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Theory of Energetic Particles in Plasmas

Prof. Herbert L. Berk
University of Texas at Austin
Institute for Fusion Studies

Dedicated to Carl Oberman and John Dawson
My personal mentors during my graduate years

Concept of a Burning Plasma

– 1. Fusion reaction needs energetic boost to overcome
– Coulomb barrier to produce fusion

$$D + T \Rightarrow He^4 + n + 17.5 \text{ Mev}$$

- **10- 25 keV plasma temperature needed!**

2. Energetic 3.5 Mev alpha particles (He^4) trapped in magnetic field and electrons absorb energy to heat plasma

3. If heating rate matches overall energy confinement time τ_E we satisfy Lawson criterion

$$P\,\tau_E \sim 10 \text{ atm sec}$$

4. Then we need to throw in fuel and extract ash; energy dynamics takes care of itself (with intelligent guidance)

Basic Plasma Physics Issue

Traditionally fusion research concentrates on increasing confinement without self-burning effects; occasionally with weak self- burn effects.

Ultimate object: obtain sustained self-burning plasma

BUT

Will burning plasma's properties be understandable?

a. Partial pressure of energetic component significant:
$P_\alpha / P_{bkg} \sim 0.3 (T_{kev} /20)^{5/2}$
MHD stability involves critical pressure limits;
But energetic particles may not behave in MHD manner

b. High alpha particle energy
~ 100 X background plasma temperature, with speed
greater than shear Alfven speed
--> Leads to the excitation of:
Universal Instability Drive
when diamagnetic drift frequency, proportional to species energy,
exceeds mode frequency.
Causes radial diffusion (underlying mechanism for
anomalous loss in standard plasma loss)

Result: Alpha particles can tap universal instability at much higher frequencies than a standard plasma and, in particular, self-excite shear Alfven-like perturbations

MHD Stability

After Uncontrolled Fusion was achieved, Controlled Fusion was thought to be a snap (within 20 years)!

Major Problem: Most obvious magnetic configurations found to be unstable: (e.g., z-pinch, simple mirrors)

Need for a method of global stability analysis: Led to:

MHD Energy Principle for perturbed energy δW_{mhd}

(Bernstein, Frieman, Kruskal, Kulsrud):

Ideal MHD fluid equations ⇒ quadratic form

$$\delta W_{MHD} = \frac{1}{2}\int d^3x \{ |\delta\vec{B}|^2 + \vec{J} \bullet (\vec{\xi} \times \delta\vec{B})$$

$$+(\vec{\xi} \bullet \nabla P)(\nabla \bullet \vec{\xi}) + \gamma P |\nabla \bullet \vec{\xi}|^2 \}$$

Linear stability if $\delta W_{MHD} > 0$ for all possible perturbations

Workhorse of the program for over 45 years

Fluid Equations for "Collisionless" Plasma?

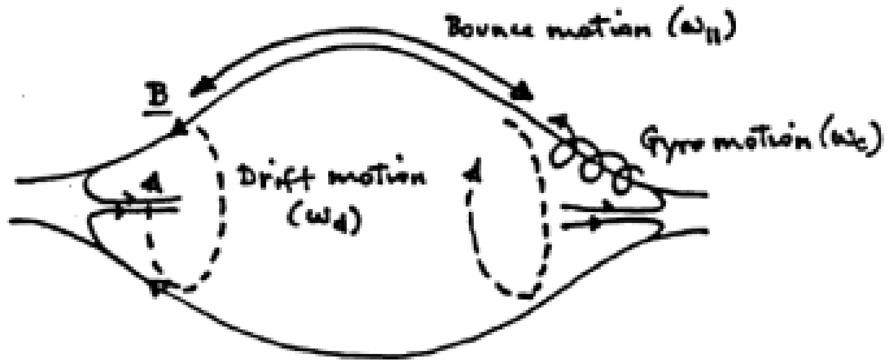
Quickly MHD theory was tuned to better reflect implications of low collisionality [pressure anisotropy; long mean free path];with improved δW theories; e.g. Chew, Goldberger, Low and Kruskal-Oberman δW theories
(Kruskal-Oberman truly kinetic rather than fluid theory)

However, stability predictions not essentially different from ideal fluid MHD theory, due to omission of crucial effects.

Mode frequency ordered larger than:

(a) diamagnetic drift frequency
leads to source of universal instability drive, since in the rest frame of one specie, other species have mean speed
$\Rightarrow$ kinetic streaming instability
(b) curvature and grad-B drifts
for energetic particles these frequencies often large and lead to surprising modifications of MHD behavior

Early Clue: Mirror Machine Hot Electron Experiments



Energetic electron plasmas frequently avoided curvature driven MHD instability
[e.g. Table Top (Perkins and Post), Elmo (Dandl),
Berkeley Hot Electron Exp (Lieberman and Lichtenberg)]

First theoretical explanation (Krall):

(a) Cold plasma reduces predicted MHD growth rate below grad-B drift instability and then hot electron drift "shorts" the charge separation of MHD – interchange instability
(b) Energetic component does not move with bulk plasma displacement, creating a stabilizing magnetic well

Effect of Passive Internal Current

Interchange Instability Drive is profoundly changed by
passive ring current that produces diamagnetic stabilizing magnetic well
Interchange drive apparent in alternate form of δW_{MHD}
(Furth, Killeen, Rosenbluth & Coppi; Greene and Johnson

$$\delta W_{MHD} = \frac{1}{2}\int d^3x \left\{ \left|\delta \bar{B}_\perp\right|^2 + \left|\delta \bar{B}_\parallel - \bar{B}\left(\bar{\xi}\cdot\nabla P\right)/B^2\right|^2 \right.$$

shear Alfvén *fast magnetosonic*

$$\left. + \gamma P\left|\nabla\cdot\bar{\xi}\right|^2 + J_\parallel(\bar{B}/B)\cdot\left(\bar{\xi}^*\times\delta\bar{B}_\perp\right) - 2\left(\bar{\xi}\cdot\nabla P\right)\left(\bar{\xi}^*\cdot\bar{\kappa}\right) \right\}$$

acoustic *kink* *interchange*

Effect of Passive Internal Current

$$\mathbf{J}_{\perp h} = \mathbf{B}\times\nabla\,\mathbf{P}_{\perp h}/|\mathbf{B}|^2$$

Primarily leads to following replacement in δW_{MHD}

$$\boldsymbol{\kappa} \Rightarrow \boldsymbol{\kappa} - \nabla\,\mathbf{P}_{\perp h}/|\mathbf{B}|^2$$

Decreasing energetic pressure profile off-sets destabilizing magnetic curvature
(Basis of many alternate and improved tokamak concepts)
BUT: hot particles must be passive and not respond to
displacement of background plasma

