

# Simulations of Alfvén eigenmodes in an ITER-like plasma

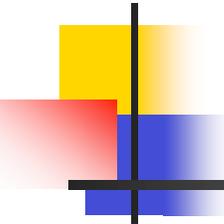
---

Y. Todo (NIFS, Japan)

6th ITPA MHD Topical Group Meeting and  
Workshop (W60) on “Burning Plasma Physics and Simulation”

July 4-6, 2005

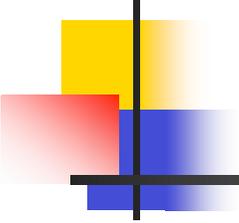
Tarragona, Spain



## Introduction

---

- For more realistic simulation of Alfvén eigenmodes, we implemented the drift model and an extended Ohm's law in the simulation code MEGA [MHD and energetic alpha particles, Y. Todo et al., Phys. Plasmas 12, 012503 (2005)].
- An ITER plasma with weakly reversed magnetic shear was investigated with the extended MEGA code.



# Outline

---

- Simulation model (Drift model)
- An Ohm's law extended with the electron Landau fluid model
- Initial condition
- Results I: ITER plasma with  $\beta_{\alpha 0}=2\%$  and weakly reversed shear
- Results II: ITER plasma with  $\beta_{\alpha 0}=1\%$  and weakly reversed shear
- Summary

# Drift model<sup>1)</sup> + current coupling model of energetic particles

$$\frac{dn}{dt} + n \nabla \cdot \mathbf{v} = 0$$

$$\frac{d}{dt} \Big|_{MHD} P + \frac{5}{3} P \nabla \cdot \mathbf{v}_{MHD} = 0$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} + \frac{1}{en} \left( \frac{1}{2} \nabla P - \mathbf{j} \times \mathbf{B} \right) = \eta \left[ \mathbf{j} - \frac{3n}{4B} \mathbf{b} \times \nabla \frac{P}{n} \right]$$

$$m_i n \left[ \frac{d\mathbf{v}_E}{dt} + \frac{d}{dt} \Big|_{MHD} (\mathbf{b} v_{||}) \right] + \nabla P = (\mathbf{j} - \mathbf{j}'_h) \times \mathbf{B}$$

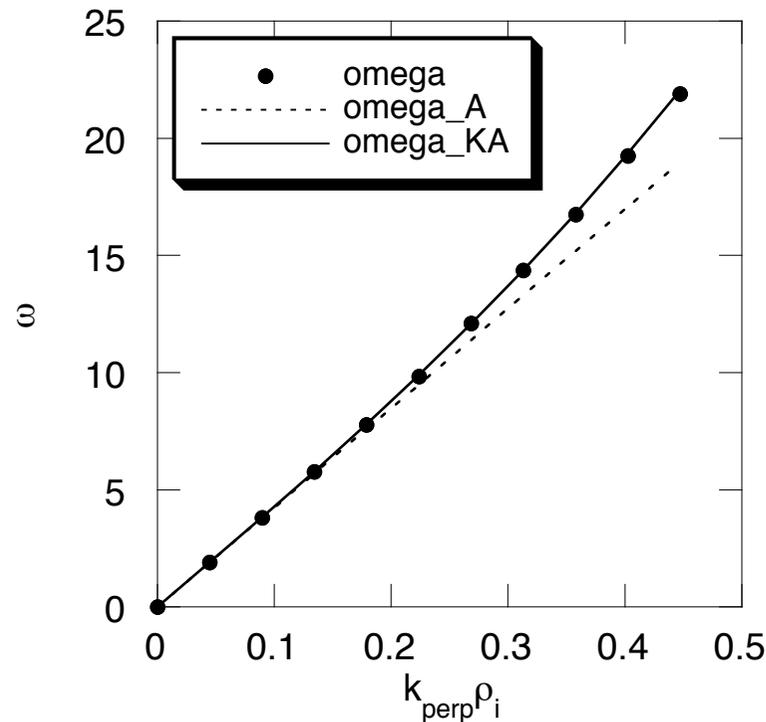
$$\frac{d}{dt} \Big|_{MHD} \equiv \frac{\partial}{\partial t} + (\mathbf{v}_E + v_{||} \mathbf{b}) \cdot \nabla$$

