CONTROL OF EXTERNAL KINK INSTABILITY

by

G.A. Navratil

Presented at
Forty-sixth Annual Meeting
American Physical Society
Division of Plasma Physics
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ACKNOWLEDGEMENTS

• **Substantial Theory Modeling in Literature – some key authors:**

• **Contributions from Experimental Groups:**
  DIII-D, Extrap-T2R, HBT-EP, JET, JT-60U, NSTX

• **Discussions with…**
  Bialek, Boozer, Bondeson, Chu, Drake, Fitzpatrick, Garofalo, Hender, Jensen, Liu, Mauel, Okabayashi, Reimerdes, Sabbagh, Shilov, Strait, Taylor, …
PRIMARY LIMITING MODE IN MAGNETIC CONFINEMENT SYSTEMS: LOW-n Kink

• Long wavelength global MHD modes driven by pressure & current gradient:
  Shift & Tilt: $n = 0$ and $1$
  Kink: $n = 1$

• ‘Classic’ Instability: Ideal conducting wall on plasma boundary stabilizes the kink mode by freezing magnetic flux value on wall surface.

• Resistive conducting wall stabilization fails on magnetic field soak-through time scale: $\tau_w$
WANT TO PRESENT A REVIEW…

...OF IMPORTANT & EXCITING SCIENTIFIC ADVANCEMENT IN MHD THROUGH THE INTERPLAY OF THEORY & EXPERIMENT BEGINNING IN EARLY 1990S:

• **Building on Basic Understanding of MHD Kink Mode Stabilized by a Conducting Wall**

• **Observation of Plasma Rotation Stabilization of Kink Mode with a Conducting Wall**

• **Development of a “Simple” Model which Describes Most [but not yet all] of Kink Mode Behavior**

• **Extension of the Model to Active Feedback Control of the Kink Mode**
Foundation of Kink Mode Stability Built on Energy Principle $\delta W$ Stability Analysis

1957 Bernstein, Frieman, Kruskal, Kulsrud

perturbed magnetic energy

$$\delta W_p = \frac{1}{2} \int d^3 x \left\{ \varepsilon_o c^2 \delta B^2 + \varepsilon_o c^2 (\nabla \times B) \cdot (\xi \times \delta B) \right\}$$

current driven - destabilizing

pressure driven - destabilizing

plasma compression

$$\delta W_v = \frac{1}{2} \int d^3 x \varepsilon_o c^2 \delta B^2$$

vacuum perturbed magnetic energy

If $\delta W_p + \delta W_v < 0$ mode is unstable
BASIC KINK MODE

• Long wavelength mode driven by pressure & current gradient

\[ \text{Cylindrical } k \sim \frac{2\pi}{L} \]

\[ \text{Toroidal: low } n = 1 \]

• Unstable when \( \delta W_p + \delta W_\infty v < 0 \)

• Dispersion Relation: \( \gamma^2 K + \delta W_p + \delta W_\infty v = 0 \), where \( K \) is kinetic fluid mass

• Define \( \Gamma_\infty^2 = \frac{[\delta W_p + \delta W_\infty v]}{K} \sim \left[ \frac{v_{Alfvén}}{L} \right]^2 \)
IDEAL WALL STABILIZES THE KINK MODE

• Ideal wall traps field in vacuum region and restoring force stabilizes the kink – EXTERNAL Kink:

  • Unstable when $\delta W_p + \delta W^d_v < 0$  
    Note: $\delta W^d_v > \delta W^\infty_v$

  • Dispersion Relation: $\gamma^2 - \Gamma^2_\infty + [\delta W^d_v - \delta W^\infty_v]/K = 0$

  • Critical Wall Distance, $d_c$, where kink stable for $d < d_c$: simple $[\delta W^d_v - \delta W^\infty_v]/K$ parameterization with $d$:

    $\gamma^2 - \Gamma^2_\infty[1 - d_c/d]/K = 0$
KINK MODE IS STABILIZED BY IDEAL WALL

\[ 0 = \gamma^2 - \Gamma_\infty^2 \left( 1 - \frac{d_c}{d} \right) \]