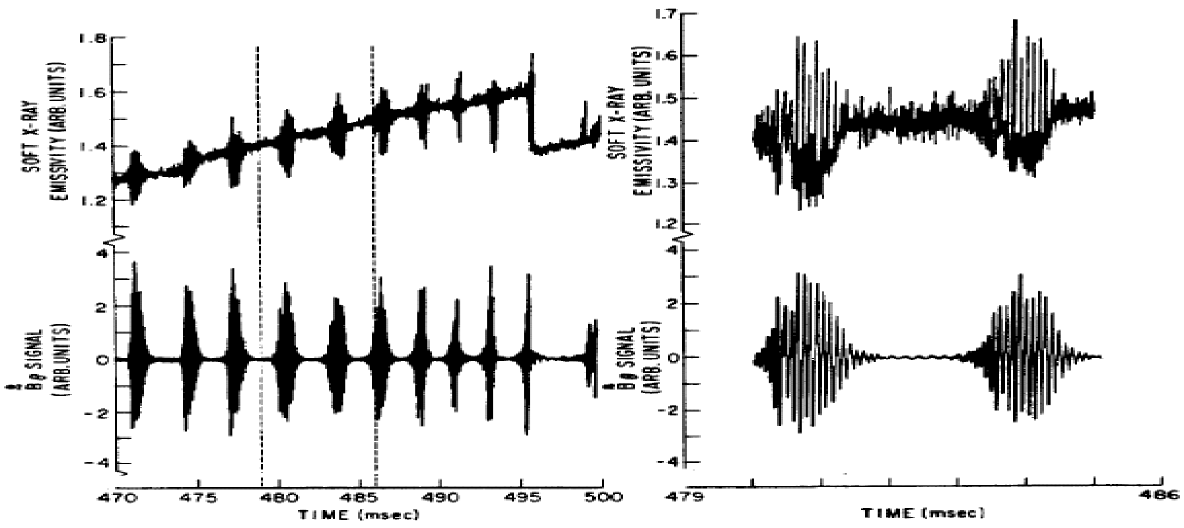
Can Energetic Particle Improve MHD Stability for Tokamak?

Hot passing particles respond to perturbations similar to background plasma

Trapped energetic particles retain stabilizing influence similar to mirror machine with caveats

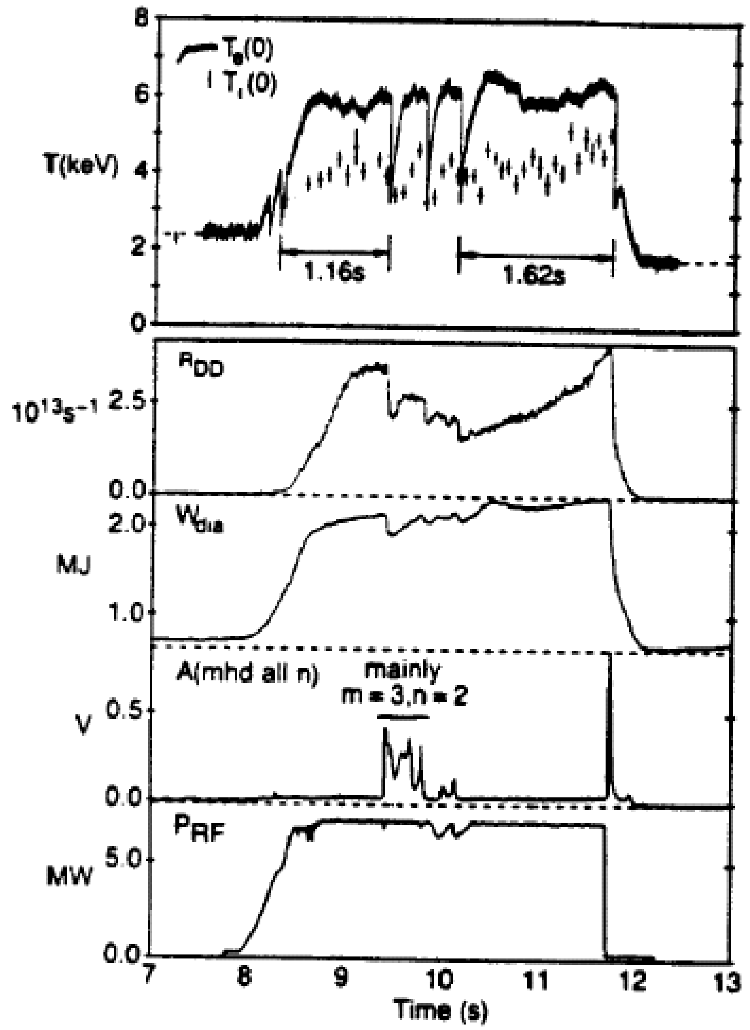
(a) grad-B drift (in vacuum field) is larger than predicted MHD growth rate
(b) drift reversal due to energetic particle diamagnetic well is avoided (Van Dam, Lee, Rosenbluth);
(c) can be basis for improved MHD operation in a rf heated or burning plasma

First Experimental Effect of Energetic Particles Fishbones on PDX (McGuire et al.)



First Theoretical Explanation (Chen, White, Rosenbluth)
Internal Kink Mode: oscillates at frequency comparable to hot particle grad- B drift frequency
Marginal Stability: a balance between dissipation from Alfven continuum (to be discussed further below) and particle resonance tapping Universal Instability Drive
A Non-perturbative instability: wave does does exist in the absence of hot particles

Sawtooth Suppression and Monster Relaxation JET



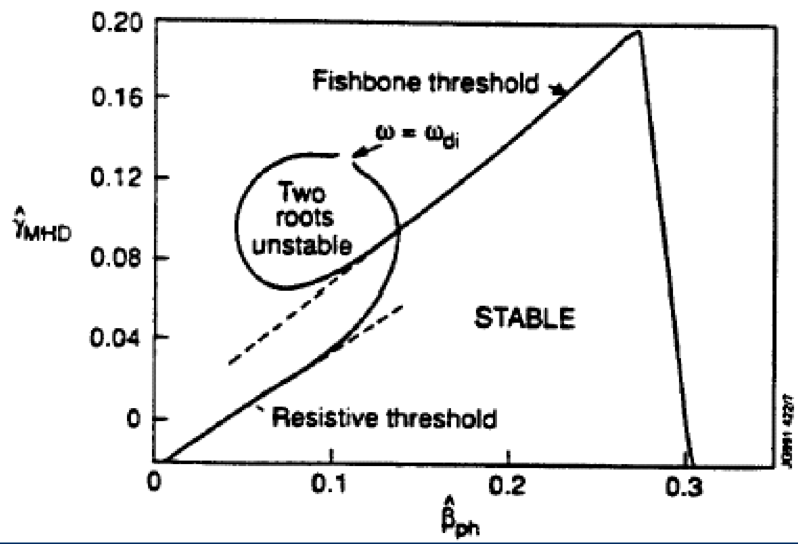
Sawtooth Activity Suppressed But Relaxation More severe

Theory for Stabilization

Coppi, Porcelli, et al.; White, Bussac, Romanelli, others)

Complete theory involves numerous effects:

- (a) resistive tearing
- (b) ideal MHD
- (c) hot particle effects (self-well, resonant universal drive)
- (d) ion diamagnetic drift (background plasma)
- • picture idealized to circular large aspect ratio tokamak (comprehensive correct quantitative study still needed)



Challenge for Burning Plasmas

Enhanced stabilization will increase the beta limits of both sawteeth oscillation, as well as MHD beta limits of other modes

When new stability limits are reached a “harder” relaxation likely arises (e.g. the monster sawtooth relaxation)

With suitable control, can we exploit the “good” (higher stored energy) without the “bad” (harder relaxation)?

