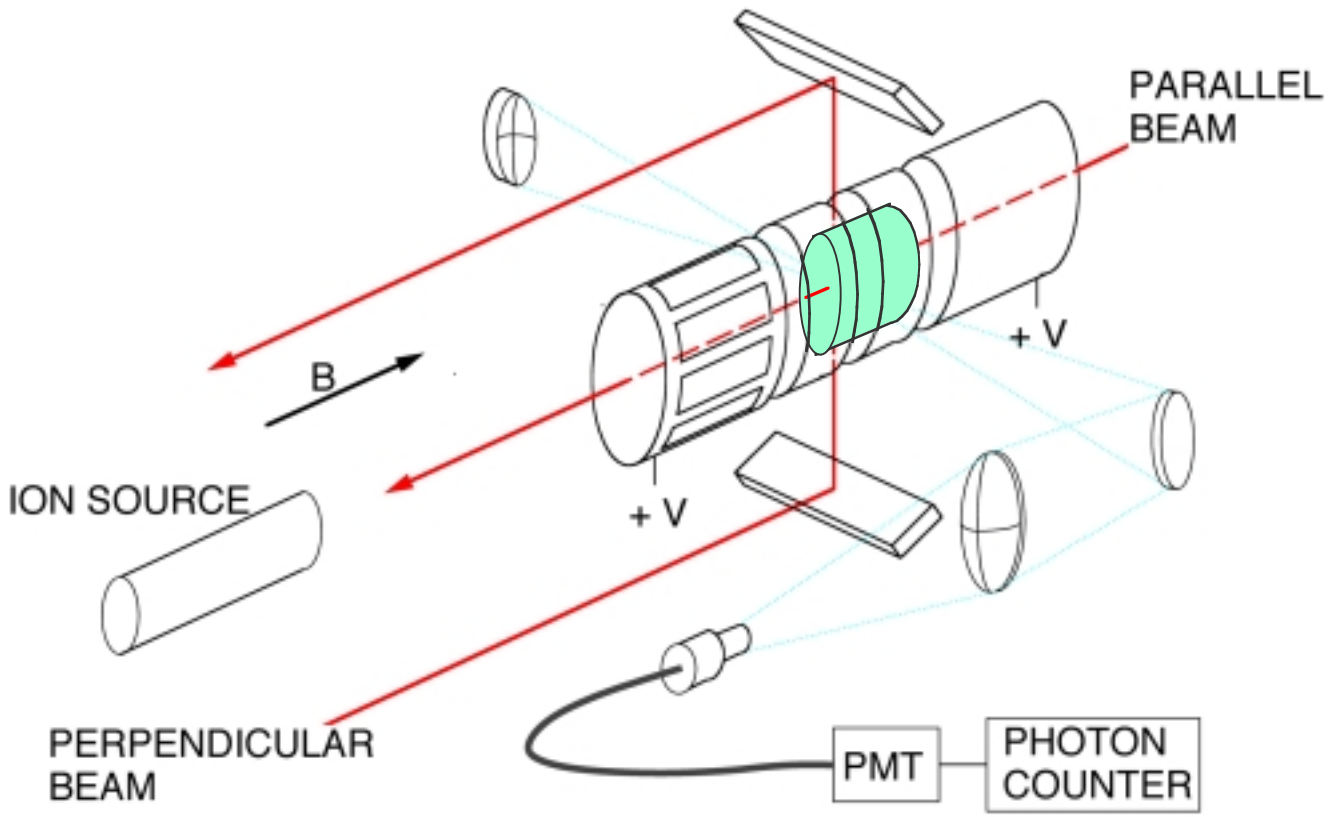
Rosenbluth and Liu ‘76
Dubin and O’Neil ‘97

Experiments on Pure Ion Plasmas

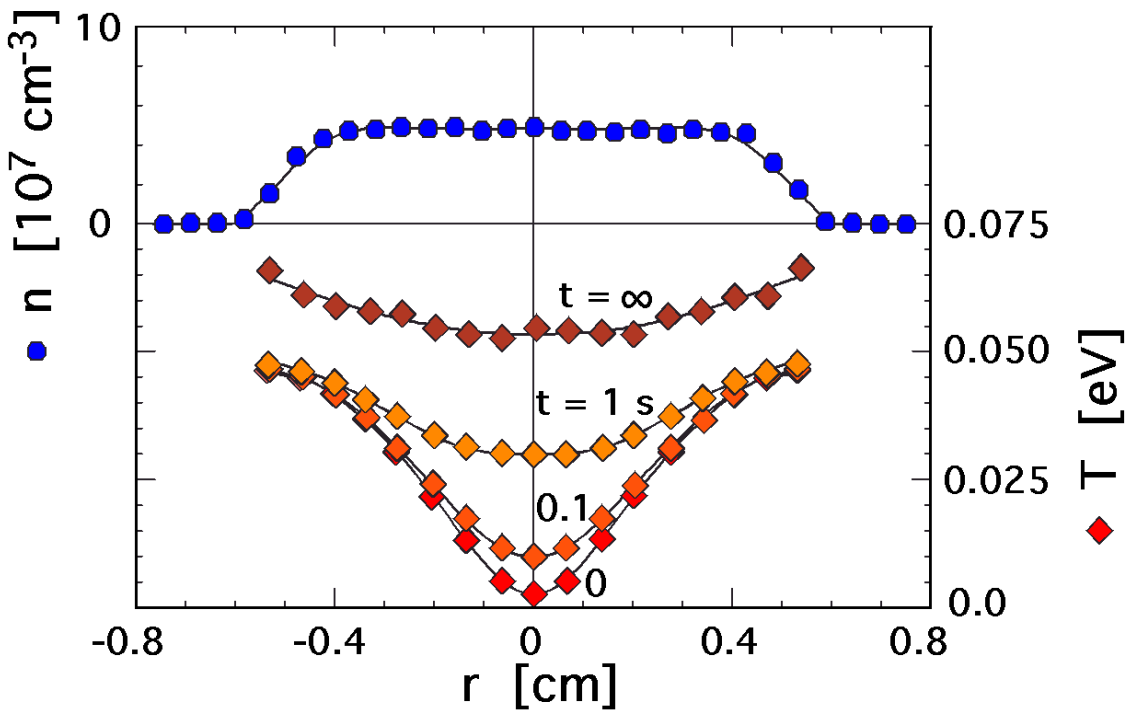
UV Laser diagnostics on Mg^+ plasma allow measurement of $f(r,\mathbf{v})$

=> $T(r)$, $n(r)$

Also, lasers can locally heat/cool the plasma



Measuring Heat Flux



$$\Gamma_q = -\frac{1}{r}\int_0^r r'dr'\left(\frac{3}{2}n\dot{T} - \dot{q}_{ext}\right)$$