Ideal Stability

Plasma-Wall Separation, \( \frac{d}{d_c} \)

Ideal mode stable

Ideal mode unstable

\( \gamma / \Gamma_\infty \)
ADJUSTABLE CONDUCTING WALL POSITION IN HBT-EP:
EXTERNAL KINK STABILIZED BY NEARBY THICK AL WALL

Weakly Coupled Wall
\[ c = 1 - \frac{\delta W_v^\infty}{\delta W_v^b} = 0.007 \]

Closely Coupled Wall
\[ c = 1 - \frac{\delta W_v^\infty}{\delta W_v^b} = 0.120 \]

Wall Retracted

Wall Near Plasma
**Resistive Wall ‘Leaks’ Stabilizing Field: $\tau_w$**

- Stabilizing field decays resistively on wall time scale $\tau_w \sim L/R$:  
  \[ \frac{d\psi_w}{dt} = -\frac{\psi_w}{\tau_w} \]

- Quadratic kink: \[ \gamma^2 - \Gamma_\infty^2[1-dc/d] = 0 \] coupled to ‘slow’ flux diffusion  
  \[ \gamma\psi_w = -\frac{\psi_w}{\tau_w} : \tau_w \gg \tau_{\text{Alfven}} \]

- Cubic Dispersion Relation with new ‘slow’ root—the RWM: \[ \gamma^2 - \Gamma_\infty^2[1-(dc/d)\gamma\tau_w/(\gamma\tau_w + 1)] = 0 \]
Resistive wall mode (RWM) is unstable

- Mode structure similar to ideal external kink
- Mode grows slowly: $\gamma \sim \tau_w^{-1}$
RWM IDENTIFIED IN REVERSED-FIELD PINCHES


- Short time scale ($\tau \sim 0.5$ ms) resistive wall added to HBTX1C RFP device.
- RWM observed growing on wall flux diffusion time scale.
PBX-M OBSERVED WALL STABILIZING EFFECT ON KINK

- Modes slowed but not stabilized - onset of RWM
- Showed key effect of plasma-wall coupling, c.
JT-60U OBSERVED n=1 RESISTIVE WALL MODE

• Slow growing kink modes appear as q decreases to 3 in agreement with RWM model

3/1 global kink structure

Growth rate agrees with simple RWM dispersion relation
RWM STABILIZED IN DIII-D BY ROTATION FOR MANY WALL-TIMES, $\tau_W$

- Normalized plasma pressure, $\beta_N$, exceeds no-wall stability limit by up to 40%.
- $n = 1$ mode grows ($\gamma \sim 1/\tau_W$) after toroidal rotation at $q = 3$ surface has decreased below $\sim 1$ kHz.

![Graphs showing normalized plasma pressure and toroidal rotation frequency over time.](image)
OUTSTANDING KINK CONTROL QUESTIONS IN MID-90’s

• Why is the kink stabilized for many wall-times when the plasma rotates?

• Why does the plasma rotation slow down?

• Is there a critical rotation speed for stability and how does it scale?

• Is the kink mode structure ‘rigid’ so that simple single-mode models can be used?

• Can these slowed growth rate kinks be stabilized by active feedback control?
Passive Control of Kink Mode:

Plasma Rotation Stabilization
ROTATION AND DISSIPATION CAN STABILIZE RWM

- **Rotation** Doppler shift: $\gamma \Rightarrow \gamma + i\Omega$ where $\Omega$ is plasma rotation.