Detailed theoretical and experimental work is required to implement the best scenario. Challenge for new computational methods (e.g. M-3D code (Park and Fu))

Tapping Universal Instability Drive

- **Hot particle diamagnetic drift frequency** higher than that of relatively high frequency waves (e. g. shear Alfven wave)

2. **Shear-Alfven Waves** are just right speed to resonate with fusion produced alpha particles ~ 10^9 cm/s

3. **Perturbative Instability:** wave exists with low damping rate when hot particles absent

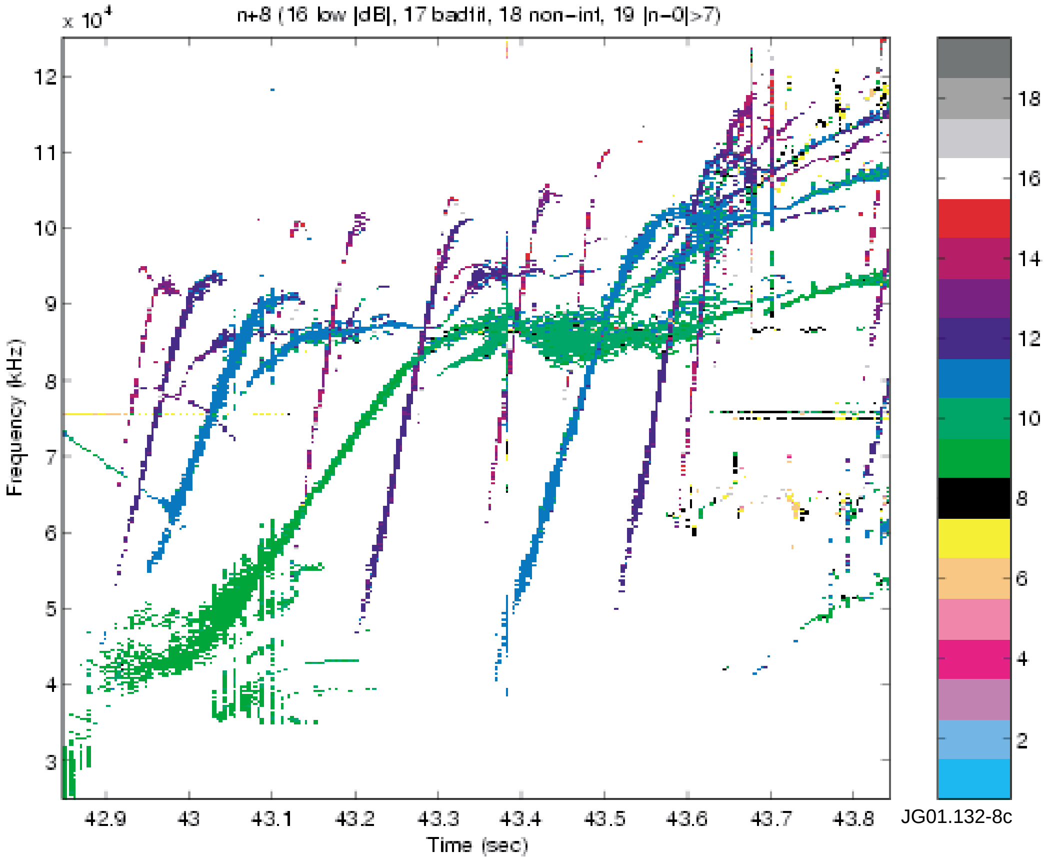
a. Ordinary Alfven wave, $\omega = k_{||} v_A(r)$; considered to be too singular (in ideal theory excited only on a single field line) and leads to damping in practice (continuum damping)

b. Need to resolve continuum:
First rigorous analysis (Rutherford and Rosenbluth)
Kinetic Alfven Wave in cylindrical approximation

c. Global Alfven Eigenmodecan be established if $[k_{||} v_A(r)]^2$ has minimum (Ross, Mahajan,Li)

d. Alfven Cascade: $[k_{||} v_A(r)]^2$ has maximum in magnetic shear-reversed profiles (JET group, Berk and Breizman)

Mode Number Identification **(JET Shot # 49382)**



Finding the Wave!

1. **Gap in the Alfven Continuum** (J. Tataronis and C. Phillips)
   in torus that does not exist in cylinder

2. **Discrete Modes Exist In Gap** (C.Z. Cheng, L.. Chen,
M.S. Chance); Toroidal Alfven Eigenmodes (TAE);
gap emerges for TAE from frequency degeneracy caused by

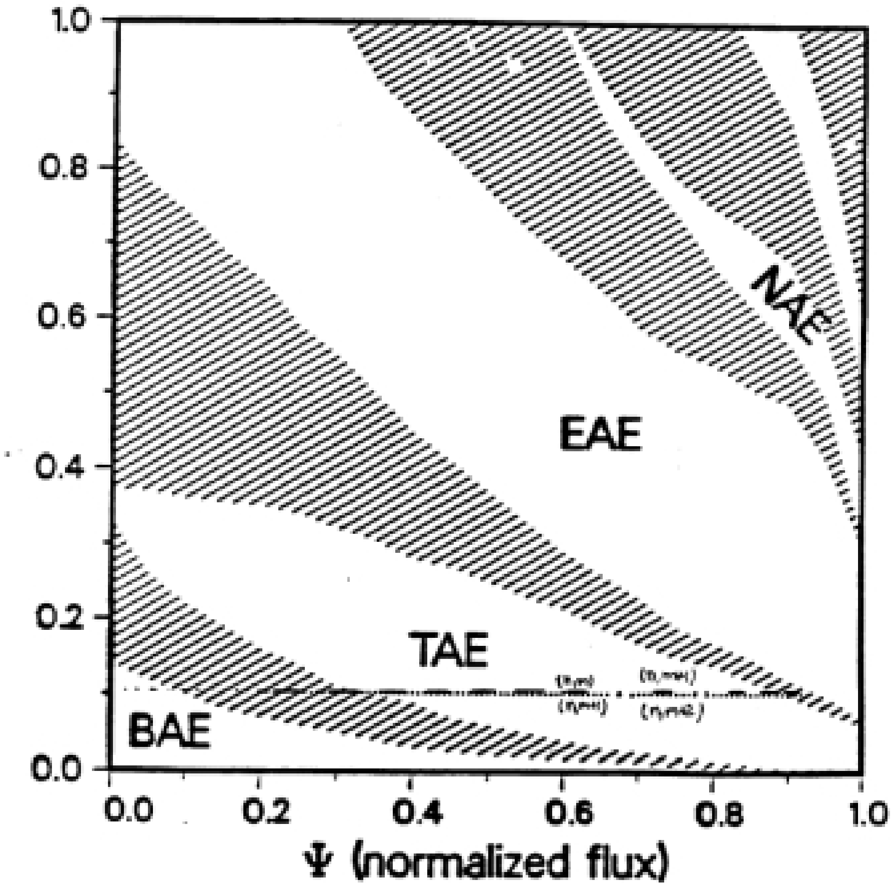
$$k_{\| m}(r) \equiv [n - m/q(r)] = -k_{\| m+1}(r) \quad ; \Rightarrow$$