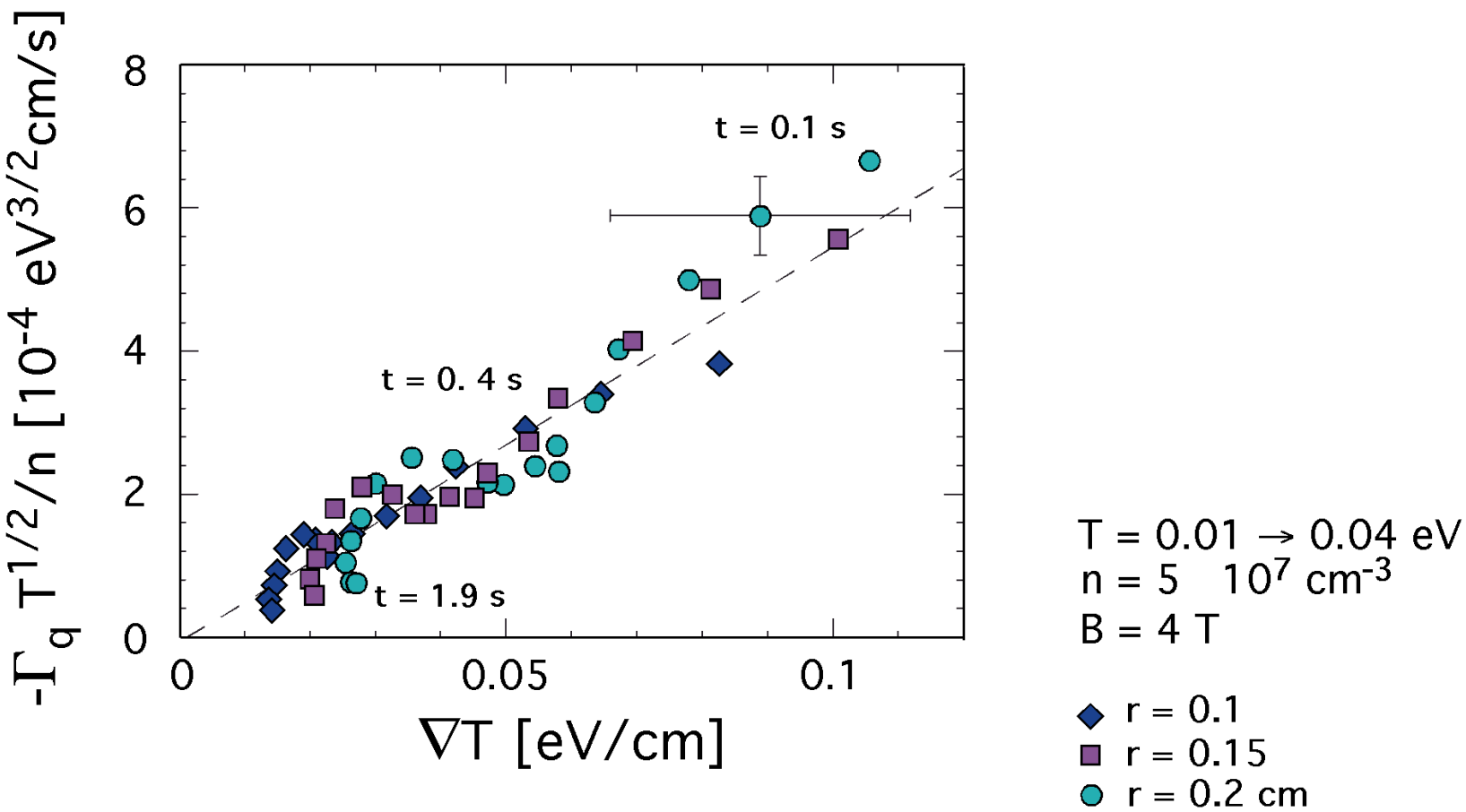
radial heat flux

measured heat sources

measured temperature change

Measured Heat flux is diffusive

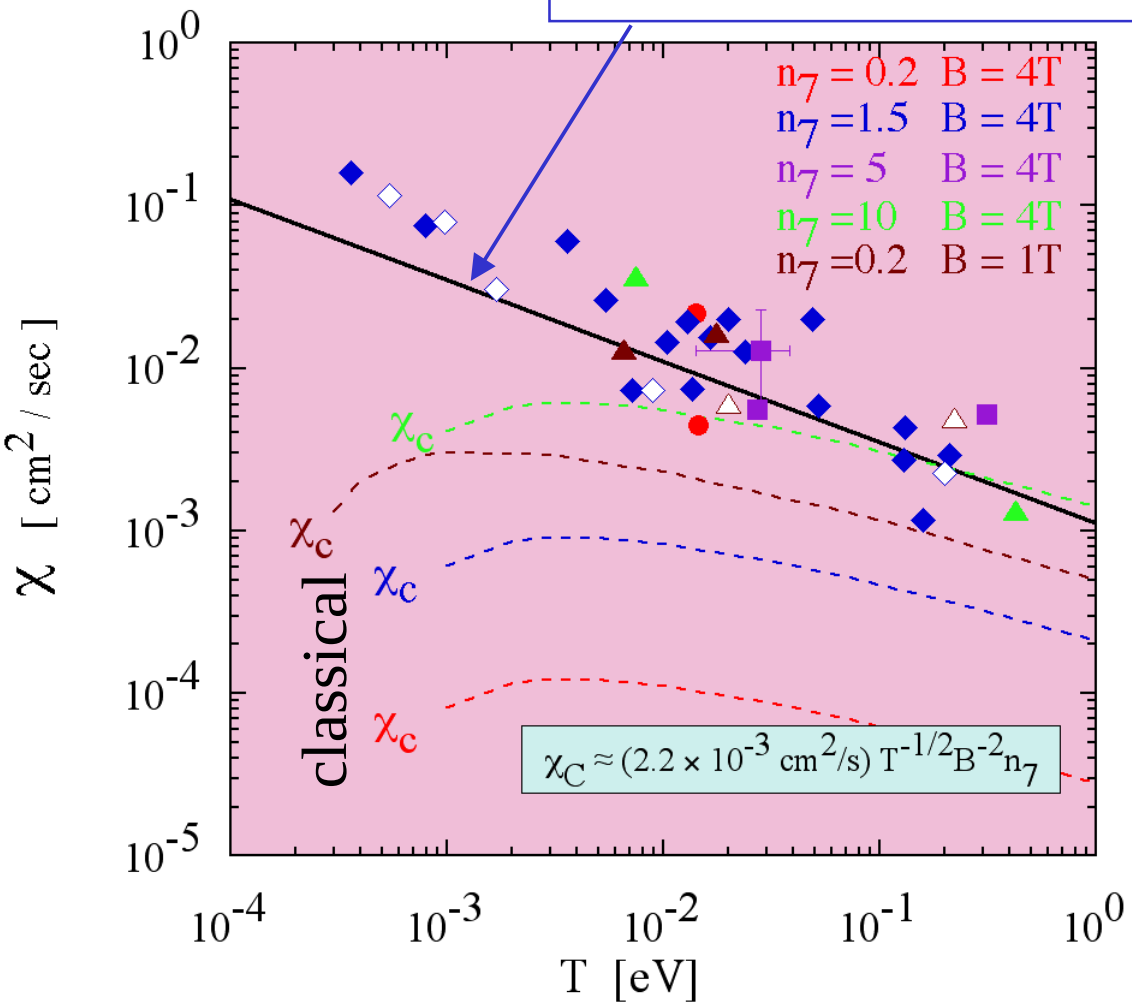
i.e. Measured heat flux is proportional to the temperature gradient



