Theory of Alfvén waves and energetic particle physics in burning plasmas

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(I) Introduction

- Energetic particles (Alpha particles and/or fast ions) integral components of current and ITER burning plasma experiments.
- \( V_{EP} \sim V_A \) (Alfvén speed) \( \Rightarrow \) Collective excitations of SAW by EPs.
- Superthermal SAW fluctuations \( \Rightarrow \) Break EP’s adiabatic invariants; J and \( \Psi (r) \).
  \( \Rightarrow \) Anomalous transports (redistribute) in EP’s \( (\varepsilon = \frac{v^2}{2}, r) \) phase space
  \( \Rightarrow \) Potentially significant adverse effects on the performance of burning plasma experiments.
(II) Linear SAW-EP Physics

(II.1) SAW spectra in toroidal plasmas

- SAW – Anisotropic electromagnetic wave in magnetically confined plasmas
  \[ \omega^2 = k^2 V_A^2 = \omega_A^2, \quad k = \frac{k_B}{B}, \quad V_g = \frac{V_A B}{B} \]
  \[ |\omega_A| \ll \Omega_i, \quad \lambda_\parallel \sim R, \quad \lambda_\perp \sim \rho_i - a. \]
  Nearly incompressible

- SAW – Fundamental oscillations in laboratory as well as solar/interstellar/magnetosphere plasmas. Important dynamic roles in, e.g., solar corona heating, accelerating aurora electrons

- In toroidal plasmas: Non-uniformities across the magnetic surfaces
  \[ k_\parallel = k_\parallel (r), \quad V_A = V_A (r) \Rightarrow \omega_A (r) \Rightarrow \text{SAW continuous spectrum} \]
(II.1) **SAW spectra in toroidal plasmas** (continued...)

- **Consequences of SAW continuum:**
  - Initial perturbations: \( \exp[i\omega_A(r)t] \Rightarrow \) perturbations with a finite width \( \Delta r \) decay via phase mixing on a time scale
    \[
    \tau_{pm} \sim \left| \frac{d\omega_A}{dr} \Delta r \right|^{-1}.
    \]
  - Driven perturbation at frequency \( \omega_0 \)
    \( \Rightarrow \) “Singularly” absorbed at the resonant layer \( \omega_0^2 = \omega_A^2(r_0) \)
    \( \Rightarrow \) Resonant absorption (continuum damping) rate \( \propto \frac{d\omega_A(r_0)}{dr} \)
    \( \Rightarrow \) H. Grad [1969]: phase-mixing and singular absorption – exact analogy with free-streaming and Landau resonance in Vlasov plasma
    \( \Rightarrow \) Kinetic \((\rho_i, m_e)\) and resistivity effects \( \Rightarrow \) regularizing the “singular” structures
    \( \Rightarrow \) Kinetic Alfvén wave, radiative damping, etc.
(II.1) **SAW spectra in toroidal plasmas** (continued…)

- **SAW frequency gaps:**
  
  ◦ Various poloidal asymmetries ⇒ break translational symmetries along $\mathbf{B}$ into corresponding **lattice symmetries**.

  ⇒ Corresponding **frequency gaps** in SAW continuum.
(II) Linear SAW-EP Physics

(II.2) Instability Mechanisms

- For SAW waves in $\beta << 1$ plasmas
  \[ \delta E \parallel \approx 0, \delta B \parallel \approx 0 \]
  \[ \Rightarrow \text{EP experiences } (\mathbf{V}_d \times \delta \mathbf{B}) \text{ force}; \quad \mathbf{V}_d = \text{magnetic drifts}. \]

- Resonance conditions –
  - Circulating particles: \[ \omega - k \parallel v \parallel - p \omega_t = 0 \]
    \[ p = \text{integers}, \quad \omega_t : \text{transit frequencies}. \]
  - Trapped particles: \[ \omega - \bar{\omega}_d - p \omega_b = 0 \]
    \[ p = \text{integers}, \quad \bar{\omega}_d : \text{toroidal precessional frequency}, \quad \omega_b : \text{bounce frequency}. \]
(II.2) **Instability Mechanisms** (continued...)