$\omega_m(\text{tae}) = k_{\| m}(r_m)\text{v}_A = -k_{\| m+1}(r_m)\text{v}_A$, at $q(r_m) = (m+1)/n$;

At finite toroidicity, TAE frequency close to ;
Other degeneracy-breaking variants (EAE, etc.)

Realistic Gap Structure in D-III-D

(M. S. Chu, J. Greene, A. D. Turnbull)



Destabilization of TAE

- drive by alpha particle now a concern

(Initial Estimate:Fu and Van Dam and many others)
Principal quantitative approach was use of NOVA code originally developed by C.Z. Cheng
(with later support by G. Fu, N. Gorelenkov, G. Kramer)

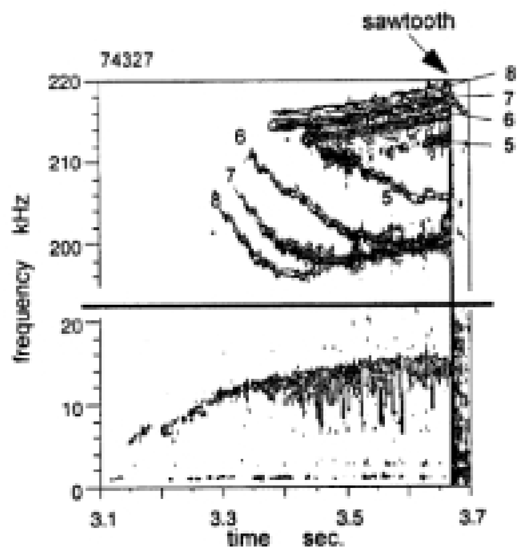
2. **Perturbative** insertion of universal hot particle instability drive needs to overcome stabilization mechanisms from background plasma.

Ion Landau Damping (tail resonance when $v_{||} = v_A / 3$, a side-band resonance), **electron collisionality,** (Gorelenkov and Sharapov)**, continuum damping radiation damping** (Mett and Mahajan; Rosenbluth)], **electron Landau damping**.

Dominant damping sources: Ion Landau damping, radiation damping, collisional damping

NON-PERTURBATIVE MODES

With enough hot particles, nonperturbative unstable modes
Emerge from the continuum (particularly susceptible in lowshear
Regime [Energetic Particle mode (EPM), (L. Chen and F. Fonca;
Gorelenkov and Cheng]
Good Experimental Candidate for this Mode has been seen on
TFTR (Bernarbei, et. al)

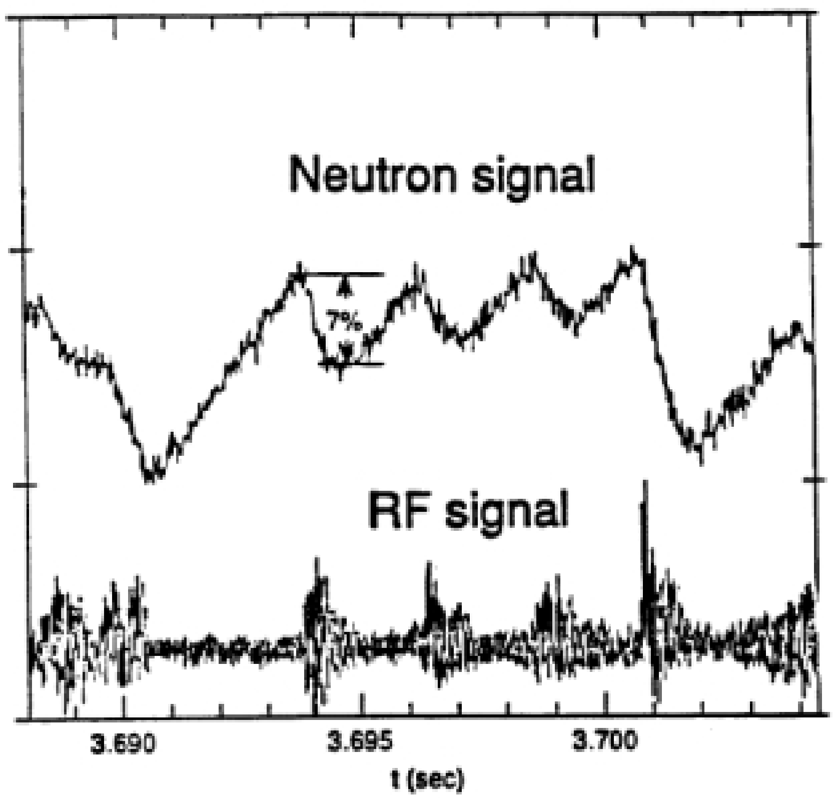


Standard Burning Plasma likely to be difficult to excite
In Advanced Tokamak, Reversed or Low Shear, Easier to Excite