- **Dissipation** represented by friction loss $(\gamma + i\Omega)\nu$, where form of $\nu$ still being actively studied by theory community:

  $$(\gamma + i\Omega)^2 - \Gamma_{\infty}^2[1-(d_c/d)\gamma\tau_w/(\gamma\tau_w + 1)] + (\gamma + i\Omega)\nu = 0$$


- **Cubic Dispersion Relation** with three roots: in region where $d < d_c$ new ‘slow’ RWM root can be damped with ‘fast’ stable kink mode roots tied to rotating plasma with usual ordering:

  $$\tau_w^{-1} \ll \Omega \ll v_{\text{Alfvén}}/L$$

- **Why is RWM Slow Root Stabilized?**

  kink energy release slowed by wall $<\text{dissipation loss of RWM in flowing plasma}$
KINK MODE GROWTH IS SLOWED BY RESISTIVE WALL AND STABILIZED BY PLASMA ROTATION

\[ 0 = (\gamma + i\Omega)^2 - \Gamma_\infty^2 + \frac{\Gamma_\infty^2 (d_c/d)\gamma \tau_w}{\gamma \tau_w + 1} + (\gamma + i\Omega) \nu_{\text{DIS}} \]

- Resistive wall mode (RWM) is unstable
  - Mode structure similar to ideal external kink
  - Mode grows slowly: \( \gamma \sim \tau_w^{-1} \)
- Dissipation + rotation stabilizes RWM
  - Mode nearly stationary: \( \omega \sim \tau_w^{-1} \ll \Omega_{\text{plasma}} \)
RWM Parameterized by “Normalized Stability Drive”, $S \approx C_\beta$

Chu, *et al.* Cubic dispersion relation parameterized by wall position, $d/d_c$:

$$
(\gamma + i\Omega)^2 + \nu(\gamma + i\Omega) - \Gamma_d^2 \left(1 - \frac{d}{d_c}\right) = \frac{\Gamma_\infty^2 - \Gamma_d^2}{\gamma \tau_w^* + 1}
$$

Fitzpatrick introduced equivalent defining “normalized stability drive”, $S$:

$$
(\gamma + i\Omega)^2 + \nu(\gamma + i\Omega) - \Gamma_{MHD}^2(S - 1) = \frac{\Gamma_{MHD}^2}{\gamma \tau_w(1 - c) + 1}
$$

where $c = (1 - \delta W_v^\infty / \delta W_v^d)$ is the kink mode coupling plasma to the wall.

- **Normalized Mode Drive, $S$**:
  
  $S = 0$ Marginally stable without wall $\Rightarrow (\delta W_p + \delta W_v^\infty) = 0$

  $S = 1$ Marginal with ideal wall at $d = d_c$ $\Rightarrow (\delta W_p + \delta W_v^d) = 0$

- **For pressure-driven kink modes, $S \approx C_\beta$**:
  
  $S = C_\beta = 0 \Rightarrow$ No-Wall Beta Limit

  $S = C_\beta = 1 \Rightarrow$ Ideal-Wall Beta Limit
DISPERSION RELATION: WEAK DISSIPATION

Model using DIII-D like parameters $\tau_w \sim 1$ ms and plasma rotation $\Omega$ varied from 0 to 5 kHz
DISPERSION RELATION: STRONGER DISSIPATION

Model using DIII-D like parameters: $\tau_W \sim 1$ ms and plasma rotation $\Omega$ varied from 0 to 5 kHz

![Graph showing Stronger Dissipation: $(\Omega/2\pi = 0.0)$](image-url)
Feedback control of NBI power keeps $\beta_N$ below stability limit (107603)

No other large scale instabilities encountered (NTM, n=2 RWM, . . . )

Ideal n=1 kink observed at the wall-stabilized $\beta$ limit

$\beta_N \sim 2 \beta_{N\text{no-wall}}$

$\beta = 3.7\%$
INCREASING $n=1$ ERROR FIELD AMPLITUDE CAUSES DECAY OF PLASMA ROTATION

Plasma Rotation (kHz) at $q = 2$

Critical Rotation for Onset of RWM

Clear rotation stabilization threshold observed
MOMENTUM CONFINEMENT DECREASES AS PRESSURE EXCEEDS NO-WALL KINK LIMIT IN DIII-D

- Energy confinement relatively unchanged
- Angular momentum confinement time, $\tau_L$, decreases with heating power
ROTATION-STABILIZED PLASMA HAS A RESONANT RESPONSE TO EXTERNAL MAGNETIC PERTURBATIONS

