TRANSPORT AND STABILITY IMPLICATIONS FOR SHAPE AND ASPECT RATIO OF STEADY-STATE, HIGH-PERFORMANCE TOKAMAKS

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I. Optimum tokamak study by Lin-Liu/Stambaugh

II. Comparison with similar study by Menard

III. Transport dependence on shape ($A, \kappa$)
I. WHAT IS THE OPTIMUM TOKAMAK?

- Lin-Liu/Stambaugh constructed equilibria with
  - Bootstrap fraction of 99%, fully aligned
  - \( P' = 0 \) at separatrix
  - Broad, nearly optimal, pressure profile
    - Edge ITB?

- Ideal ballooning \( \beta \) limit found using BALOO
  - Bulk of plasma has second stability access
  - Ballooning limit occurs at a point near edge
  - Wall stabilization assumed for kinks

- Systematic shape study spanned
  - \( 1.5 \leq \kappa \leq 6.0 \)
  - \( 1.2 \leq A \leq 7.0 \)
HIGH BETA, HIGH ELONGATION, HIGH BOOTSTRAP EQUILIBRIUM

- $A = 1.6$, $\kappa = 4.0$, $\delta = 0.5$, $\beta_T = 73\%$, $\beta_N = 8.0$, $\beta_P = 1.6$

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**Graphs and Diagrams:**

- Various plots showing profiles and distributions of quantities such as $P$, $J_\psi$, and $J_\phi$.

**Annotations:**

- $X =$ Total
- $O =$ Bootstrap
- $+$ = Diamagnetic
- $*$ = Pfirsch-Schlüter

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**Footer:**

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SYSTEMATIC STUDY OF $\beta_N$-LIMIT VERSUS R/a AND $\kappa$ FOR $f_{BS} = 0.99$

$\beta_N$ is optimal at $\kappa = 3–4$

$\beta_N$ dependence close to $A^{-1/2}$

Symbols are Calculations
Lines are Fits
STRONG SHAPING ($\delta$) IS NEEDED TO TAKE FULL ADVANTAGE OF HIGH ELONGATION

- Beta increases with $\delta$ for $\kappa \geq 3$

![Graph](https://example.com/graph.png)
KEY RESULTS OF LIN-LIU/STAMBAUGH STUDY

- Trade-off between fusion power and bootstrap current at a given normalized beta

\[ \beta_T \beta_P = 25 \left( \frac{1 + \kappa^2}{2} \right) \left( \frac{\beta_N}{100} \right)^2 \]

- Shape dependence of ideal ballooning stable beta

\[ \beta_N = 10 (b_0 + b_1 \kappa + b_2 \kappa^2 + b_3 \kappa^3) \coth \left( \frac{d_0 + d_1 \kappa}{A^m} \right) \frac{1}{A^n} \]

\[ b_0 = -0.7748 \quad d_0 = 1.8524 \]
\[ b_1 = 1.2869 \quad d_1 = 0.2319 \]
\[ b_2 = -0.2921 \quad m = 0.6163 \]
\[ b_3 = 0.0197 \quad n = 0.5523 \]
II. IDEAL WITH-WALL BALLOONING LIMIT FOR FULLY SELF-SUSTAINED EQUILIBRIA NEARLY SAME BETWEEN MENARD’S AND LIN-LIU’S STUDIES
III. TRANSPORT DEPENDENCE ON SHAPE ($A$, $\kappa$)

- While empirical confinement scaling relations of the form

$$\tau = 0.0562 I^{0.93} n^{0.41} B^{0.15} P^{-0.69} R^{1.97} m^{0.19} \kappa^{0.78} A^{-0.58}$$

$$\tau = 0.028 I^{0.83} n^{0.49} B^{0.07} P^{-0.55} R^{2.11} m^{0.14} \kappa^{0.75} A^{-0.3}$$

are fully predictive, the trade-offs between different parameters due to operational constraints are not readily apparent

★ e.g., safety factor constraint relates $I$, $\kappa$, $A$, and $R$

- Casting confinement scaling relations in terms of dimensionless parameters allows the shape and aspect ratio dependences to be easily determined once the operational constraints are specified
  - Can choose kinetic plasma physics parameters like $\rho_*$ and $v_*$
  - Can choose MHD parameters like $q$ and $\beta_N$
Dependence of Transport on $q$ and $\kappa$

- Experiments on DIII–D resolved the ambiguity between the $\kappa$ and $q$ scalings of transport by comparing
  - $q$ scan at fixed $\kappa$
  - $\kappa$ scan at fixed $q$
  - $\kappa$ scan at fixed $I$

- For H–mode plasmas, the change in confinement for the above three scans was explained by the unified scaling
  \[ B \tau \propto q_{95}^{-1.4 \pm 0.6} \kappa^{2.2 \pm 0.6} \]

- Note that the $q$ and $\kappa$ scalings of normalized confinement are different than the $I_p$ and $\kappa$ scalings of $\tau$
  - Converting dimensionless parameter scalings for H–mode plasmas on DIII–D to engineering parameter scalings gives
  \[ \tau \propto I_p^{0.76 \pm 0.14} \kappa^{0.65 \pm 0.16} \]
MEASURED q AND κ SCALINGS OF H–MODE TRANSPORT ARE WEAKER THAN PREDICTION FROM IPB98(y,2)

\[ B_T \propto q^{-\alpha_q} \]

\[ B_T \propto \kappa^{-\alpha_\kappa} \]

\( q \) scanned in two ways

\( \kappa \) scanned at fixed \( q \)

- IPB98(y,2) Prediction
- EGB Prediction

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OPERATIONAL CONSTRAINTS ARE CRITICAL WHEN PROJECTING ASPECT RATIO SCALING OF TRANSPORT

- Aspect ratio affects many important dimensionless parameters
  - $q$, $\beta_N$, $\nu_*$, $f_{BS}$, etc.