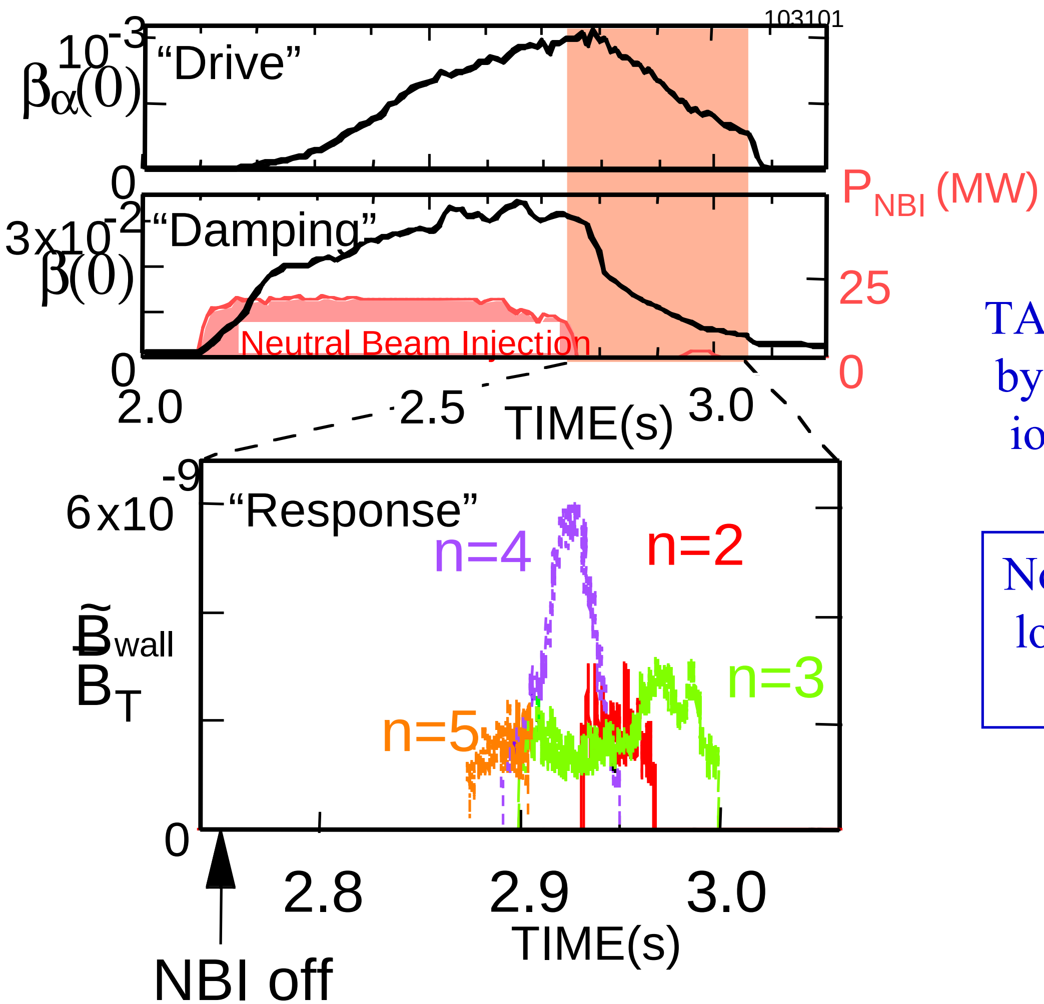
Effect of TAE's?

Benign or "Malignant"? Experimental evidence for both

Rapid Losses (K. L. Wong, et. al. TFTR; Heidbrink, et. al. DIII-D)



Alpha Driven TAEs Appear After Neutral Beam Pulse in Weak Shear Plasma (TFTR)



TAE primarily damped by radiative and beam ion Landau damping

No observed alpha loss due to alpha-Driven TAE

courtesy R. Nazikian

Nonlinear Theory for Perturbative Modes

(Berk and Breizman)

- Weak instabilities are driven by resonant particles, which nonlinearly mix around the resonant phase space islands

2. Bulk of plasma responds linearly while resonant particles satisfy nonlinear equation similar to that of a pendulum in two - dimensional phase space (constant magnetic moment and energy in rotating frame $E' = E - \omega P_{\phi}/n$ allows 6-d phase space to be studied)

3. Consistent theory for mode amplitude and frequency developed and studied both analytically and numerically

(a) Special form for near-threshold response

(b) Numerical techniques to study large deviations from threshold

(c) Analytic techniques far from linear threshold conditions (dynamical nonlinear BGK modes)

4. Scenario for global losses when resonances overlap;
much work remains to be done

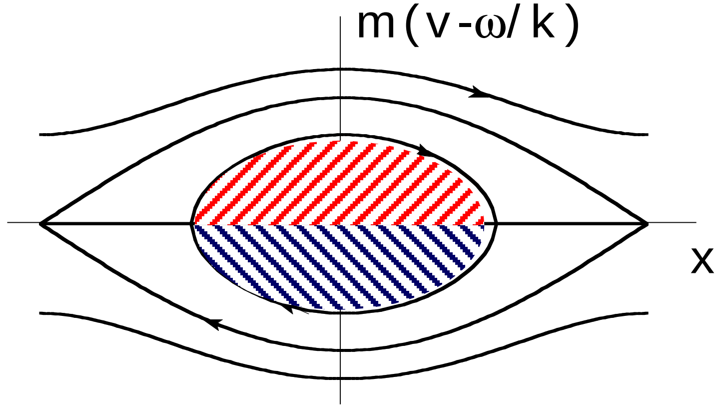
Key Element in Theory

Interaction of energetic particles with unstable waves

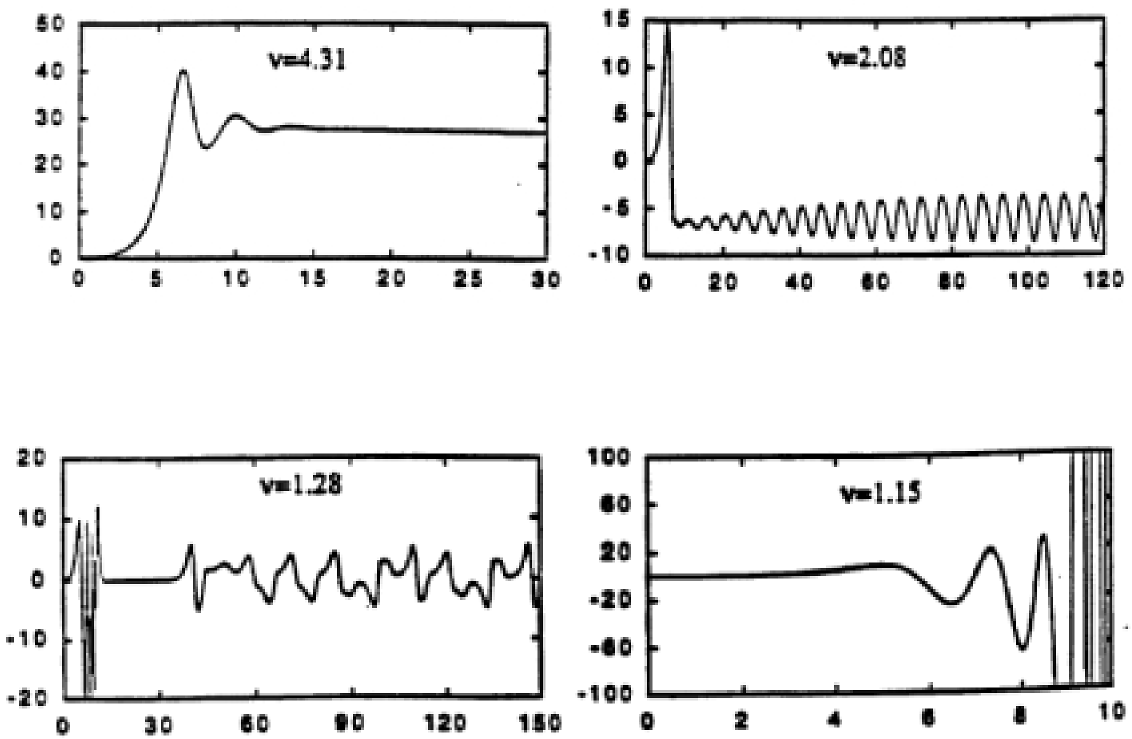
- Pendulum equation for particles in an electrostatic wave:

$$m\ddot{x} = eE\cos(kx - \omega t)$$

- Wave-particle resonance condition $\omega - kv = 0$
- Phase space portrait in the wave frame