- Weakly damped oscillator responds when driven near resonant frequency: $\omega \sim 0$ for RWM

- Amplitude of response to $n=1$ perturbation increases strongly for $\beta > \beta_{\text{no-wall}}$

- Damping rate decreases for $\beta > \beta_{\text{no-wall}}$

- "Error field amplification" by marginally stable RWM can cause slowing of rotation (A. Boozer, PRL 2001)
RESONANT FIELD AMPLIFICATION (RFA) OBSERVED IN JET

Sequence of External n=1 field pulses applied as $\beta$ is increased – showing characteristic increasing RFA response with $\beta$
Resonant field amplification increases at high $\beta_N$

- Plasma response to field from initial RWM stabilization coil pair
  - AC and pulsed $n = 1$ field perturbations
- RFA increase consistent with DIII-D
  - DIII-D RFA: 0-3.4 G/kA-turn
- Stable RWM damping rate 300s$^{-1}$
DIII–D HAS A VERSATILE COIL SET TO STUDY RESISTIVE WALL MODE DAMPING PHYSICS

- Inside vacuum vessel: Faster time response for feedback control
- Closer to plasma: more efficient coupling

- 12 “picture-frame” coils
- Single-turn, water-cooled
- 7 kA max. rated current
- Protected by graphite tiles
DIRECT MEASUREMENT OF THE RWM DISPERSION RELATION OBTAINED WITH ACTIVE MHD SPECTROSCOPY

- Apply a rotating low amplitude $n=1$ field:

$$I_c(t) = I_c e^{i\omega_{\text{ext}} t}$$

⇒ Plasma response increases significantly when beta exceeds the no-wall limit.

- Measure plasma response at different frequencies in multiple identical discharges.
PLASMA RESPONSE HAS A RIGID STRUCTURE WHICH IS INDEPENDENT OF THE EXTERNAL FREQUENCY

- Phase difference among $B_r$ arrays independent of frequency
- Phase of plasma response changes from leading to lagging the external field as frequency increases in plasma flow direction.
SINGLE MODE MODEL DESCRIBES INTERACTION BETWEEN THE RWM AND AN EXTERNAL APPLIED FIELD

- Single mode RWM model in slab geometry [Garofalo, Jensen, Strait, *Phys Plasmas* 9 (2002) 4573] yields relation between the perturbed radial field at the wall, $B_s$, and currents in the control coils, $I_{ext}$,

$$\tau_w \frac{dB_s}{dt} - \gamma_0 \tau_w B_s = M_{sc}^* I_{ext}$$

- Dispersion relations predict (complex) RWM growth rate $\gamma_0 = \gamma_{RWM} + i \omega_{RWM}$ in the absence of external currents

- Solve for plasma response contribution: $B_s = B_s^{plas} + B_s^{ext}$

- Predicted plasma response to an externally applied field rotating with $\omega_{ext}$:

$$B_s^{plas}(t) = \frac{\gamma_0 \tau_w + 1}{(i\omega_{ext} \tau_w - \gamma_0 \tau_w)(i\omega_{ext} \tau_w + 1)} M_{sc}^* I_c e^{i\omega_{ext} t}$$

- Here, $M_{sc}$ is the effective mutual inductance describing the resonant component of the applied field at the wall due to coil currents $I_c$. 
• Use predicted frequency dependence of the plasma response,

\[
B_{s}^{\text{plas}}(t) = \frac{\gamma_{0}\tau_{w} + 1}{(i\omega_{\text{ext}}\tau_{w} - \gamma_{0}\tau_{w})(i\omega_{\text{ext}}\tau_{w} + 1)} M_{sc}^* I_{c} e^{i\omega_{\text{ext}}t}.
\]

• Fit \(\gamma_{0}\) and \(M_{sc}\) to match measurements for two plasma pressures \(\beta_{N} = 2.4\) & \(\beta_{N} = 2.9\).