- Expansion free energy
  - Growth rate \( \dot{\rho} \frac{\partial F_{EP}}{\partial P_{\phi}} \sim n \frac{\partial F_{EP}}{\partial r} \)
    
    \( n \): toroidal mode number

- Instability drive maximizes around
  
  \( k_{\perp} \rho_{EP,d} , k_{\perp} \rho_{EP,b} \sim \mathcal{O}(1) \)

- Background plasmas provide additional kinetic damping.
(II.3) Stability Properties

- To nullify/minimize continuum damping
  ⇒ localize SAW excitations inside the gaps and/or around $\frac{d \omega}{dr} = 0$.

- EP pressure perturbations ⇒ instability drive ⇒ coupled to SAW vorticity equation via $B$ curvature.

- Perturbations generally consist of singular (inertial) and regular (ideal MHD) mode structures
  ⇒ Generic Fishbone Dispersion Relation

\[ i \sqrt{\Lambda^2(\omega)} = \delta \hat{\omega}_f + \delta \hat{\omega}_k. \]
(II.3) **Stability Properties** (continued...)

- **Generic Fishbone Dispersion Relation**

  \[ i \sqrt{\Lambda^2(\omega)} = \delta \hat{w}_f + \delta \hat{w}_K. \]

  - \( \Lambda^2(\omega) \): inertial-layer contributions due to thermal particles
  - \( \delta \hat{w}_f, \delta \hat{w}_K \): background MHD and EP contribution in the regular regions.
  - \( \Lambda^2(\omega) = 0 \): accumulation points of SAW continuum.

- **Example:** Toroidal AE (TAE) near the lower accumulation point \( \omega_\ell \).
  - \( \Lambda^2(\omega) \Rightarrow \omega_\ell^2 - \omega^2 \), formally
(II.3) Stability Properties (continued...)

• Two types of modes –

  ○ Gap Mode (AE) \( \Rightarrow \text{Re}(\Lambda^2) < 0 \Rightarrow \text{Re}(\delta \hat{W}_f + \delta \hat{W}_K) > 0. \)
    \( \Rightarrow \) “localization” of AE in the frequency gap.
    \( \Rightarrow \) \( \text{Re}(\delta \hat{W}_K) \): Non-resonant EP effects.
    \( \Rightarrow \) various effects in \( \text{Re}(\delta \hat{W}_f + \delta \hat{W}_k) \) can lead to AE “localization” in various gaps \( \Rightarrow \) AE “zoology”!!

  ○ Continuum mode (EPM) \( \Rightarrow \text{Re}(\Lambda^2) > 0 \Rightarrow \) EPM inside the SAW continuum
    \( \Rightarrow \) EPM existence: \( \text{Im}(\delta \hat{W}_k) > \sqrt{\Lambda^2} \).
    EP instability drive > continuum damping
    \( \Rightarrow \omega_{EPM} \): EP’s characteristic dynamic frequencies;
    \( \omega_t, \omega_d, \omega_b. \)

• Similar pictures could also emerge around the upper SAW accumulation point
(II.3) **Stability Properties** (continued...)

- “Classical” example of EPM: Fishbone instability.
  - \( \Lambda^2 \approx \omega^2 \quad \Rightarrow \quad i\omega = \hat{\omega}_f + \hat{\omega}_K \).
  - \( \omega \sim \omega_{db} \)

- Lower-frequency SAW gap
  - \( |\omega| \sim |\omega_{*i}| \sim |\omega_{ii}| \) of thermal ions
  \( \Rightarrow \) (ideal MHD) accumulation point (at \( \omega = 0 \)) shifted by thermal ion kinetic effects
  \( \Rightarrow \) New low-frequency gap!
    - Diamagnetic drift: KBM
    - Parallel ion compressibility: BAE
    - \( \nabla T_i \) and wave-particle resonance: AITG
  \( \Rightarrow \) unstable SAW accumulation point
  \( \Rightarrow \) “localization” \( \Rightarrow \) unstable discrete AITG mode!
Experimental Observations of AEs

- TAE well documented [Heidbrink et al.]
- Reverse shear AE (RSAE/AC) [Nazikian et al.]
  \[ \Rightarrow \text{up to } n \sim O(40) \Rightarrow k_\theta \rho_i \sim 0(1) \]
  \[ \Rightarrow \text{demonstrate the destabilization of RSAE/AC via the AITG mechanism.} \]