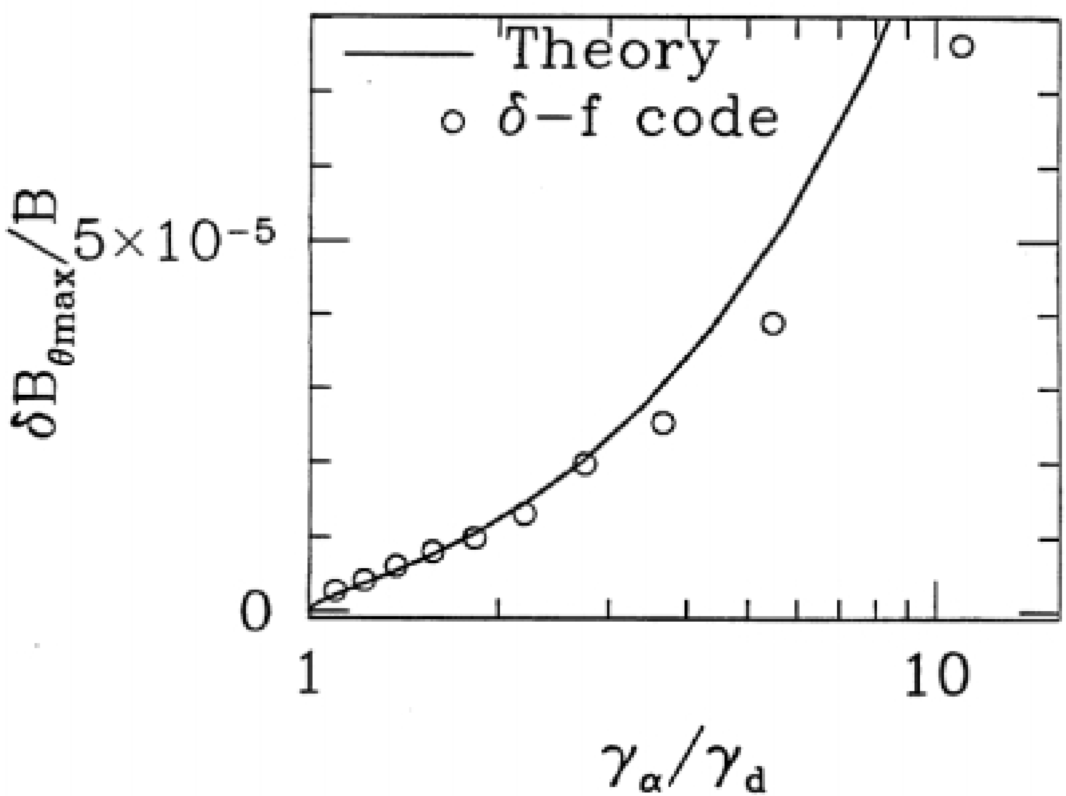


Solutions of Threshold Equations



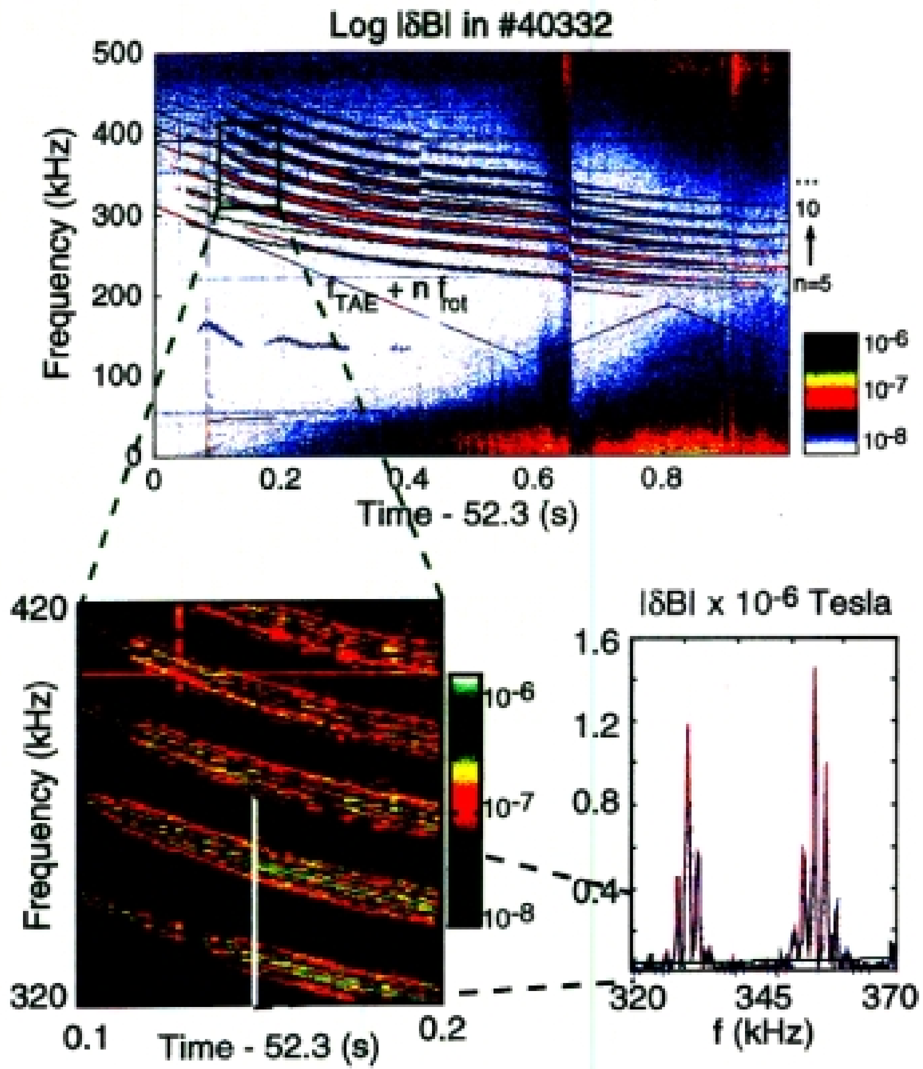
Comparison of Nonlinear Theory with Particle Simulation