• Good agreement:
  - Indicates single-mode approach is applicable.
  - Yields measurement of damping rate and mode rotation frequency:
    \[\gamma_{0} = (-157 + i80) \text{s}^{-1} \text{ for } \beta_{N} = 2.4\]
    \[\gamma_{0} = (-111 + i73) \text{s}^{-1} \text{ for } \beta_{N} = 2.9\]
2D MARS CODE SOLVES FOR $\gamma$ and $\omega$ OF KINK MODES

- New version MARS-F: MARS + feedback in vacuum region

$$\frac{\partial}{\partial t} = \gamma = \gamma - i\omega, \quad \frac{\partial}{\partial \phi} = \text{in}$$

Eq. Of Motion
$$\rho(\gamma + i\Omega) \vec{v}_1 = -\nabla \rho_1 + j_1 \times B_0 + J_0 \times b_1 - \nabla \cdot \Pi_1 - \rho U(v_1)$$

Ohm’s Law
$$(\gamma + i\Omega) b_1 = \nabla \times (v_1 \times B_0 - \eta j_1) + (b_1 \cdot \nabla \Omega) R^2 \nabla \phi$$

Ampere’s Law
$$j_1 = \nabla \times b_1$$

Pressure Eq.
$$(\gamma + i\Omega) p_1 = -(v_1 \cdot \nabla) p_0 - \Gamma p_0 \nabla \cdot v_1$$

Density Eq.
$$(\gamma + i\Omega) \rho_1 = -(v_1 \cdot \nabla) \rho_0 - \Gamma \rho_0 \nabla \cdot v_1$$

Dissipation From Landau Damping

Plasma Rotation

TWO MODELS HAVE BEEN USED IN MARS TO SIMULATE DISSIPATION EFFECT OF LANDAU DAMPING ON MHD MODES

- Parallel sound wave damping model based on Hammet/Perkins’s approximation

\[
\nabla \cdot \mathbf{\Pi} = \kappa_{||} \sqrt{\pi} |k| v_{thi} |\rho v_1| \mathbf{\hat{b}} \mathbf{\hat{b}}
\]

Scale factor \( \kappa_{||} \sim 0.5 \)

- Kinetic damping model \((\omega^* = 0, \omega_b = 0)\) from Bondeson and Chu

\[
\Delta W_{MHD} = \Delta W_p (\xi, \gamma = 0) + \Delta W_k (\xi)
\]

\[
\Delta W_k (\xi) = \sum_j (\Delta W_{T_j} + \Delta W_{c_j})
\]

\[
\Delta W_c = \int_{\text{circulating}} d\Gamma \left( -\frac{\partial f}{\partial E} \right) \frac{\omega}{\omega - (nq - m')\omega_t} \left| \exp(i\chi_m)H \right|^2
\]

\[
\Delta W_T = \int_{\text{trapped}} d\Gamma \left( -\frac{\partial f}{\partial E} \right) \frac{\omega}{\omega + m'\omega_b} \left| \exp(i\chi_m)H \right|^2
\]

Compressional Energy

Landau Resonance
MARS PREDICTIONS OF $\Omega_{\text{crit}}\tau_A$ IN QUALITATIVE AGREEMENT WITH MEASUREMENTS ON DIII-D AND JET

- In DIII-D $\Omega_{\text{crit}}\tau_A \sim 0.02$ with weak $\beta$ dependence
- In JET $\Omega_{\text{crit}}\tau_A \sim 0.005$ with weak $\beta$ dependence
- Both damping models predict $\Omega_{\text{crit}}$ within a factor of 2
\Omega_{\text{crit}} \tau_A \text{ follows } 1/(4q^2) \text{ Bondeson-Chu theory in NSTX}

\[ \frac{1}{1/(4q^2)} \]

- **Experimental** \( \Omega_{\text{crit}} \)
  - Stabilized profiles: \( \beta > \beta_{\text{N \, no-wall}} \) (DCON)
  - Profiles not stabilized cannot maintain \( \beta > \beta_{\text{N \, no-wall}} \)
  - Regions separated by \( \frac{\omega_\phi}{\omega_A} = 1/(4q^2) \)