$$\mathbf{v}_{pi} = \frac{1}{2enB} \mathbf{b} \times \nabla P$$

$$\mathbf{E}_{\perp} + \mathbf{v}_E \times \mathbf{B} = 0$$

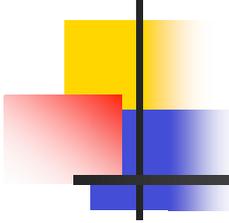
1) R. D. Hazeltine and J. D. Meiss, "Plasma Confinement" (Addison-Wesley Publishing Company, 1992).

## Dispersion relation of the kinetic Alfvén wave: comparison of the simulation results with theory



$$\omega^2 = k_{\parallel}^2 v_A^2 \left[ 1 + k_{\perp}^2 \rho_i^2 \left( \frac{3}{4} + \frac{T_e}{T_i} \right) \right]$$
$$k_{\perp} / k_{\parallel} = 1/3$$

The dispersion relation of the kinetic Alfvén wave is well reproduced with the drift model simulation.



# An Ohm's law extended with the electron Landau fluid model

---

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} + \frac{1}{en} \left( \frac{1}{2} \nabla P - \mathbf{j} \times \mathbf{B} \right) = \eta_{LF} \mathbf{j}_{\parallel}$$

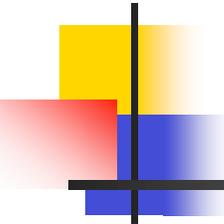
$$\eta_{LF} = \frac{m_e}{n_e e^2} \sqrt{\frac{\pi}{2}} v_{te} |k_{\parallel}|$$

$$|k_{\parallel}| \approx \frac{1}{2qR_0}$$

The effective resistivity is given by the Landau fluid model [G. W. Hammet et al. Phys. Fluids B 4, 2052 (1992).]

The parallel wave number is a reasonable approximation for Alfvén eigenmodes.

The damping rate of the n=4 TAE in the TFTR D-T plasma investigated with this code is  $5 \times 10^{-3} \omega$ , which is a half of the NOVA-K results [G. Y. Fu et al., Phys. Plasmas 5, 4284 (1998)]. This suggests that a more careful modeling is needed for a quantitative prediction.



# Guiding center approximation for energetic particles

---

$$\mathbf{u} = \mathbf{v}_{\parallel}^* + \mathbf{v}_E + \mathbf{v}_B$$

$$\mathbf{v}_{\parallel}^* = \frac{v_{\parallel}}{B^*} [\mathbf{B} + \rho_{\parallel} B \nabla \times \mathbf{b}]$$

$$\mathbf{v}_E = \frac{1}{B^*} [\mathbf{E} \times \mathbf{b}]$$

$$\mathbf{v}_B = \frac{1}{q_h B^*} [-\mu \nabla B \times \mathbf{b}]$$

$$\rho_{\parallel} = \frac{m_h v_{\parallel}}{q_h B}$$

$$\mathbf{b} = \mathbf{B} / B$$

$$B^* = B(1 + \rho_{\parallel} \mathbf{b} \cdot \nabla \times \mathbf{b})$$

$$m_h v_{\parallel} \frac{dv_{\parallel}}{dt} = \mathbf{v}_{\parallel}^* \cdot [q_h \mathbf{E} - \mu \nabla B]$$

# An extended Grad-Shafranov Equation

[E. V. Belova et al. Phys. Plasmas 10, 3240 (2003)]

Fast ion current density without ExB drift:

$$\mathbf{j}'_h = \int q_h (\mathbf{v}'_{\parallel} + \mathbf{v}_B) f_0(P_\varphi, \varepsilon, \mu) d^3v - \nabla \times \int \mu f_0(P_\varphi, \varepsilon, \mu) \mathbf{b} d^3v$$

parallel + curvature drift + grad-B drift      magnetization current

Find the stream function the fast ion current:

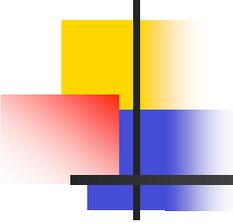
$$j'_{h,R} = -\frac{1}{R} \frac{\partial K}{\partial z} \quad j'_{h,z} = \frac{1}{R} \frac{\partial K}{\partial R}$$

An extended Grad-Shafranov equation:

$$R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial z^2} = -\mu_0 R^2 \frac{d}{d\psi} p - \mu_0 R j'_{h,\varphi} - \frac{I}{(2\pi)^2} \frac{dM}{d\psi}$$

$$I \equiv 2\pi R B_\varphi$$

$$M \equiv I - 2\pi K$$



# Initial distribution of alpha particles

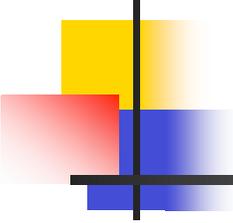
---

$$f(P_\varphi, v) = h(P_\varphi)g(v)$$

$$h(P_\varphi) = \sum_{n=0} a_n \left( \frac{P_{\varphi, \max} - P_\varphi}{P_{\varphi, \max} - P_{\varphi, \min}} \right)^n$$

$$g(v) = \frac{1}{v^3 + v_c^3} \quad (v \leq v_\alpha)$$
$$= 0 \quad (v > v_\alpha)$$

**Here,  $a_n$  is chosen using the least square method so that the alpha particle pressure is close to the one specified.**



## The $\delta f$ method

---

The energetic ion pressures are calculated using the particle weight:

$$P_{h\parallel}(\mathbf{x}) = P_{h\parallel 0}(\mathbf{x}) + \sum_i^N m_h v_{\parallel i}^2 w_i S(\mathbf{x} - \mathbf{x}_i) ,$$

$$P_{h\perp}(\mathbf{x}) = P_{h\perp 0}(\mathbf{x}) \frac{B(\mathbf{x})}{B_0(\mathbf{x})} + B(\mathbf{x}) \sum_i^N \mu_i w_i S(\mathbf{x} - \mathbf{x}_i) .$$

The evolution of the particle weight is given by,

$$\frac{d}{dt} w_i = -\alpha V_i \frac{d}{dt} f_0(\varepsilon, \mu, P_\varphi) = -\alpha V_i \left[ \frac{d\varepsilon}{dt} \frac{\partial f_0}{\partial \varepsilon} + \frac{dP_\varphi}{dt} \frac{\partial f_0}{\partial P_\varphi} \right]$$

[ $\alpha$  : normalization factor,

$V_i$  : phase space volume which the  $i$ -th particle occupies]

