A Simple Approximate Q scaling Suitable For Comparing Devices

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A simple scaling for fusion gain is desirable for comparing devices

Separate $Q$ into factors

Size: $B \cdot R$

Shape: $\hat{S}$

Plasma Physics: $\tau_E$, $(\beta^*/\beta)$, safety factor

$$Q_{DD} = \frac{P_{fusion}}{P_{IN}} \approx C_f \int [T_{ini}]^2 dV_{PIN}, \text{ where } C_f = \frac{1}{2} \frac{<\sigma v>}{T_i^2} \xi_f$$

Introduce $\beta$ to replace $n_i T_i$:

$$Q_{DD} \approx \left( \frac{\beta^*}{\beta} \right)^2 \frac{V \beta^2 B^4}{2 \mu_0 P_{IN}} \text{ and use } V = (2 \pi R) \pi a^2 \kappa$$

Use a slight modification of DIII-D/JET scaling for confinement, with an enhancement factor

$$\tau_E = 1.1 \times 10^{-4} \text{H} \frac{I_p R^{3/2}}{\sqrt{P_{IN}}} \frac{\sqrt{\kappa}}{\sqrt{1.8}}$$
Introduce shape parameter to remove plasma current

\[ \hat{S} \equiv q_{\psi} \frac{\mu_0 I_p}{2\pi a B} \]

By analogy: \( \varepsilon = \frac{a}{R} = q_{cyl} \frac{\mu_0 I_p}{2\pi a B} \), \( \hat{S} \) is a generalized inverse aspect ratio. Once \( \varepsilon \) is chosen, \( (\ell_i \hat{S}) \) is bounded by \( n=0 \) stability.

\( \frac{\hat{S}}{q} \) is, of course, simply \( \frac{I}{a B} \), but I don’t know how to interpret the latter. I do know the meanings of \( \hat{S} \) and \( q \).

Combine these and assume \( T_e = T_i \),

\[ Q_{DD} = \text{Constant} \cdot R^2 B^2 \left( \frac{\hat{S}^2}{q^2} \right) \left( \frac{H^2}{\beta^*} \right)^2 \]

\( \) (Nuclear Physics SIZE SHAPE PLASMA PHYSICS)
Fit D-D fusion reactivity for several tokamaks.

<table>
<thead>
<tr>
<th>Tokamak</th>
<th>DIII–D (double-null)</th>
<th>DIII–D (single-null)</th>
<th>TFTR</th>
<th>JT-60U</th>
<th>JET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge</td>
<td>87977</td>
<td>88964</td>
<td>68522</td>
<td>17110</td>
<td>26087</td>
</tr>
<tr>
<td>B (T)</td>
<td>2.15</td>
<td>2.15</td>
<td>5.00</td>
<td>4.40</td>
<td>2.80</td>
</tr>
<tr>
<td>R (m)</td>
<td>1.67</td>
<td>1.69</td>
<td>2.50</td>
<td>3.05</td>
<td>2.95</td>
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<tr>
<td>^S</td>
<td>1.42</td>
<td>1.03</td>
<td>0.35</td>
<td>.50</td>
<td>0.76</td>
</tr>
<tr>
<td>(β*/β)</td>
<td>1.26</td>
<td>1.14</td>
<td>1.73</td>
<td>1.41</td>
<td>1.34</td>
</tr>
<tr>
<td>H</td>
<td>2.2</td>
<td>2.6</td>
<td>1.6</td>
<td>1.9</td>
<td>2.4</td>
</tr>
<tr>
<td>H·(β*/β)</td>
<td>2.8</td>
<td>3.0</td>
<td>2.8</td>
<td>2.7</td>
<td>3.2</td>
</tr>
<tr>
<td>q</td>
<td>4.2</td>
<td>3.7</td>
<td>3.8</td>
<td>4.0</td>
<td>3.8</td>
</tr>
<tr>
<td>τE(s)</td>
<td>0.40</td>
<td>0.43</td>
<td>0.19</td>
<td>0.54</td>
<td>1.30</td>
</tr>
<tr>
<td>β (%)</td>
<td>6.7</td>
<td>5.8</td>
<td>1.0</td>
<td>1.5</td>
<td>2.2</td>
</tr>
<tr>
<td>Qdd*</td>
<td>0.0020</td>
<td>0.0016</td>
<td>0.0021</td>
<td>0.0037</td>
<td>0.0051</td>
</tr>
</tbody>
</table>

Notice that the product of peaking and enhancement factors, H·(β*/β), shows little variation.
On Average $H \cdot (\beta^*/\beta) \approx 3$. Fit using this value:

$$Q_{dd}^{\text{fit}} = 1.4 \cdot 10^{-3} B^2 R^2 (S/q)^2$$
Note that all the data are at $q \approx 4$ and all are transient discharges.

But all DT designs (except Ignitor) intend to operate at $q=3$.

Based on DIII-D experience, I think $H \cdot (\beta^*/\beta) \approx 1.5$ is a better guess for $q=3$.

(This is also consistent with the difference in JET transient and stationary plasmas.)

Using $Q_{DT} \approx 200 \cdot Q_{DD}$ my estimate based on the fit and the assumptions above is

$$Q_{DT} = 0.105 \frac{R^2 B^2}{q^2} \left( \frac{S^2}{q^2} \right)$$

Assume cost scales with size, $R^2 B^2$

Then the **bang for the buck** is $Q/(BR)^2$ leading to the next figure.
I think this is how $Q$ will scale between devices. I do not claim that the numerical factor, 0.105, is more than approximate.
At $q = 3$, scaled to $R \cdot B = 20$, 72475 would be 20 MA for $Q > 12$. However experience suggests $\beta \cdot \tau_E$ optimizes at $q \approx 4 \Rightarrow 15$ MA would be better.
Opinions

• If the mission is to test technology then a superconducting device is reasonable. Eddy current heating of superconducting coils appears to limit shaping. Large $B^2R^2$ would appear the only solution. Since $B$ is limited to low values (compared to copper) the device will be just-plain-big.

• If the mission is to learn about the physics of a burning plasma then the situation changes considerably. As an experiment it should try to optimize "bang for the buck". Great gains can be made by increasing $\hat{S}$.

• Perhaps a plasma like shot 72475 is too aggressive. This was the only shot like this. (It was also the only attempt.) But 87977 is a shape that is run day in and day out on DIII-D.

• The engineering task for strong shaping is challenging. For such dramatic potential gain we should try much harder to increase $\hat{S}$.