Thermal Conduction: theory and experiment

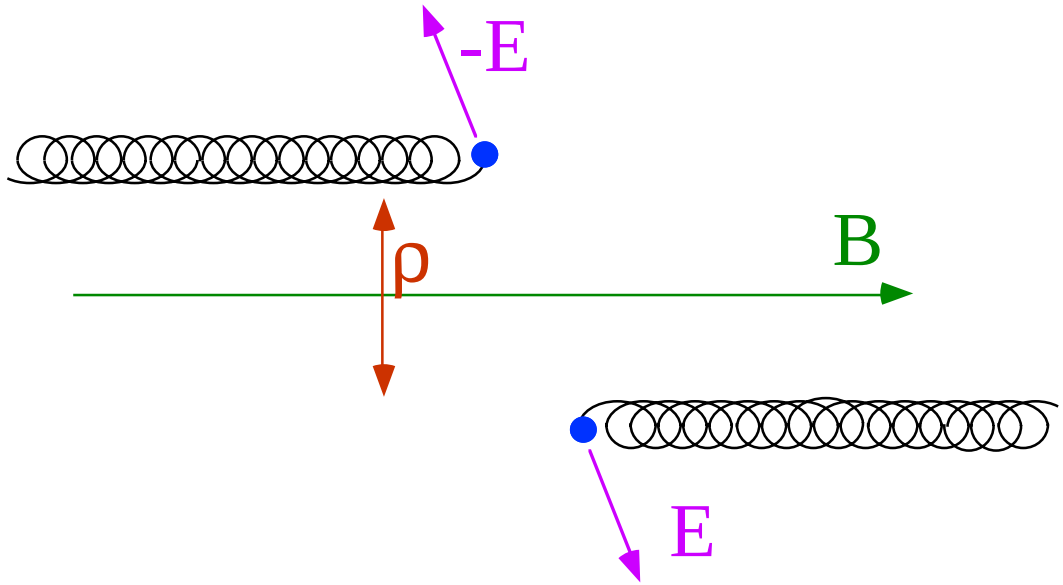
κ is up to 300x classical

$$\kappa / n = 1.2\, \nu_c \lambda_D^2 = 0.097 \frac{e^2}{m\bar{v}}$$



Hollmann et al., PRL 1999

Test Particle diffusion:



Coulomb interaction

$$\Delta x = \int_{-\infty}^{\infty} dt\, E_y(t) \frac{c}{B}$$

Integration along unperturbed orbits:

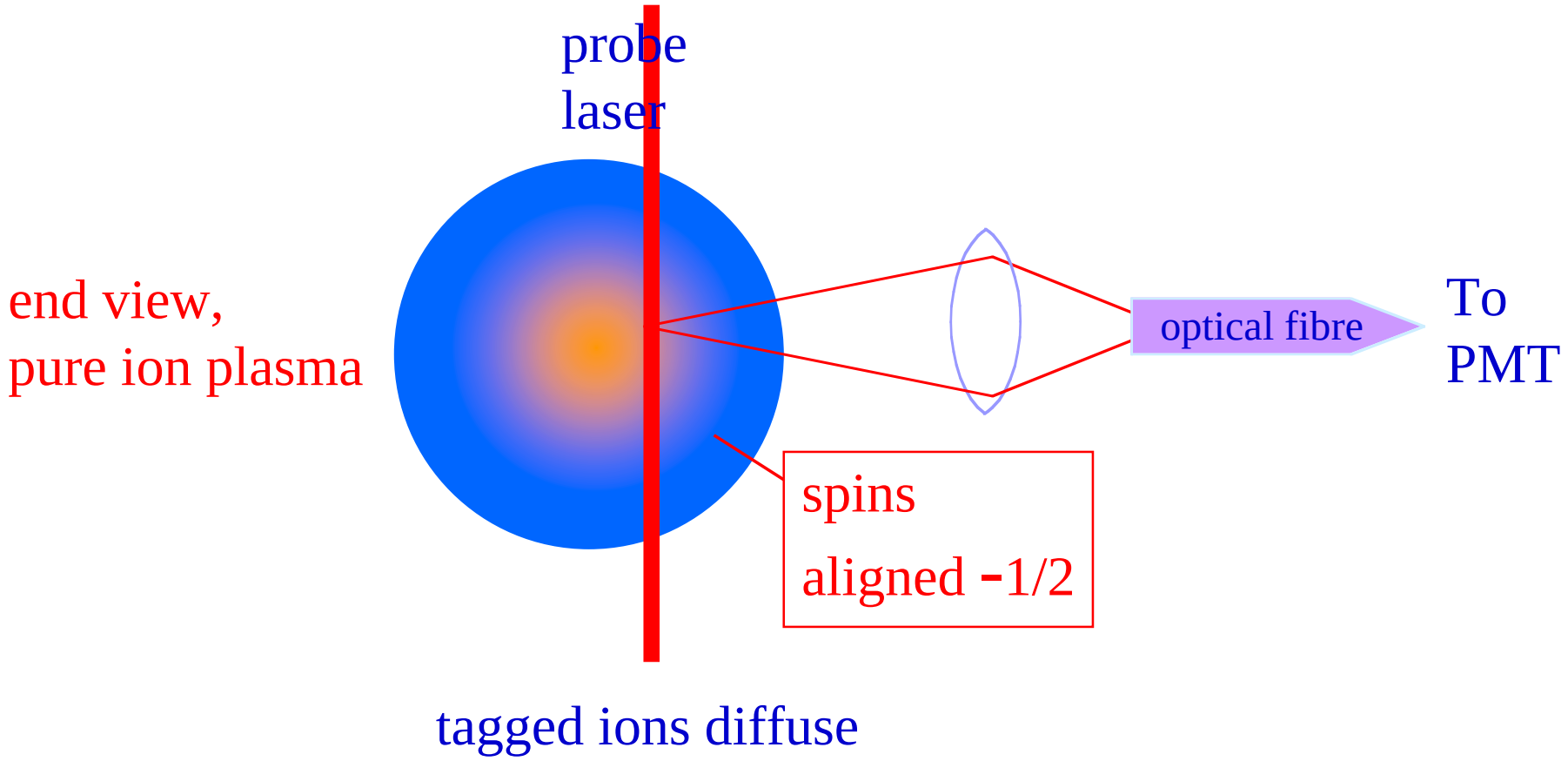
Relative z velocity v_z = constant

$$\Rightarrow D^{IUO} \sim v_c r_c^2 \quad (\sim 3\, D^{class})$$

Anderegg et. al., PRL 1997

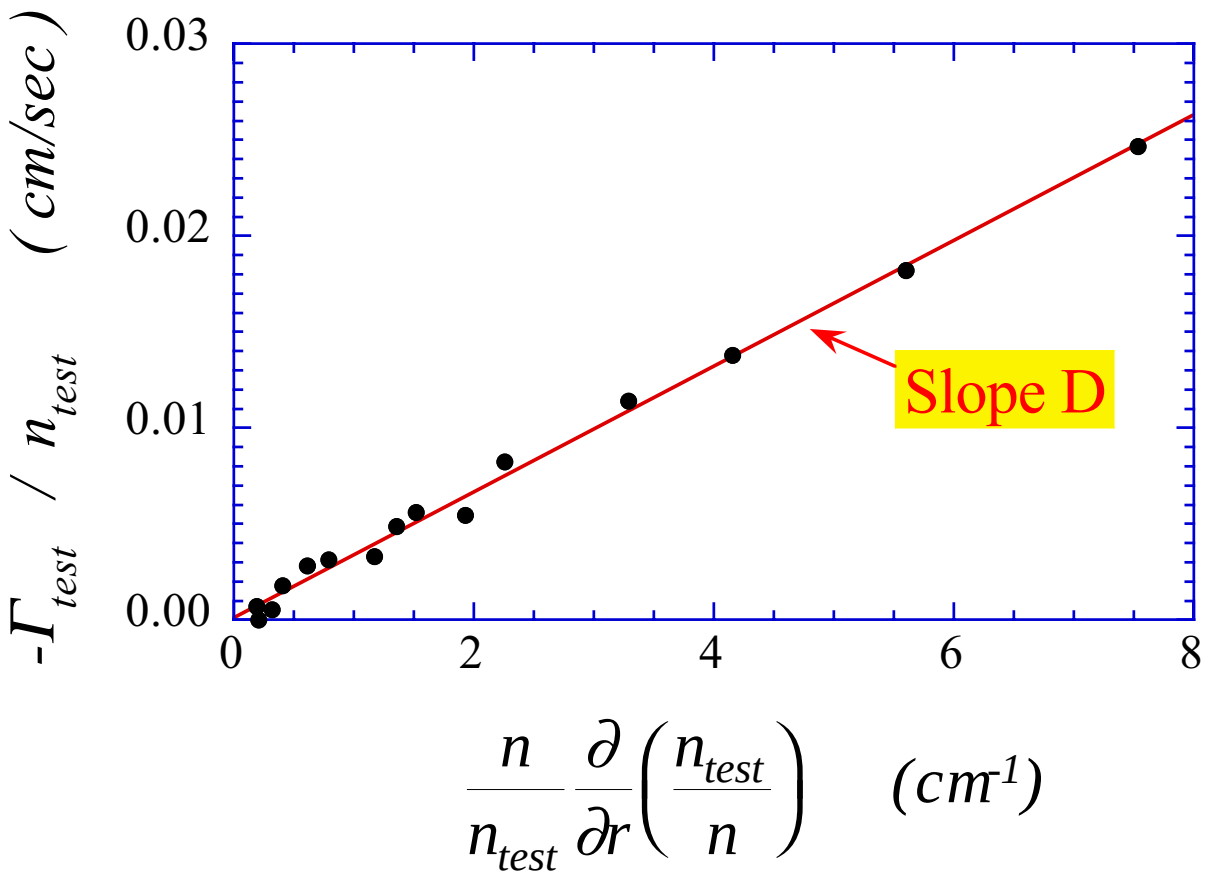
Lifshitz and Pitaevskii, *Physical Kinetics*