# An ITER plasma with weakly reversed shear (based on ITER technical basis)

$$\beta_\alpha = \beta_{\alpha 0} \exp[-(r/0.45a)^2]$$

$$q(r/a = 0) = 3.5$$

$$q(r/a = 0.7) = 2.2 = q_{min}$$

$$q(r/a = 1.0) = 5.3$$

$$R = 6.35\text{m}, \alpha = 1.85\text{m}$$

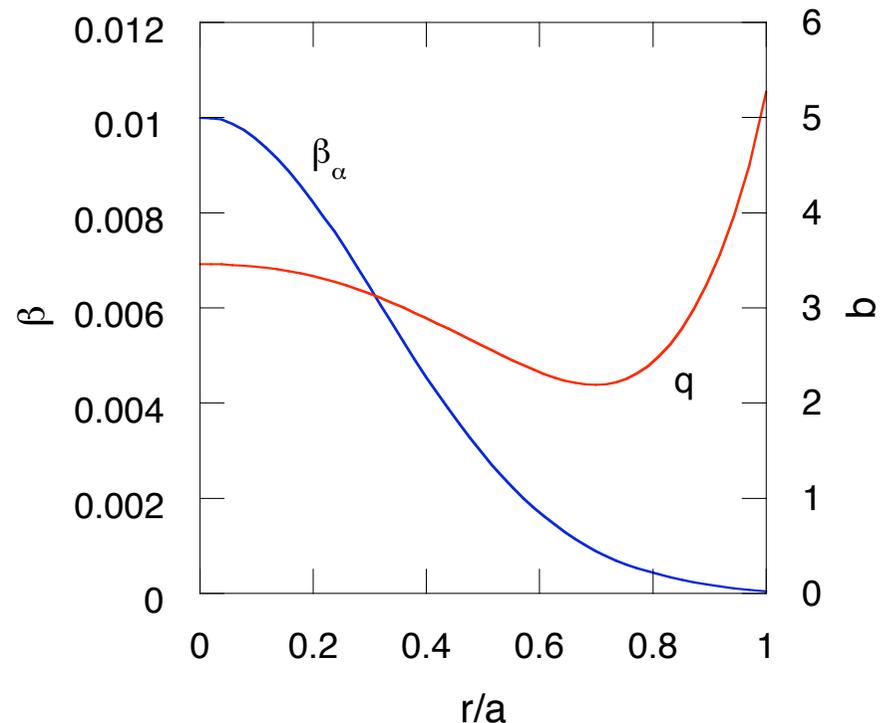
$$\kappa = 1.85, \delta = 0.4$$

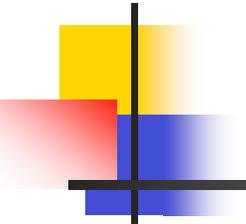
$$B = 5.18\text{T}, n_e = 6.7 \times 10^{19} \text{m}^{-3}$$

$$\beta_{thermal} = 2.5\%$$

$$T_e = 12.3\text{keV}$$

$$v_\alpha = 1.48v_A \text{ (corresponds to 3.5MeV)}$$





Two cases were investigated.