Observation of sea of RSAE/AC Alfvén Eigenmodes in DIII-D

R. Nazikian, et al., PRL 96, 105006, 2006

FIR scattering

Observation of sea of RSAE/AC Alfvén Eigenmodes in DIII-D

R. Nazikian, et al., PRL 96, 105006, 2006
A “Sea of Core Localized Alfvén Eigenmodes” Observed in DIII-D Quiescent Double Barrier (QDB) plasmas

- Bands of modes $m=n+l$, $l=1, 2, \ldots$
- Neutral beam injection opposite to plasma current: $V_{\parallel} \approx 0.3 V_A$

R. Nazikian, et al., PRL 96, 105006, 2006
(III) Nonlinear SAW-EP Physics

(III.1) Nonlinear Physics of AE

- Weak instabilities \( \Rightarrow \frac{\gamma}{\omega} \sim 0(10^{-2}) \Rightarrow \) weak nonlinear perturbations.

(i) Wave-Trapping Physics [Berk, Breizman, et al.]

- Single linear TAE + nonlinear resonant EP
  \( \Rightarrow \) analogy to the single-wave bump-in-tail paradigm

- \( \omega_{TAE} \Rightarrow \omega_{pe} ; P_\phi \Rightarrow v \)

- Include background dissipation and restoring \( F_b \) via collisions (or \( F_{EP} \) via source inputs)

- Wave trapping of resonant EPs
  \( \Rightarrow \) hole/clump production in \( F_b \) \( \Rightarrow \) sidebands generation
  \( \Rightarrow \) Theoretical explanation of JET observations of pitchfork splitting of \( \omega_{TAE} \) [Fasoli, et al.]
Pitchfork splitting of TAE in JET

(III) Nonlinear SAW-EP Physics

(III.1) Nonlinear Physics of AE (continued…)

(ii) Nonlinear Frequency Shifts

- Single TAE $\Rightarrow (n = 0, m = 0)$ zonal flows/fields and/or $(n = 0, m = \pm 1)\delta B$ and $\delta n$.
  - radially local nonlinear equilibrium modifications.
  - narrowing of TAE frequency gap and/or lowering $\omega_{TAE}$
  - enhancing continuum/radiative damping.

- Simulations (Todo et al.): $n=0$ perturbations effective in lower TAE saturation amplitudes
TAE-induced Losses of Fast Ions
IFS-NIFS collaboration, 9th IAEA TCM on Energetic Particles (2005)

MHD nonlinearity reduces the saturation level of the dominant (n=4) mode and generates a zonal flow (n=0)

Wave energy burst in quasilinear simulation

Wave energy burst with MHD nonlinearity included

Future work: examine the effects of MHD nonlinearity on a longer (experimentally relevant) time interval.
(III) Nonlinear SAW-EP Physics

(III.1) Nonlinear Physics of AE (continued…)

(iii) Nonlinear Downward Frequency Cascading
  o Multiple TAEs ⇒ nonlinear ion Landau damping
    ⇒ Cascading to lower-frequency, more stable TAEs.
    ⇒ Enhancing effective continuum/radiative damping.

(iv) Additional Considerations
  o Each toroidal-n mode: O(nq) AEs localized at different radial locations
  o Different-n AEs have nearly degenerate frequencies.
    ⇒ Within the TAE frequency gap: dense populations of AEs
      (“lighthouses”) with “unique” frequencies and radial locations.
    ⇒ Significant multiple-TAE nonlinear interactions
    ⇒ Diffusive redistribution of $F_{EP}(\varepsilon, p_\phi(r)|\mu)$
    ⇒ AE – avalanche: turbulence spreading
(III) Nonlinear SAW-EP Physics

(III.2) Nonlinear Physics of EPM

- Stronger instability drive (to overcome continuum damping)
  \[ \gamma / \omega \sim O \left( 10^{-2} - 10^{-1} \right) \]
- \( \omega_{EPM} \sim \) characteristic EP dynamic frequencies
- EPM in-situ at where drive \( \alpha_{E_p} \propto \beta_{E_p} \)' maximizes.
  \[ \Rightarrow \text{EPM rapidly redistribute } F_{E_p}(\varepsilon, \phi) \]

(i) Fishbone Paradigm

- n=1 internal kink
- \( \omega \sim \bar{\omega}_{db} \)
- Simulations [Fu et al.] : Rapid radial redistribution of \( F_{E_P} \)
saturation and downward frequency chirping.
Hybrid MHD-GK simulations of fishbones

As flattening region of distribution function increases, the mode frequency chirps down.