Experiment: ions are tagged and probed using lasers

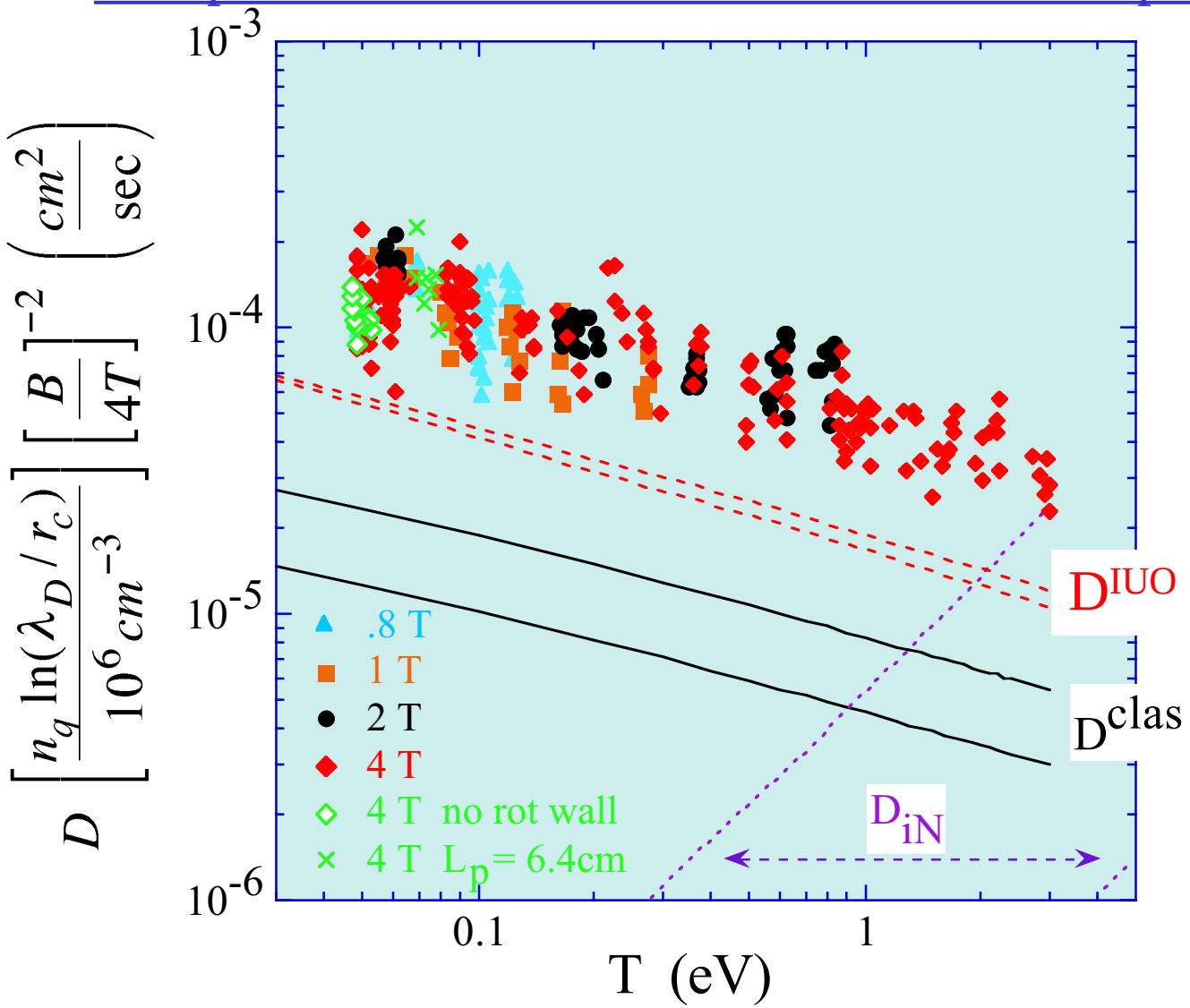


Measured Test Particle flux satisfies Fick's law

$$\Gamma_{test} = -Dn\frac{\partial}{\partial r}\left(\frac{n_{test}}{n}\right)$$



Test particle diffusion coefficient versus Temperature



Measured diffusion ~ 10x classical, ~ 3 x D^{IUO}

Add velocity diffusion to orbits

$$\Delta x = \int dt \frac{c}{B} E_y(t)$$

$$= -\frac{ec}{\pi^2 B} \int d^3k \frac{ik_y}{k^2} e^{i\mathbf{k}\cdot\mathbf{r}} \times \int_0^\infty dt\, e^{i k_z \mathrm{v}_z t - k_z^2 D_v t^3/3}$$

Velocity diffusion coefficient

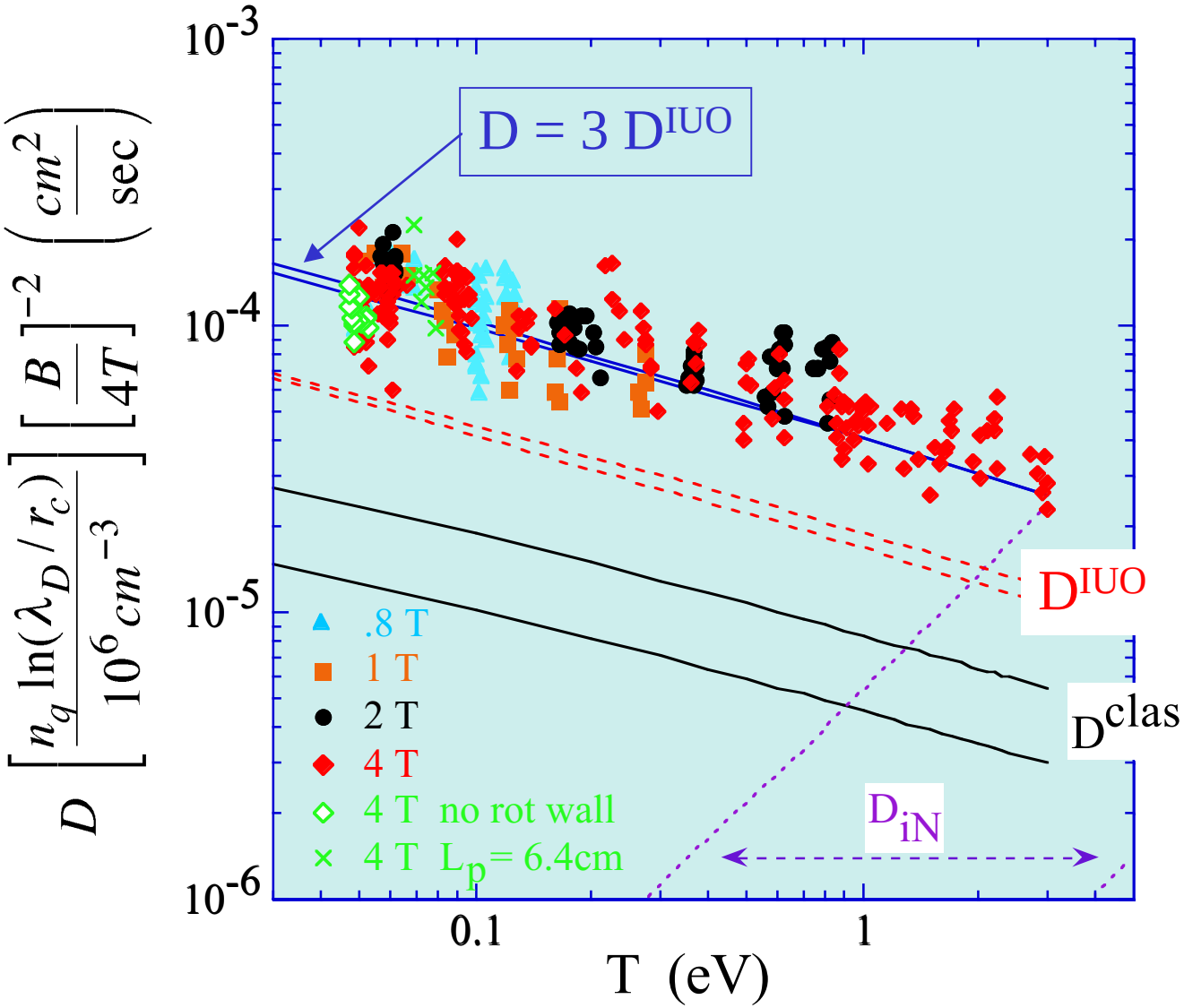
$$\cancel{\frac{\pi\delta(k_z)}{|\mathrm{v}_z|}} \; for\ D_v = 0;$$

$$\frac{3\pi\delta(k_z)}{|\mathrm{v}_z|} \; for\ D_v \to 0 \;\; !!!$$

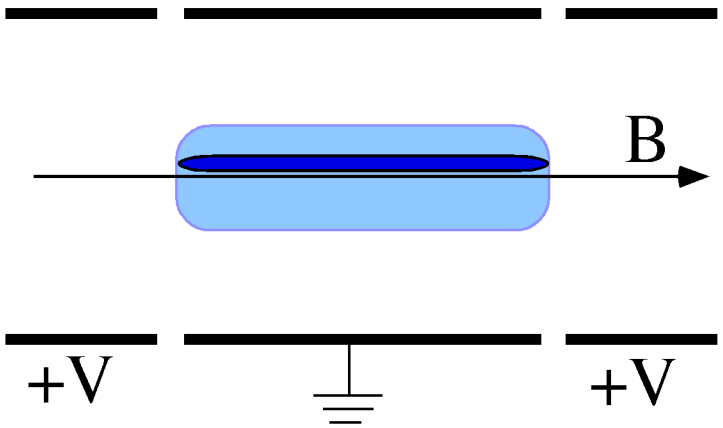
Result: $D \sim 3\, D^{IUO} \sim 9\, D^{class}$

Dubin, PRL '97

Theory that includes effect of collisional caging agrees with experiments!