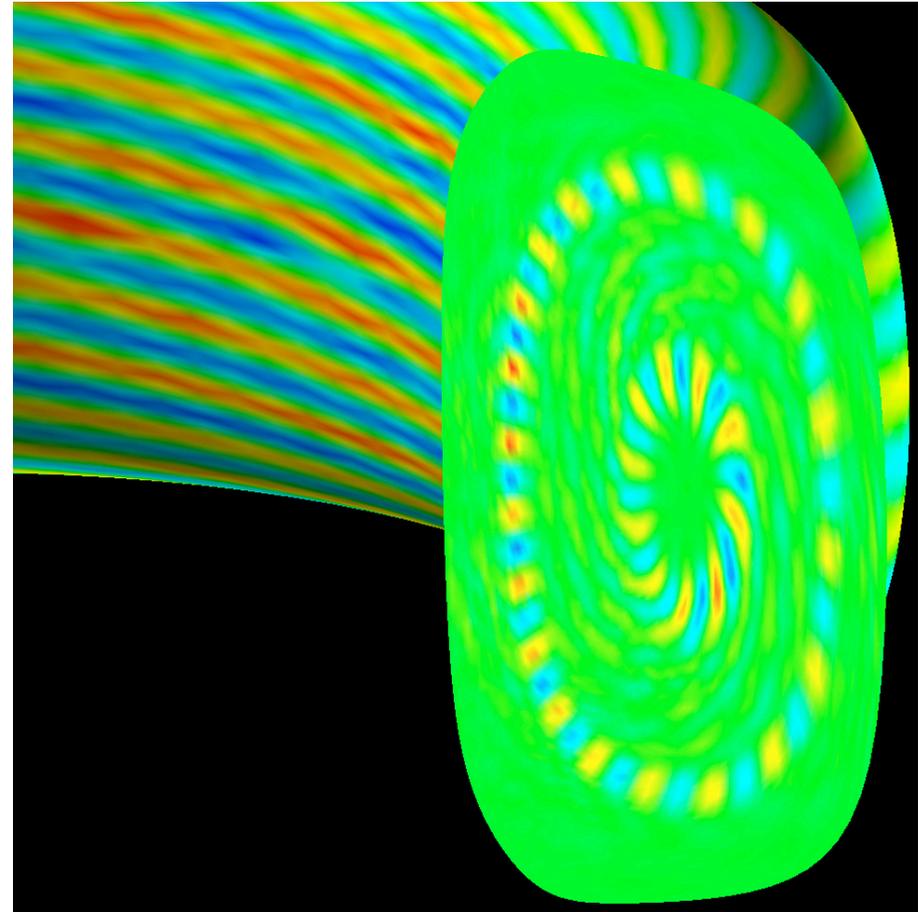
---

- $\beta_{\alpha 0}=2\%$

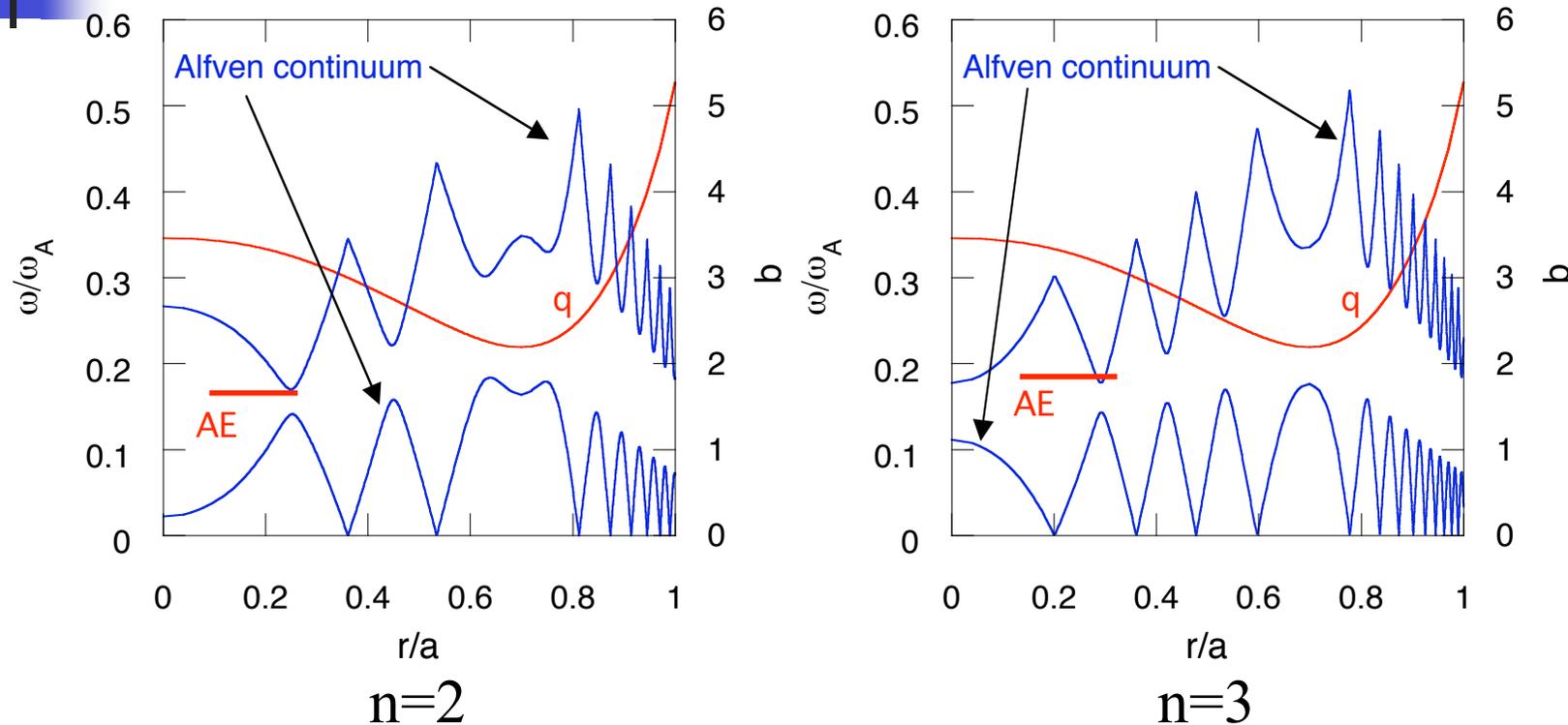
- $\beta_{\alpha 0}=1\%$

# Two types of unstable modes are observed ( $\beta_{\alpha 0}=2\%$ )

1. Low  $n$  ( $n=2,3$ ) Alfvén eigenmodes at  $r/a \sim 0.3$ . The frequency is  $\omega_{n=3} = 0.19 \omega_A$  and  $\omega_{n=2} = 0.17 \omega_A$ .