Gorelenkov, Chen, White and Berk



Solid curve- Theory, Circles- simulation

TAEs driven by minority tail ions



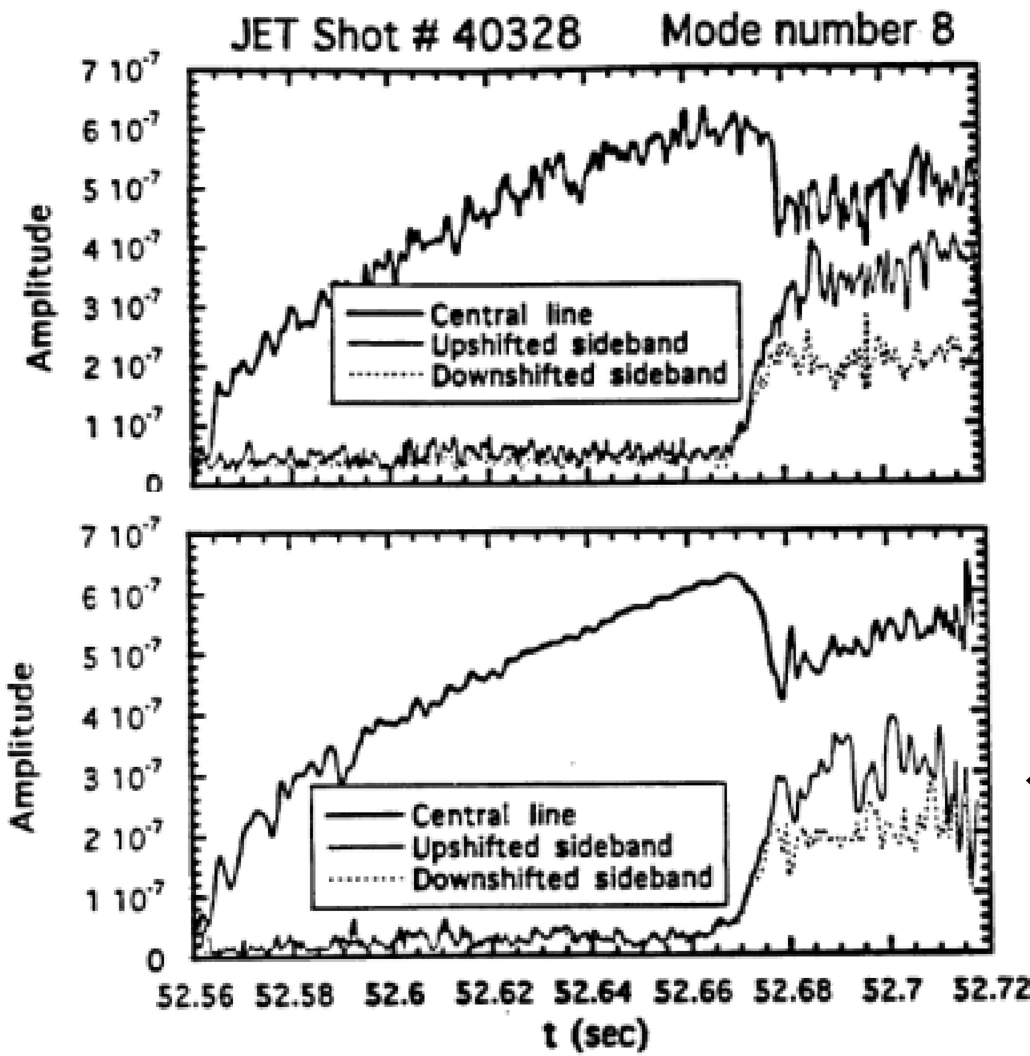
Symmetric splitting

Courtesy of
- Fasoli

and

R. Heeter

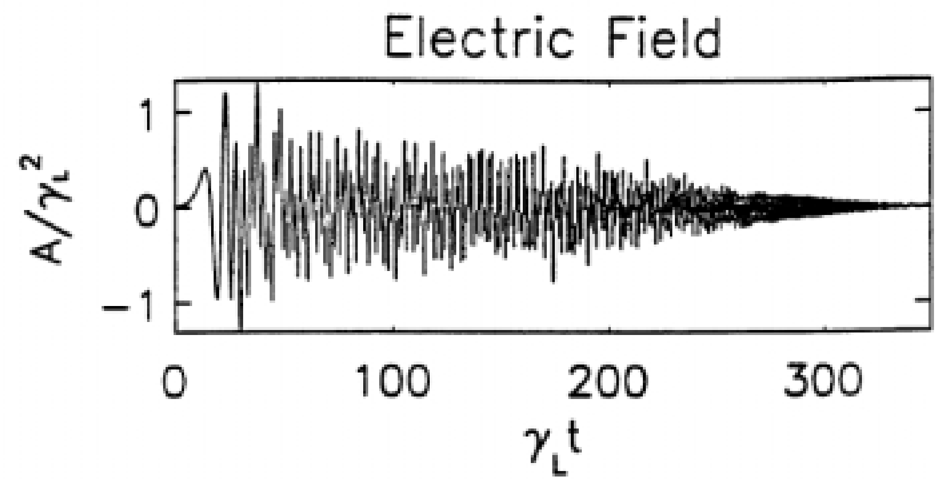
Comparison Jet Shot and Theory



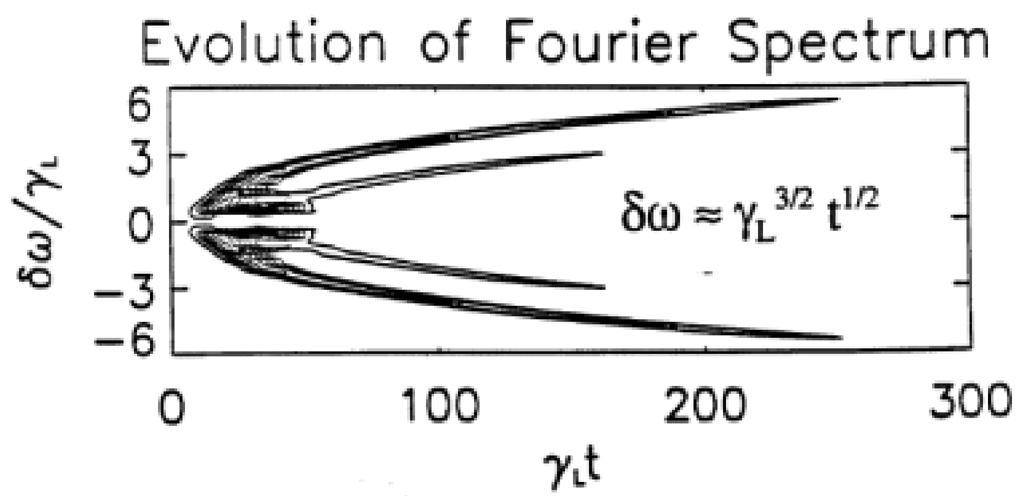
Analysis by (Fasoli, Breizman, Sharapov, Heeter