2D ExB Theory of Collisional Diffusion



Bounce frequency ω_b large

$\omega_b / [r\, d\omega_{ExB}/dr] >> 1$

bounces N_b >>1

=> Ions behave like charged rods

$$\frac{d\mathbf{r}_i}{dt} = \frac{c}{B}\mathbf{E}(\mathbf{r}_i) \times \mathbf{z}$$

All times scale as B: $t = \bar{t}B / c$ => $\frac{d\mathbf{r}_i}{d\bar{t}} = \mathbf{E}(\mathbf{r}_i) \times \mathbf{z}$

Units of diffusion are distance^2/time

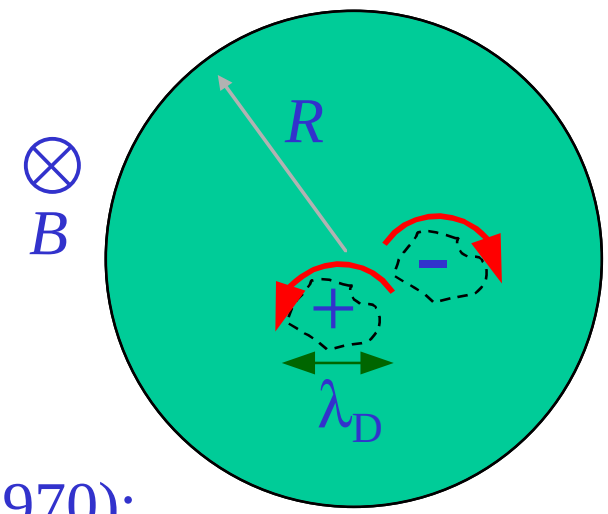
$D^{2D} \sim 1/B >> D^{classical}$

Taylor/McNamara,
Dawson/Okuda

‘Diffusion’ in shear-free homogeneous 2D guiding center plasma

Taylor/McNamara (1970):
N rods, |charge|/length q

$$D^{TM} = \frac{qc}{2B}\sqrt{\frac{N}{\pi}}$$

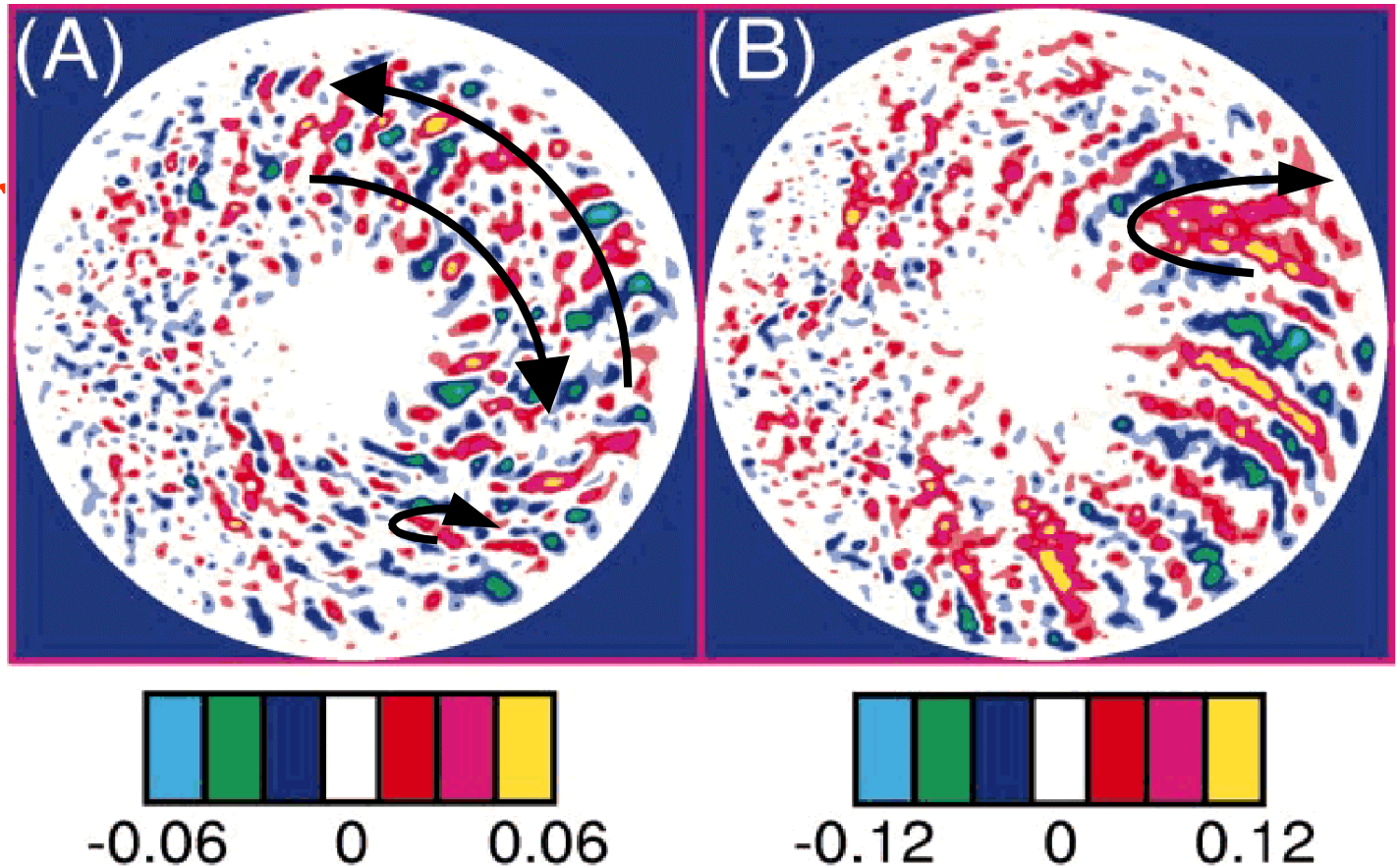


Taylor/McNamara, Dawson/Okuda (1970):
Debye shielding limits size of largest fluctuations

$$D^{DO} \sim D^{TM}\frac{\lambda_D}{R}$$

Shear reduces transport in tokamak plasmas

With shear



Without shear

Shear in ExB flow rips apart the turbulent eddies

(simulations) Lin et al., Science 281 (1998)

Shear reduction of *collisional* transport in a 2D guiding-center plasma

Fluctuations are not from turbulence but rather from intrinsic graininess (collisions)