- **Drift Kinetic Theory**
  - Trapped particle effects significantly weaken stabilizing ion Landau damping
  - Toroidal inertia enhancement more important
    - Alfven wave dissipation yields \( \Omega_{\text{crit}} = \frac{\omega_A}{4q^2} \)
MODE FREQUENCY AND DAMPING CANNOT BE FIT SIMULTANEOUSLY

- Both damping models predict \( \gamma_{RWM} \) too low.
- Kinetic damping predicts mode frequency \( \omega_{RWM} \).
- Further work on damping [e.g. neoclassical viscosity] models being explored.
• Why is the kink stabilized for many wall times when the plasma rotates? Dissipation (viscosity) of slow RWM in rotating plasma: Chu-Bondeson, & Fitzpatrick models give qualitative agreement with experiment.

• Why does the plasma rotation slow down? Resonant Field Amplification (RFA) of ‘error’ fields in rotationally stabilized plasma near marginal stability ⇒ Error field reduction allows ideal wall limit stabilization by rotation!

• Is there a critical rotation speed and how does it scale? Yes, qualitative agreement with sound-wave & kinetic models for $\Omega_{\text{crit}} \tau_A$; BUT quantitative detail not yet complete: $\gamma$ & $\omega$ not yet consistent with dissipation models.

• Is kink mode structure ‘rigid’ so simple single mode models can be used? Yes – mode is remarkable robust even in multi-mode RFP plasmas.

• Can these slowed growth rates kinks be stabilized by active feedback control?
Active Control of the Kink Mode:

Feedback Stabilization Using Externally Applied Fields
ACTIVE CONTROL OF THE RESISTIVE WALL MODE SEEN ON HBTX

• In 1989 the RFP device HBTX observed the first simple feedback experiment on a m=1/n=2 RWM [B. Alper, Phys. Fluids 1990]

• Phased currents in sine & cosine helical coils outside resistive wall

• The 1/2 mode amplitude was reduced 50% [< 20G]

• Supported proposal by Bishop [Plas. Phys. Cont. Fus. 1989] to use an active “intelligent shell” for RWM control in the RFP.
FEEDBACK LOGIC FOR RWM FEEDBACK STABILIZATION

Smart Shell

Feedback cancels the radial flux from MHD mode at wall sensor

Explicit Mode Control

Feedback cancels the flux from MHD mode at plasma surface

\[ A = G \delta \Psi \]

Feedback cancels the radial flux from MHD mode at wall sensor

\[ A = G (\delta \Psi - M I_c) \]

Feedback cancels the flux from MHD mode at plasma surface
“Smart-Shell” Feedback Successfully Implemented on HBT-EP

Smart-Shell:

- 30 independent radial field flux loops
- 30 independent, overlapping control coils
- Locally prevents flux penetration through wall segments
- Effectively increases wall time and wall coupling
“Smart-Shell” Feedback Successfully Implemented on HBT-EP

CONFIGURATION OF SENSOR COIL AND ACTIVE SADDLE COIL
TOROIDAL ARRAYS IN EXTRAP T2R FOR MULTI-MODE FEEDBACK

Sensor coil array: 64x4=256 saddle coils. Each coil has 90° poloidal, 360/64=5.125° toroidal extent.

Active coil array: 16x4=64 saddle coils. Coils are "m=1" pair connected into 16x2=32 independently driven coils. Total surface coverage is 50%.

The feedback scheme is based upon detection and control of Fourier harmonics ($b_{m,n}$):

$$I_{m,n} = K_{m,n} b_{m,n}$$

where $K_{m,n}$ is a gain.

$$I_{j,k} = 2 \text{Real} \{ S I_{m,n} \exp[i(m \theta_j + n \phi_k)] \} \quad \text{[Inverse FFT]}$$
FEEDBACK CONTROL EXTENDS THE LIFETIME & SUPPRESSES MULTIPLE RWMs IN THE EXTRAP T2R RFP

- Discharge lifetime extended with feedback.
- RWM amplitudes suppressed by feedback.
THEORY AND MODELING TOOLS PROVIDE FOUNDATION FOR FEEDBACK DESIGN AND ANALYSIS