- Future experiments between DIII–D and NSTX/MAST will directly measure aspect ratio scaling of transport

- For steady-state, high-performance tokamaks, the aspect ratio scaling of confinement is more easily projected by substituting operational constraints for engineering parameters in scaling relations

\[
\begin{pmatrix}
I_p \\
B \\
n \\
P
\end{pmatrix}
\leftrightarrow
\begin{pmatrix}
\rho_* \\
\beta_N \\
f_{BS} \\
f_{GR}
\end{pmatrix}
\]
INCLUDING EFFECT OF ASPECT RATIO ON $\beta_N$, $\kappa$ CHANGES R/a
DEPENDENCE OF NORMALIZED CONFINEMENT

- Confinement scaling relations converted to dimensionless parameters

\[
\text{IPB98}(y,2) \quad B_\tau \propto \rho_*^{-2.7} A^{1.2} \kappa^{2.4} \beta_N^{1.2} f_{BS}^{-2.1} f_{GR}^{0.0}
\]
\[
\text{EGB} \quad B_\tau \propto \rho_*^{-3.3} A^{1.8} \kappa^{2.2} \beta_N^{2.1} f_{BS}^{-1.7} f_{GR}^{-0.6}
\]

- Include optimum tokamak scalings $\beta_N \propto A^{-1/2}$, $\kappa \propto A^{-1/2}$

\[
\text{IPB98}(y,2) \quad B_\tau \propto \rho_*^{-2.7} A^{-0.6} f_{BS}^{-2.1} f_{GR}^{0.0}
\]
\[
\text{EGB} \quad B_\tau \propto \rho_*^{-3.3} A^{-0.3} f_{BS}^{-1.7} f_{GR}^{-0.6}
\]
FUSION GAIN OPTIMIZES FOR $A = 2.2–3.0$ FOR STEADY-STATE, HIGH-PERFORMANCE TOKAMAKS AT STABILITY LIMIT

\[ \begin{align*}
\text{Fixed } & \beta_N, \kappa, f_{\text{BS}}, f_{\text{GR}} \text{ path} \\
\rho_{*}^{-1} & \propto a^{1/2} B^{1/2}
\end{align*} \]

\[ \begin{align*}
\text{Fixed } & f_{\text{BS}}, f_{\text{GR}} \text{ path with } \beta_N, \kappa \propto A^{-1/2} \\
\rho_{*}^{-1} & \propto a^{1/2} B^{1/2} A^{1/4}
\end{align*} \]

- Assume $B = B_c (1-A^{-1})$ where $B_c$ (= field at centerpost) is fixed
Recirculating power fraction optimizes for $A = 2.4–2.8$ at stability limit

$$Q = \frac{P_F}{P_{CD}} = \frac{\gamma_{CD} P_F}{n R (1 - f_{bs})} = \frac{\gamma_{CD} \beta_N^2 \kappa B_c^2 \left(1 - \frac{1}{A}\right)^2 R_a^2}{f_{GR} R (1 - f_{bs})}$$

- $n = f_{GR} \frac{1}{\pi a^2}$
- $B_c =$ field at centerpost (fixed maximum from stress)
- $f_{bs} = c_{bs} \beta_p / \sqrt{A} = \frac{c_{bs}}{20} \sqrt{A} \ q_{cyl} \ \beta_N$
- Express $\beta_N (A)$ as $\beta_{NO} A^{-\alpha}$ and $\kappa$ as $\kappa_O A^{-\phi}$
- Optimize the function $\frac{A^{-2\alpha} \left(1 - \frac{1}{A}\right)^2 A^{-\phi}}{\left(1 - \frac{c_{bs}}{20} q_{cyl} \beta_N A^{1/2 - \alpha}\right)}$
  for $\alpha = 1/2; \ \phi = 1/2 \ \ A_{max} = 2.3$
KEY RESULTS OF TRANSPORT DEPENDENCE ON SHAPE (A, $\kappa$)

- Transport dependence on elongation and safety factor are weaker than IPB98(y,2) relation but close to EGB relation.

- Transport dependence on aspect ratio is more apparent when the operational constraints ($\beta_N$, $\kappa$, $f_{BS}$, $f_{GR}$) are directly incorporated into the confinement scaling relation.

- For “optimum tokamak”, fusion gain is optimized between aspect ratio of 2.2 and 3.0 (depending upon which confinement scaling relation is used).

- If stability limit and elongation are assumed independent of aspect ratio, then fusion gain optimizes at higher R/a.