Diffusion in presence of shear can be calculated rigorously using Boltzmann/Klimontovitch theory
(Dubin + Jin, Phys. Lett. A 2001)

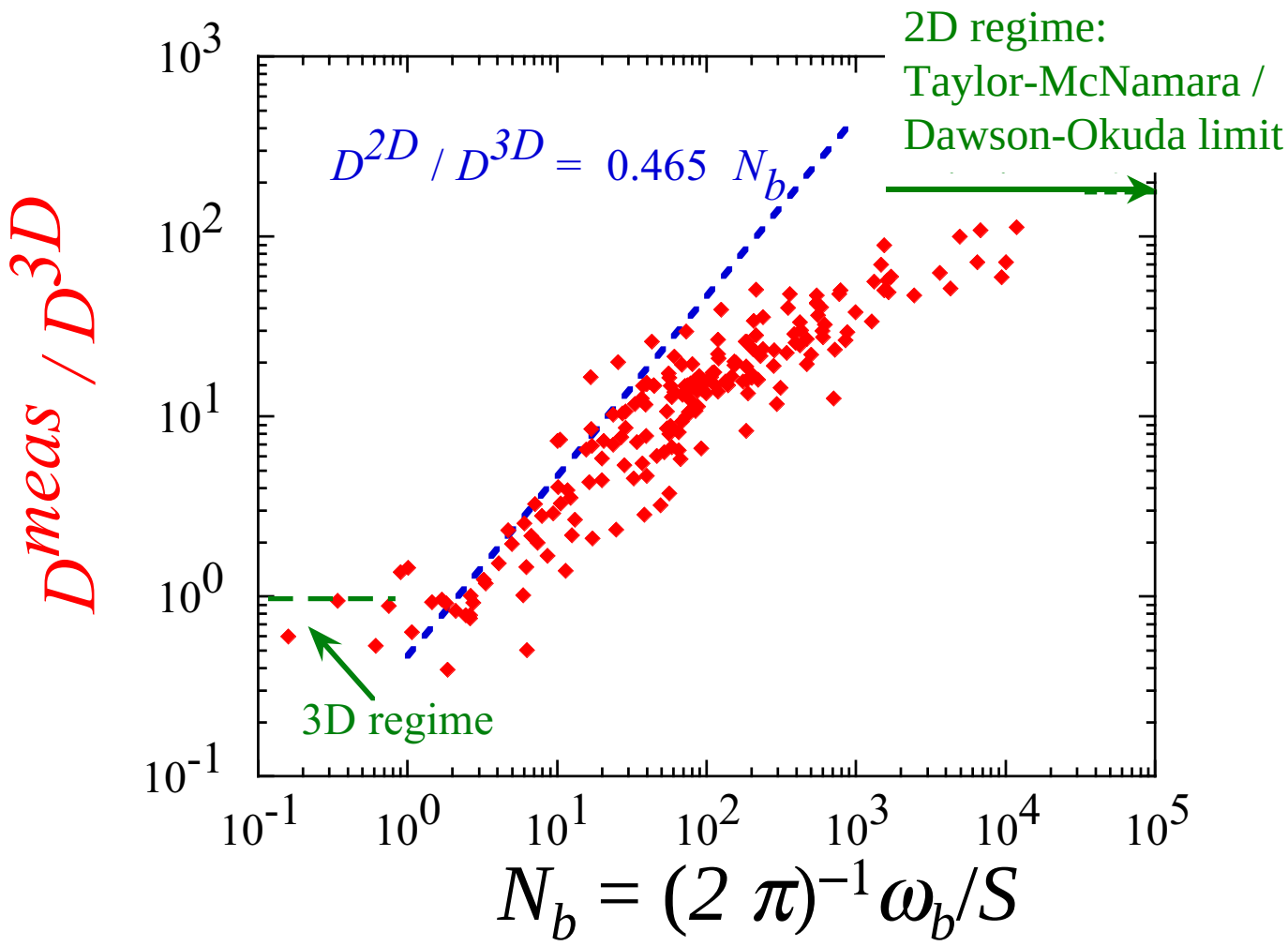
$$\boxed{D^{2D} = \frac{4\pi^2 q^2 c^2 n_{2D}}{B^2 |S|}\left(\ln^2(\Lambda_B) + \pi^2 \ln \Lambda_K\right)}$$

Diffusion in 2D regime can be measured experimentally.
For parameters of experiment,

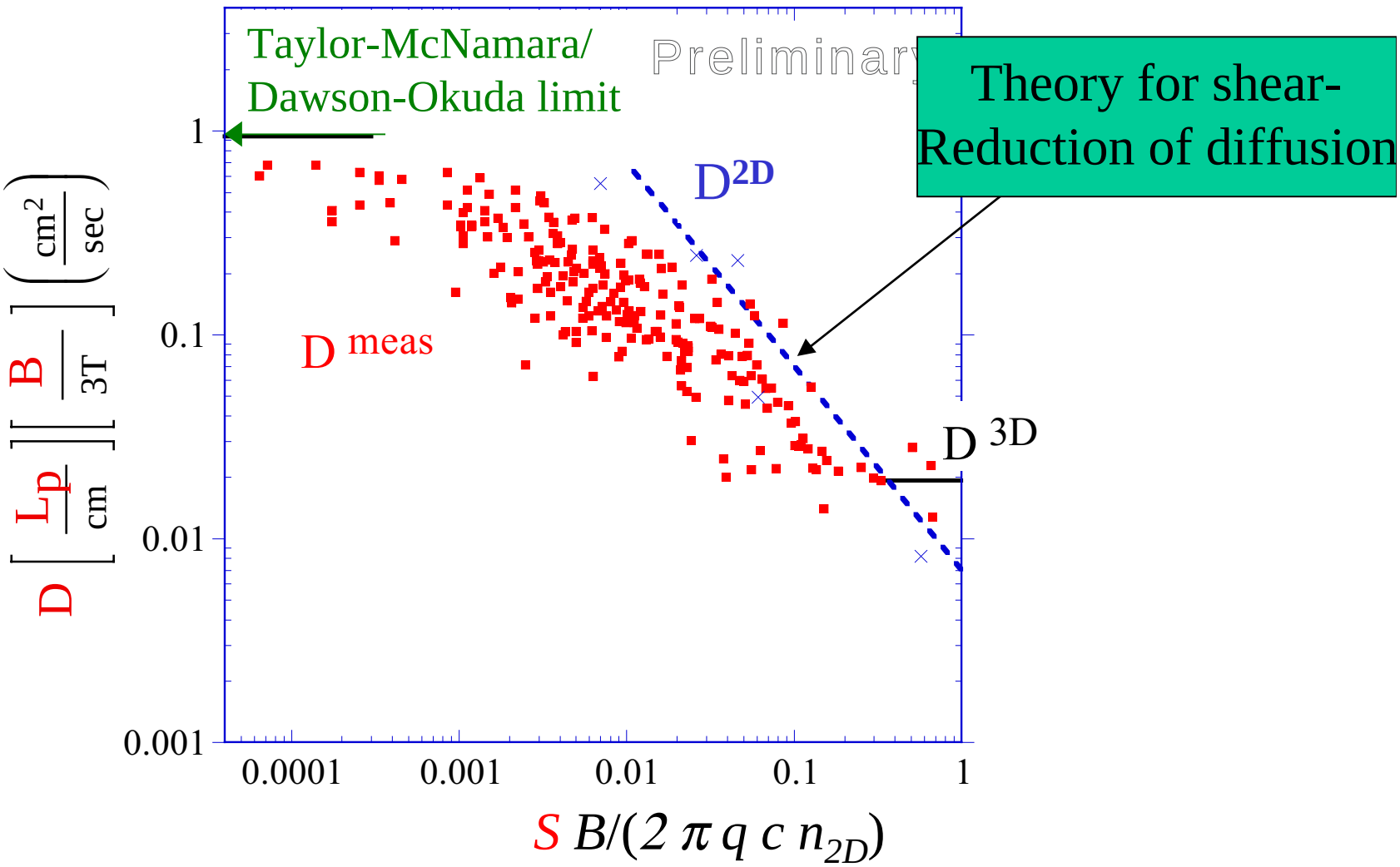
$$D^{2D} \cong 0.5 \; D^{3D} \boxed{\frac{\omega_b}{2\pi |S|}} \rightarrow N_b$$