⇐Simulation result

Solution Beyond Explosion
simulated by N. Petviashvili

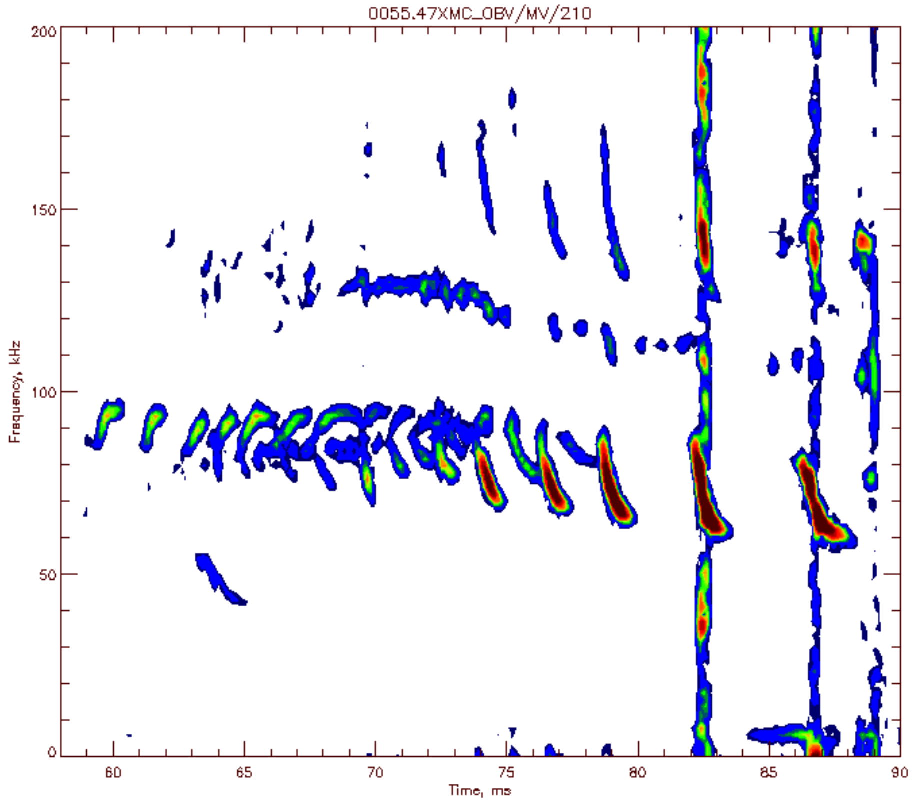


Mode Lasts many inverse damping times



Mode frequency changes in time

Upward and Downward Chirping of TAE mode in MAST



(Culham Laboratory, U. K. courtesy of Mikhail Gryaznevich)

Predictions of TAE Stability in Burning Plasmas

A completely satisfactory code that makes reliable stability predictions for burning plasmas not yet completed

Difficulties

Mode structure can extend globally throughout plasma,
but spatial fine structure (approaching ion Larmor radius) is important

MHD theory not satisfactorily account for continuum damping nor radiation damping of “kinetic Alfven waves

Alpha particle response requires integral equation description when perturbation theory breaks down

Near degeneracy of eigenvalues requires improvements in perturbation theory that has been used

PPPL Tae Stability Codes

Despite difficulties, the PPPL suite of codes has been aggressively used to assess stability of burning plasmas

Nova and Nova-K (Cheng, Fu) (Nova finds basic global mode structure and Nova-K accounts for perturbations due to drive and damping mechanisms).

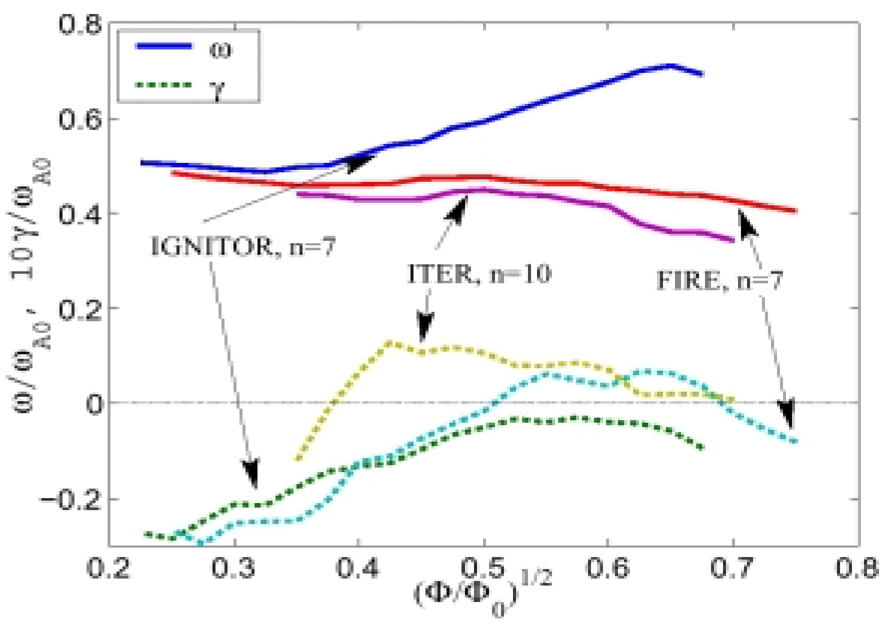
HINST code (Cheng, Gorelenkov) is a local ballooning mode code that includes much of above needed physics, but only at one radial surface.

TAE Stability Results

original ITER had significant instability at high T_{eo}
> 24 keV as predicted in Nova/Nova-K (Fu)

Nova-NovaK Results (Kramer):
ITER FEAT, FIRE and IGNITOR stable at their nominal temperatures (T_{ep} ~ 20, 12, 10 kev respectively);
stability often due to edge damping mechanisms affecting internal drives

HINST Results (Gorelenkov)
ITER FEAT and FIRE unstable: IGNITOR stable



Conclusions

Confinement physics in burning plasmas may be profoundly altered from today's plasmas, especially when operating at higher temperatures (≥ 20keV).

Strong likelihood of enhanced MHD stabilization, but then global crashes are possible from either being overly-MHD stabilized or due to losses arising from resonant Alfvenic instabilities that tap the alpha particle contribution to the Universal Instability Drive.

New fundamental insights into wave-particle interaction have been achieved, but extensive quantitative work still needs to be performed, especially to understand how global losses arise (can benign TAE oscillations arise in burning plasma? saturation level low for one resonance but many resonances can lead to overlap and "avalanches").

Profound challenges remain for theoretical and experimental understanding of how best to operate in a burning plasma regime, particularly in developing methods to maintain benefit of improved conditions (the problem of control exists in present-day experiments; we will need to learn to be more astute in solving the control problems in burning plasma experiments).

Innovation at PPPL

1. Use alpha particles in more efficient manner

a. electrons are a parasite, object is to really heat ions (Furth)

b. alpha channeling proposed to heat ions directly (Fisch-Rax)

c. NSTX may be indicating that channeling arises through energetic particle interaction with compressional Alfven wave and break-down of magnetic moment adiabaticity, which heats ions (Gates, White, and NSTX team

2. State-of-the-art numerical studies bringing new opportunities to plasma fusion research (Tang)
Strong interplay between energetic alpha particle behavior with MHD theory ripe for a new level of numerical investigation [e.g., with M3D code (Park and Fu)]