• **1-D Models**
  • Lumped parameter circuit modeling
  • Instructive, but qualitative
    A. Boozer PoP 1998, 1999, 2004
    M. Okabayashi, N. Pomphey and R. Hatcher, NF 1998
    T. Jensen and A. Garofalo, PoP1999

• **MHD Models**
  • With finite wall resistivity
  • Ideal MHD mode interacts with resistive wall geometry
    A. Bondeson and Y. Liu MARS-F code 2D
      [2D plasma model + toroidal rotation + dissipation]
    M. Chance and M. Chu GATO + VACUUM code 2D
      [2D MHD plasma model – no rotation]
    J. Bialek and A. Boozer VALEN/DCON code 3D
      [simple plasma model – rotation not yet implemented]
VALEN CODE BASED ON SET OF COUPLED CIRCUIT EQUATIONS WITH UNSTABLE PLASMA MODE

• These equations are implemented in VALEN:

\[ L_w I_w^w + M_{wp} I_d^w + M_{wp} I^p = \Phi_w \]
\[ M_{pw} I_w^w + LI_d^d + LI^p = \Phi \]
\[ LI^p (1 + s) = \Phi \]
\[ \frac{d\Phi_w}{dt} + R_w I_w^w = 0 \]
\[ \frac{d\Phi}{dt} + R_d I_d^d = 0 \]

No inertial term so ‘fast’ Alfvén time scale for flux release by kink \( \tau_A \approx L/R_d \) modeled by thin resistive shell on plasma surface.

• Coefficients are determined by 3D geometry of conductors and plasma mode shape determined from DCON

• Mode strength controlled by parameter: \( s \)
VALEN COMBINES DCON KINK MODE WITH 3D FINITE ELEMENT ELECTROMAGNETIC CODE

- RWM dispersion relation with full 3D coupling effects
- Single n=1 DCON mode without rotation
VALEN AND MARS BENCHMARKING STUDIES FOR ITER EQUILIBRIA IN GOOD AGREEMENT

\[ \beta y \quad (1/\text{sec}) \]

- VALEN
- MARS
- Ideal Kink range

Graph showing the relationship between \( \beta y \) and \( \beta_N \) with no-wall and ideal-wall limits. The graph compares VALEN and MARS results with new and old branches.
OBSERVED OPEN LOOP RWM GROWTH RATES AGREE WITH VALEN PREDICTION

\[ \gamma T_w \]

- VALEN prediction
- No Feedback
- No Rotation

No-wall limit

Ideal-wall limit

C_\beta
DIII–D INTERNAL CONTROL COILS ARE PREDICTED TO PROVIDE STABILITY AT HIGHER BETA

- Inside vacuum vessel: Faster time response for feedback control
- Closer to plasma: more efficient coupling
FEEDBACK WITH I-COILS IN DIII-D INCREASES STABLE PLASMA PRESSURE TO NEAR IDEAL-WALL LIMIT

- VALEN code prediction

![Graph showing normalized growth rate vs. CB]

\[ \gamma \tau_w \]

- VALEN code:
  - DCON MHD stability
  - 3D geometry of vacuum vessel and coil geometry

- \( \tau_w \) is the vacuum vessel flux diffusion time (~ 3.5 ms)
FEEDBACK WITH I-COILS IN DIII-D INCREASES STABLE PLASMA PRESSURE TO NEAR IDEAL-WALL LIMIT

- C-coil stabilizes slowly growing RWMs

- External C-Coil:
  - Control fields must penetrate wall
  - Induced eddy currents reduce feedback

- $\tau_w$ is the vacuum vessel flux diffusion time (~3.5 ms)
FEEDBACK WITH I-COILS IN DIII-D INCREASES STABLE PLASMA PRESSURE TO NEAR IDEAL-WALL LIMIT

- I-coil stabilizes RWMs with growth rate 10 times faster than C-coils

- Internal I-Coils:
  - Improved coil/plasma coupling
  - Improved spatial match to RWM field structure