Preliminary Experiments:

Diffusion *increases* from 3D level as collisions become 2-Dimensional

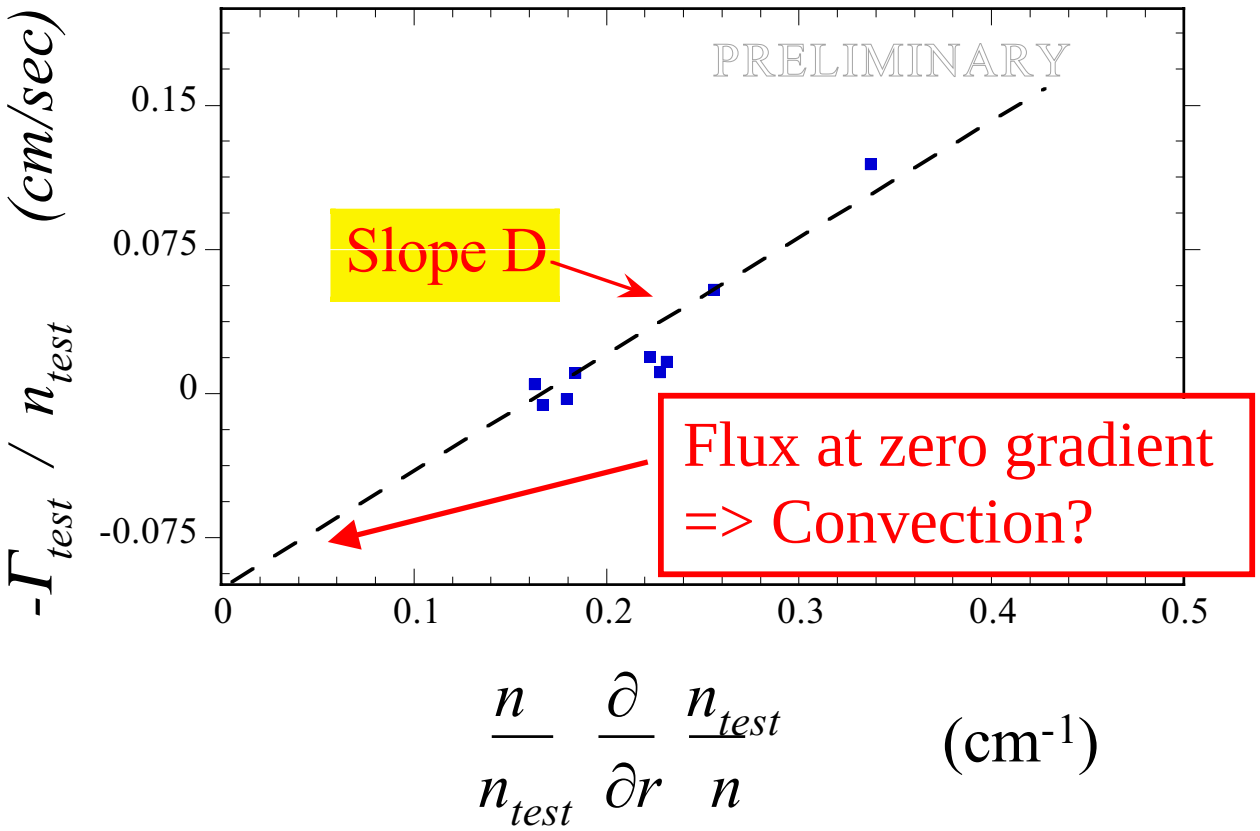


Diffusion decreases as ExB shear increases

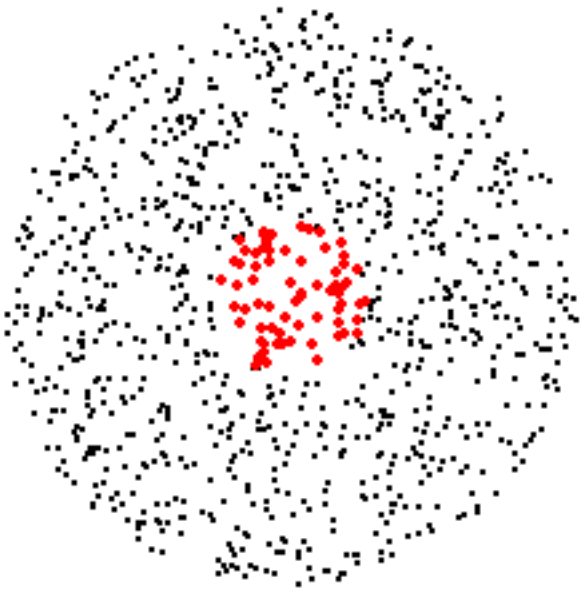


But….

For smallest shears, flux doe not appear to be completely diffusive



Is this evidence of convective cells?



Particle in cell simulation of 2D ExB plasma: test particles in red (viewed in rotating frame)

Can we directly view convective cells in the experiment?

conclusions

- When $r_c << \lambda_D$, long-range collisions dominate collisional transport (in neutral plasmas too!)
- measurements of transport in nonneutral plasmas allow *precision* tests of theory
- heat conduction is *independent of* ***B***
- ***ExB*** drift diffusion ~ 10x D^{class}
- ***In 2D regime, maximum ExB*** diffusion given by Taylor/McNamara/Dawson/Okuda
- 2D collisional diffusion reduced in presence of shear