2. A middle  $n$  ( $m/n=20/9$ ) and low frequency mode (pressure driven MHD mode) at  $q=q_{\min}=2.2$ ,  $r/a \sim 0.7$ . The frequency is  $\omega = 6 \times 10^{-3} \omega_A$ .

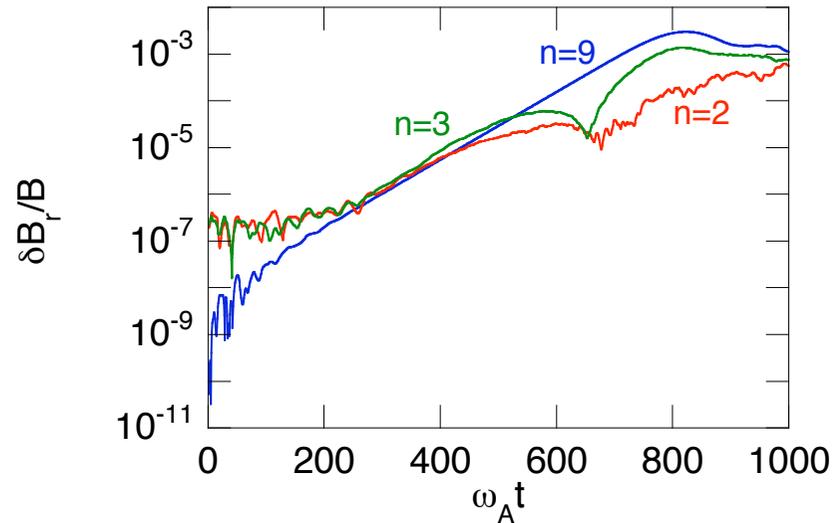


# Alfven continua and Alfven eigenmodes

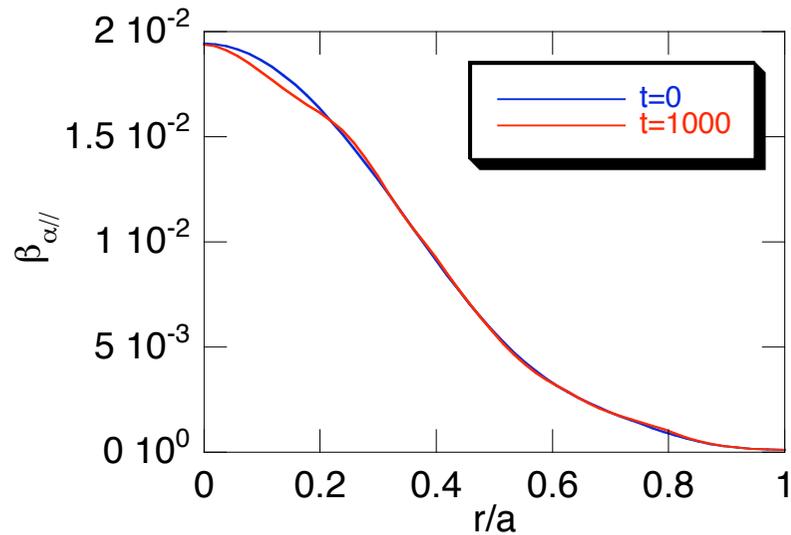


1. The frequency of each eigenmode is close to the upper continuum.
2. The eigenmodes are spatially localized in one side of the respective gaps.