(III.2) Nonlinear Physics of EPM (continued...)

(ii) EPM at the TAE range

- Abrupt Large Event (ALE) in observed JT - 60U [Shinohara et al.]
- Simulations [Vlad et al.] : $n = 1$ EPM redistributes $F_{EP}$ radially
  ⇒ results consistent with ALE.
(III.2) **Nonlinear Physics of EPM** (continued...)

(iii) **EPM – Avalanche paradigm**

- Strong EP drive $\Rightarrow$ EPM localized at $\beta_{EP}$’ max
- Convective radial transport of EP
- Radial propagation of EPM turbulence via couplings between poloidal harmonics
  $\Rightarrow$ Propogation of EPM “unstable” front (EPM-Avalanche)
(III.2) **Nonlinear Physics of EPM** (continued...)

(iv) **Analytical description** [Zonca et al.]

\[
D_{EPM}^\ell (-i\omega + \partial_t, \partial_r, r) A(r, t) = \delta \hat{W}_k^{n\ell} \left( \partial_t, \partial_r, r, |A|^2 \right) A(r, t)
\]

⇒ Radial convective amplification
⇒ Source propagation

o Consistent with simulations

(v) **Additional Considerations**

o EPM has stronger n dependences \((\bar{\omega}_d \alpha n)\) ⇒ narrow unstable spectrum in n
o Single-n dynamics dominates the initial rapid convective phase
o Reduced instability drive ⇒ AE dynamics.
(IV) ITER Applications

- $\alpha$ particles + fast ions $\Rightarrow$ unstable AE and/or EPM in ITER in various scenarios. [Gorelenkov et al.; Vlad et al.]

- Unstable n spectrum: $n_{\text{max}} \sim O(10-20)$
  - Dense AE “lighthouse” spectrum in $(\omega,r)$
  - Significant implications to the nonlinear AE physics!
TAE Instability in ITER
(N.N. Gorelenkov, et al., NF 45, 226, 2005)

- Neutral Beams Have Comparable Linear Drive as Alpha particles
- Local flat shear region at \( r/a \sim 0.5 \) can establish global mode

![Graph showing unstable modes in ITER](image)

- Global nature of the TAE can cause alpha loss
- Nominal plasmas are close to thresholds for alphas losses based on quasilinear marginal stability postulate (Gorelenkov,'05)
- Reversed shear scenario plasmas is more TAE unstable with \( n \) from 1 to 7 and with \( \sim 2\% \) growth rate.
- The most unstable modes are localized at the strongest fast ion pressure gradient
Global Hybrid MHD-Gyrokinetic simulations of ITER operation scenarios: SC2(normal shear), SC4 (reversed shear), SCH (hybrid scenario).
- Assuming only fusion alphas, AE are marginally unstable in all scenarios.
- Only SC4 (reversed shear) shows significant broadening of the alpha particle profiles at nominal values of alpha particle power density.
- EPM are excited in SCH above a threshold ~1.6 the nominal value of alpha particle power density.

ITER SC4 – nominal $\beta_\alpha$ (AE/AC)

ITER SCH – 3.3 x nominal $\beta_\alpha$ (EPM)
(V) Summary and Discussions

- Linear physics well at hand.
- Still need comprehensive linear code to accurately evaluate the stability properties.
- ITER (alpha + fast ions) ⇒ SAW excitations ⇒ consequences on EP transports remain uncertain.
- Key nonlinear physics mechanisms identified and some “verified” either by customized simulations and/or experimental observations.
- Multi-n simulations up to n ~ 0(10-20) with accurate background kinetic damping, realistic geometries, and boundaries needed to push forward this area.
- In the longer time scales, interactions between SAW-EP dynamics and Drift/Alfvén-thermal particles dynamics will emerge ⇒ challenging multi-scale physics.
(V) Summary and Discussions
(continued)

- SAW EP research ⇒
  - Intellectually challenging (complexities in geometries and nonlinearities) and programmatically important
  - Strong and healthy positive interplays among experiments, theory and simulations!!
  - Electron-fishbones via $\omega = \overline{\omega}_d$ resonance [this Conference] also shed interesting physics insights.