- $\tau_w$ is the vacuum vessel flux diffusion time (~3.5 ms)
FEEDBACK EFFICACY DEMONSTRATED BY GATING OFF THE GAIN FOR 20 MS AT TIME OF EXPECTED RWM ONSET

Without feedback, slow Ip ramp rate (0.5 MA/s) destabilizes slowly growing RWM

With feedback, beta collapse avoided

n=1 mode starts up during feedback off period, stabilized after feedback is turned back on

n=1 mode detected on poloidal field probes and SXR arrays, decoupled from driver coils
FEEDBACK WITH INTERNAL CONTROL COILS HAS ACHIEVED HIGH $C_\beta$ AT ROTATION BELOW CRITICAL LEVEL PREDICTED BY MARS

- Trajectories of plasma discharge in rotation versus $C_\beta$

- No feedback plasma approaches limit and disrupts

- C-coil feedback plasma crosses limit & reaches higher pressure
FEEDBACK WITH INTERNAL CONTROL COILS HAS ACHIEVED HIGH $C_\beta$
AT ROTATION BELOW CRITICAL LEVEL PREDICTED BY MARS

- With near zero Rotation, $C_\beta$ is near the maximum set by existing control system characteristics: bandwidth & processing time delay.

- I-coil feedback plasma reaches near zero rotation.

MARS /VALEN prediction with measured amplifier time response for zero rotation.

Ideal wall limit

No wall limit

MARS prediction

Stable (without feedback)

Unstable

Rotation (km/s)

Time
FEEDBACK WITH INTERNAL CONTROL COILS HAS ACHIEVED HIGH $C_\beta$
AT ROTATION BELOW CRITICAL LEVEL PREDICTED BY MARS

- Combination of low rotation and feedback reaches $C_\beta$ is the ideal wall-limits

MARS/VALEN prediction with measured power supply time response for zero rotation

MARS prediction Stable (without feedback)

Stable (without feedback)

Unstable

ideal wall limit

no wall limit

Rotation (km/s)

$C_\beta$

0.0

1.0

ideal wall limit

MARS prediction Stable (without feedback)

Stable (without feedback)

Unstable

ideal wall limit

no wall limit

Rotation (km/s)

$C_\beta$

0.0

1.0

MARS prediction Stable (without feedback)

Stable (without feedback)

Unstable

ideal wall limit

no wall limit

Rotation (km/s)

$C_\beta$

0.0

1.0

MARS prediction Stable (without feedback)

Stable (without feedback)

Unstable

ideal wall limit

no wall limit

Rotation (km/s)
RWM FEEDBACK ASSISTS IN EXTENDING $\beta_n \sim 4$ ADVANCED TOKAMAK DISCHARGE MORE THAN 1 SECOND

- High performance plasma approaches $\beta \sim 6\%$
- Without feedback plasma disrupts due to RWM
SUMMARY & CONCLUSIONS

• Basic Physics of the Wall Stabilized Kink Mode:

  + Dissipation (viscosity) models in MARS give qualitative agreement with experiment for critical rotation thresholds.

  + Resonant Field Amplification critical for RWM dynamics since $\omega \sim 0 \Rightarrow$ can slow rotationally stabilized plasma near marginal stability and error field reduction allows ideal limit stabilization by rotation.

  + Qualitative agreement with kinetic damping models BUT complete quantitative details still not complete: predicted $\gamma$ and $\omega$ not self-consistent with experiment.

  + Rigid mode model is a useful tool for analysis.
SUMMARY & CONCLUSIONS

• Can these slowed growth rates kinks be stabilized by active feedback control? YES!
  + Feedback stabilization of the RWM has been demonstrated significantly above the no-wall pressure limits for 100s of wall times.
  + 2D MARS+F and 3D VALEN+DCON provide quantitative tools to design and assess optimized feedback control systems: Coil location & geometry, feedback loop transfer function, and noise and power requirements.
  + Tools and a predictive physical model are in hand for application of kink mode control to next generation Burning Plasma experiments.