# Saturation levels and alpha particle transport ( $\beta_{\alpha 0}=2\%$ )

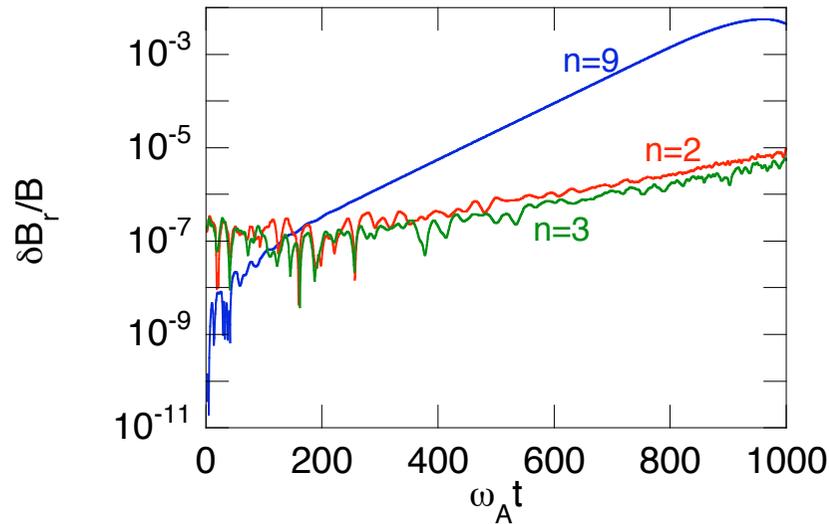


The saturation levels are  $\delta B_r/B \sim 10^{-3}$ .



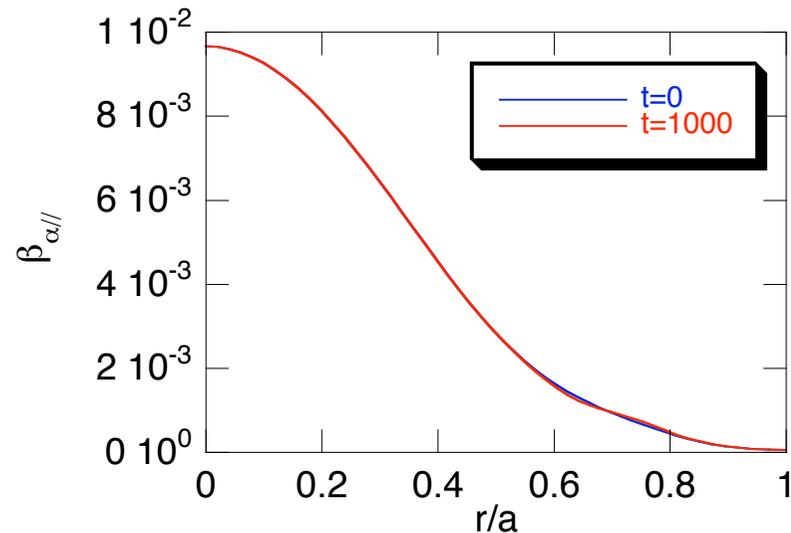
The reduction in alpha particle beta value is  $|\delta\beta_{\alpha}| \sim 6 \times 10^{-4}$ .

# Saturation levels and alpha particle transport ( $\beta_{\alpha 0}=1\%$ )



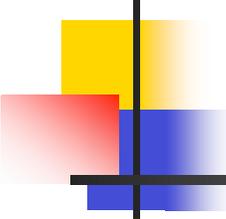
The saturation level of the  $n=9$  mode is

$$\delta B_r/B \sim 5 \times 10^{-3}.$$



The reduction in alpha particle beta value is

$$|\delta \beta_{\alpha}| \sim 10^{-4}.$$



## Summary

---

- The simulation code for MHD and energetic alpha particles (MEGA) has been extended with the drift model. The electron Landau fluid model is also employed to extend the Ohm's law.
- An ITER plasma with weakly reversed magnetic shear was investigated.
- Alfvén eigenmodes with  $n=2$  and  $3$  are unstable near the plasma center. An MHD instability with  $n=9$  takes place at  $q=q_{\min}=2.2$ .
- The reduction in alpha particle beta value is  $|\delta\beta_{\alpha}|\sim 6\times 10^{-4}$  for  $\beta_{\alpha 0}=2\%$  and  $|\delta\beta_{\alpha}|\sim 10^{-4}$  for  $\beta_{\alpha 0}=1\%$ .